# Radioactivity



Assume a radioactive material $Q(t)$ decays at rate proportional to the amount present at any particular time $\Rightarrow$

$$\boxed{\frac{dQ}{dt} = -rQ}$$ , where $-r<0$ is the decay rate

This DE has the form of (A) "$1^{st}$ order linear DE with constant coefficient" $\Rightarrow$ so we can use (A*) to find the quantity $Q(t)$ available at time $t \Rightarrow$

$$\boxed{Q(t) = \frac{b}{a} + \left(Q_0 - \frac{b}{a}\right)e^{at}}$$ (A*) where

$Q_0 = Q(0)$ is the initial amount of this material

$a = -r$ and $b = 0$ so $\frac{b}{a} = 0$

$$\boxed{Q(t) = Q_0 e^{-rt}}$$ is the solution of this problem

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**Half-life time $\tau$:** is the time takes the material to decay by the half of its initial

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|---|---|---|
| $Q(\tau) = \frac{1}{2} Q_0$ | $-r\tau = \ln\frac{1}{2}$ | Note that |
| $Q_0 e^{-r\tau} = \frac{1}{2} Q_0$ | $-r\tau = -\ln 2$ | $-r<0 \Rightarrow r>0 \Rightarrow \tau > 0$ |
| $e^{-r\tau} = \frac{1}{2}$ | $\boxed{\tau = \frac{\ln 2}{r}}$ | |

17.4

Exp (Q14 - section 1.2)

Radium-226 has half-life of 1620 years.
Find the time period during which a given amount of this material is reduced by one-quarter.

$$\tau = \frac{\ln 2}{r} \Rightarrow r = \frac{\ln 2}{\tau} = \frac{\ln 2}{1620}$$

$Q(t) = Q_0 e^{-rt}$ is the amount available at time $t$

we need to find the time $t^*$ s.t

$Q(t^*) = \frac{3}{4} Q_0$ since $\frac{1}{4}$ is reduced

$$Q_0 e^{-rt^*} = \frac{3}{4} Q_0$$

$$e^{-rt^*} = \frac{3}{4}$$

$$-rt^* = \ln \frac{3}{4}$$

$$t^* = \frac{\ln \frac{3}{4}}{-r} = \frac{\ln(0.75)}{-\frac{\ln 2}{1620}} = -1620 \frac{\ln(0.75)}{\ln 2}$$

$$\approx 672.4 \text{ years}$$