Birzeit University
Mathematics Department
Math332
Homework (Nonhomogeneous BVPs)

Instructor: Dr. Ala Talahmeh **First Semester 2019/2020**

Exercise #1. Solve the problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + (4t-8)\sin 2x, \quad 0 < x < \pi, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 0, \quad u(\pi,t) = 0, \ t > 0, \\ u(x,0) = 0, \quad u_t(x,t) = 0, \ 0 < x < \pi. \end{cases}$$

Ans.

$$u(x,t) = \big(2\cos 2t - \frac{1}{2}\sin 2t + t - 2\big)\sin 2x.$$

Exercise #2. Solve the problem

$$\frac{\partial^2 u}{\partial x^2} + \sin x \cos t = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < \pi, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 0, \quad u(\pi,t) = 0, \ t > 0, \\ u(x,0) = 0, \quad u_t(x,t) = 0, \ 0 < x < \pi. \end{cases}$$

Exercise #3. Solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = u(1,t) = 3, \ t > 0, \\ u(x,0) = x + 4, \quad 0 < x < 1. \end{cases}$$

Exercise #4. Solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = \sin t, \quad u(1,t) = 0, \ t > 0, \\ u(x,0) = 0, \quad 0 < x < 1. \end{cases}$$

Exercise #5. Solve the problem

$$\frac{\partial^2 u}{\partial x^2} + 2t + 3tx = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = t^2, \quad u(1,t) = 1, \ t > 0, \\ u(x,0) = x^2, \quad 0 < x < 1. \end{cases}$$

Exercise #6. Solve the problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x\cosh t + (1-x)\sinh t, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = \sinh t, \quad u(1,t) = \cosh t, \ t > 0, \\ u(x,0) = x, \quad u_t(x,0) = 1 - x + \sin 2\pi x. \ 0 < x < 1. \end{cases}$$

Ans.

$$u(x,t) = \frac{1}{2\pi} \sin 2\pi t \ \sin 2\pi x + (1-x)\sinh t + x\cosh t.$$

Exercise #7. Solve the problem

$$\frac{\partial u}{\partial t} = 9\frac{\partial^2 u}{\partial x^2} + \sum_{n=1}^{\infty} \frac{\sin nx}{n^2}, \quad 0 < x < \pi, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 0, \quad u(\pi,t) = 0, \ t > 0, \\ u(x,0) = 6\sin 4x, \quad 0 < x < \pi. \end{cases}$$

Ans.

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{1 - e^{-9n^2 t}}{9n^4} \right) \sin nx + 6e^{-144t} \sin 4x.$$

Good Luck