Birzeit University
Mathematics Department
Math332
Third Exam

Instructor: Dr. Ala Talahmeh **First Semester 2019/2020**
Name:.. **Number:**...............
Time: 100 min **Date: 14/12/2019**

Problem#1 [8+12 points].

(a) Find the **Fourier transform** of

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(b) Solve the following problem using the **Fourier transform** leaving the solution in **integral form**

$$\begin{cases} u_t = 3u_{xx}, & -\infty < x < \infty, \quad t > 0, \\ u(x,0) = f(x), \end{cases}$$

where f is given in part (a).

[Cont....]

Problem#2 [25 points]. Use the **Laplace transform** to solve the problem:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = \sinh t, \quad \lim\limits_{x\to\infty} u(x,t) = 0, \ t \geq 0, \\ u(x,0) = 0, \quad u_t(x,0) = e^{-x}, \quad x > 0. \end{cases}$$

Hint. The following formulas may be useful.

$$\mathcal{L}\{e^{kt}\} = \frac{1}{s-k}, \quad s > k. \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad s > 0.$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}, \quad s > 0. \ \ \mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}, \quad s > 0.$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}, \quad s > |k|. \ \ \mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}, \quad s > |k|.$$

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Important: Please do ONE problem only.

Problem#3 [25 points]. Solve the problem

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = t\cos x, \quad 0 < x < \pi, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u_x(0,t) = u_x(\pi,t) = 0, \quad t \geq 0, \\ u(x,0) = 2 + \cos 3x + \cos x, \quad 0 < x < \pi. \end{cases}$$

Problem#4 [25 points]. Solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0,$$

subject to the boundary and initial conditions

$$\begin{cases} u(0,t) = u(1,t) = 3, \ t > 0, \\ u(x,0) = x + 4, \quad 0 < x < 1. \end{cases}$$

Hint: Let $u(x,t) = v(x,t) + \psi(x)$.

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Good Luck