Birzeit University
Mathematics Department
Math 337

Third Exam -solution First Semester 2021/2022

Q1: (77 points)) Prove or disprove the following

1) If G is an abelian group of order 20 then G has an element of order 5

True: By Cauchy's theorem of finite abelian groups

2) If G is abelian then G/N is abelian for any $N \lhd G$.

True: Let $xN, yN \in G/N$, then $xNyN = xyN = yxN = yNxN$

3) If G is cyclic then G/N is cyclic for any $N \lhd G$.

True: Let $G =< g >$, then gN generates G/N, since if $xN \in G/N$, then there is an $n \in Z$ such that $x = g^n$, so $xN = g^n N = (gN)^n$

4 If $N \lhd G$, and G/N is abelian then G is abelian.

False: Let $G = A_4, N = Z(A_4)$

5 If $G/Z(G)$ is abelian then G is abelian.

False: same in 4

6 If $G/Z(G)$ is cyclic then G is abelian.

see notes or book

7 If G, H are groups and, $f : G \to H$ is a group homomorphism and $a \in G$ with $f(a) = b$, $|a| = 4$, then $|b| = 4$

False: take any group with at least two elements and $f : G \to G, f(x) = e, \forall x \in G$

8 If G, H are groups and, $f : G \to H$ is a group isomorphism and $a \in G$ with $f(a) = b$, $|a| = 4$, then $|b| = 4$

True: see notes

9 For any group G, $Z(G) \lhd G$.

True: see notes

10 If G is a group and H a subgroup of $[G : H] = 2$, then H is a normal subgroup of G

True: see notes

11) $A_n \lhd S_n$

True: see notes or use 10

Q2: (30 points))

1) State and prove Cauchy's Theorem for finite abelian groups.

see notes

2) State and prove the first isomorphism theorem for groups (FTOGH)

see notes

3) Let G be a group, A a subgroup of B and both normal subgroups of G. Prove that $(G/A)/(B/A) \cong G/B$

$f : G/A \to G/B$ by $f(gN = gB)$ and show it is well defined onto group hom with $ker(f) = B/A$