Birzeit University
Mathematics Department
Math332
Homework
Application of Laplace transform

Instructor: Dr. Ala Talahmeh **First Semester 2019/2020**

Exercise #1. Use the **Laplace transform** to solve the problem:

$$9\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 3, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 0, \quad u(3,t) = 3 - 3t, \ t > 0, \\ u(x,0) = x, \quad u_t(x,0) = -x, \ 0 < x < 3. \end{cases}$$

Ans. $u(x,t) = (1-t)x$.

Exercise #2. Use the **Laplace transform** to solve the problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 0, \quad u(1,t) = 0, \ t > 0, \\ u(x,0) = 2\sin 3\pi x, \quad 0 < x < 1. \end{cases}$$

Ans. $u(x,t) = 2e^{-9\pi^2 t}\sin 3\pi x$.

Exercise #3. Use the **Laplace transform** to solve the problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 0, \quad u(1,t) = 0, \ t > 0, \\ u(x,0) = 0, \quad u_t(x,0) = \sin \pi x, \quad 0 < x < 1. \end{cases}$$

Ans. $u(x,t) = \frac{1}{\pi}\sin \pi x \sin \pi t$.

Exercise #4. Use the **Laplace transform** to solve the problem:

$$4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x > 0, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = \sin \pi t, \quad \lim_{x\to\infty} u(x,t) = 0, \ t > 0, \\ u(x,0) = 0, \quad u_t(x,0) = 0, \quad x > 0. \end{cases}$$

Ans.

$$u(x,t) = \sin \pi\Big(t - \frac{x}{2}\Big)\mathcal{U}\Big(t - \frac{x}{2}\Big).$$

Exercise #5. Use the **Laplace transform** to solve the problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x > 0, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = 1, \quad \lim_{x\to\infty} u(x,t) = 0, \ t > 0, \\ u(x,0) = e^{-x}, \quad u_t(x,0) = 0, \quad x > 0. \end{cases}$$

Ans.

$$u(x,t) = \Big(1 - \cosh(t-x)\Big)\mathcal{U}(t-x) + e^{-x}\cosh t.$$

Good Luck