Chapter $\underline{9}$ ⇝ Hypothesis tests

↳ Always we have two Hypothesis

1. the null Hypothesis ⇝ H_0

↳ الفرضية الصفرية ← تمثل وضع المجتمع

2. the alternating Hypothesis ⇝ H_a / H_1

↳ الفرضية البديلة ← تمثل رأي/ اعتقاد الباحث

↳ we start by setting H_a ⇝ where H_a is the researcher claim

then we write H_0 ⇝ wher H_0 is the opposite of H_a

Ex ⇝ The mean weight of high school students is 70 kg. A sample of 50 students is selected. This sample produced a mean of 73. Test the claim is different from 70.

اختبار ادعاء — H_a

use ⇝ $\alpha = 0.05$, $\sigma = 20$

الحل ⇝ $H_0 : \mu = 70$

$H_a : \mu \neq 70$

ملاحظة ← H_a ← لا يمكن ان تكون $= / \leq / \geq$

$> / < / \neq$ لازم

we have three forms of Hypotheses

① upper tail test

$H_0 : \mu \leq \mu_0$

$H_a : \mu > \mu_0$

انتبه للإشارة ←
لمعرفة الاتجاه عند ال Ha

μ_0 ⇝ Constant ثابت

اشارة اكبر ← upper

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② lower tail test

$H_0 : \mu \geq \mu_0$

$H_a : \mu < \mu_0$

اشارة اقل ← lower

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③ Two tailed test

$H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

اشارة لا تساوي ← two

The porpos is to decide wether Ho is accepted or not
How? ⇝ we take a sample, and make some calculations

↳ if the sample supports Ho ⇝ we accepte Ho

↳ if the sample not support Ho ⇝ we reject Ho

9.3 ⇝ population Mean → σ known

→ One tailed test
1. lower tail test
2. upper tail test

→ $\boxed{\text{Test statistic}}$ for Hypothesis tests about a population mean : σ is known

مهم أوي

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{X}$ = sample mean
μ_0 = hypothesis mean
σ = pop. standard deviation
n = sample size

This approach called Critical value approach

To test the hypothesis

1. write the hypothesis H_0 / H_a
2. Find the test statistic " أهم نقطة "
3. test using Critical value approach
4. Conclusin

" بنروح على الجدول "

	lower-tail test	upper-tail test	two-tailed test
Hypothesis	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test statistic	$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
Rejection Rule Critical value approach	reject H_0 if $Z \leq -Z_\alpha$	reject H_0 if $Z \geq Z_\alpha$	reject H_0 if $Z \leq -Z_{\alpha/2}$ or $Z \geq Z_{\alpha/2}$
Rejection Rule P-value approach	reject H_0 if P-value $\leq \alpha$	reject H_0 if P-value $\leq \alpha$	reject H_0 if P-value $\leq \alpha$

الهدف من هذا السابق

← التأكد من ان الفكرة السائدة عن المجتمع
صحيحة ام خاطئة

مهم ① ← ملاحظة مهمة ← H_a ← مستحيل تكون =

② ← احنا بنرفض ونقبل H_0 مش H_a

∴ الهدف H_0 ← يعني الـ H_a هي وسيلة

مهم ← α ← تسمى significance level

Ex ~ $H_0: \mu \leq 40$ given $n = 25$
$H_a: \mu > 40$ $\bar{x} = 42$
$\sigma = 8.2$
$\alpha = 10\%$

① This upper-tail

② Compute test statistic value

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{42 - 40}{\frac{8.2}{\sqrt{25}}} = \frac{2}{\frac{8.2}{5}} = 2 \times \frac{5}{8.2} = 1.2195$$

③ write rejection Rule "Critical value approach"

reject Ho if $Z \geq Z_\alpha$

④ Find the Critical value "Z_α"

the Critical value is Z_α

$\alpha = 10\% = \frac{10}{100} = 0.1$

Z = is the value of Z when the area to the right of Z is 0.1 " فوق Z"

لمساحة لي فوق = 0.1 / المساحة لي تحت = 0.9

$\boxed{Z_{0.1} = 1.28}$ ← للحصول للجدول

0.9000 الاقرب لها = 0.8997

⑤ what is your Concluding ?

يعني هل انا رح ارفض Ho ام اقبلها

The rule is reject Ha if

$Z \geq Z_\alpha$

Nav $Z = 1.219$

$Z_\alpha = Z_{0.1} = 1.28$

$\boxed{1.219 \geq 1.28}$? ⇝ This is false

so ⇝ we dont reject Ho

يعني ما دام المعادلة خاطئة ← لا نرفض Ho

يعني حكي الباحث <u>غلط</u>

Example: student say that they study on averge 5 hours per day, A Claim that the real average is much less then that. A sample of 30 students was taken, the sample mean was 4.6 hours, Assume that the pop. stand. Dev. = 2.1 at 10% significance level, test the Claim

solution

1. $H_0 \quad \mu \geq 5$ → lower tail test
   $H_a \quad \mu < 5$

$\bar{x} = 4.6$
$\sigma = 2.1$
$\alpha = 0.1$
$n = 30$

2. $$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{4.6 - 5}{\frac{2.1}{\sqrt{30}}} = -1.04$$

$Z = -1.04$ test statistic

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3. Critical value

$\boxed{1 - 0.1 = 0.9}$

$-Z_\alpha = -Z_{0.1} = -1.28$ حسب الجدول

بالرجوع للجدول 1.28 اقرب اشي عليها

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4. Reject Ho if $Z < -Z_\alpha$

$-1.04 < -1.28$? → false

5. So Don't Reject Ho

Exampl → 9.3 مثال ص → The mean weight of high school students is 70 kg, A sample of 50 student is selected. This sample produced mean of 73. test the Claim is different from 70

use $\alpha = 0.05$, $\sigma = 20$

تعرفي من الكلمة اول
two tailed ∴

solution

1 → $H_0 : \mu = 70$ → two tailed test

$H_a : \mu \neq 70$

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2 test statistic

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{73 - 70}{\frac{20}{\sqrt{50}}} = 1.06$$

$\bar{X} = 73$

$\sigma = 20$

$n = 50$

$\mu_0 = 70$

---

3 Critical value

$\pm Z_{\frac{\alpha}{2}} = \pm Z_{\frac{0.05}{2}} = \pm Z_{0.025}$ → $1 - 0.025 = 0.9750$

بالرجوع للجدول = 1.96 مباشرة

So the Critical values $= \pm 1.96$

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4 Reject if Ho $Z \leq -Z_{\frac{\alpha}{2}}$ or $Z \geq Z_{\frac{\alpha}{2}}$

$1.06 \leq -1.96$ or $1.06 \geq 1.96$

both are false

5 → So we don't reject Ho

ملاحظة ← اذا كانت وحدة فيهن صح معناها صح ونرفض
الـ Ho

9.2 ⇝ P-value

↳ P-value approach only when σ is known

→ P-value → a probability (area "مساحة") value found using Z-table based on test statistic and type of the test

لـ P عبارة عن مساحة يتم إيجادها عن طريق ما يلي

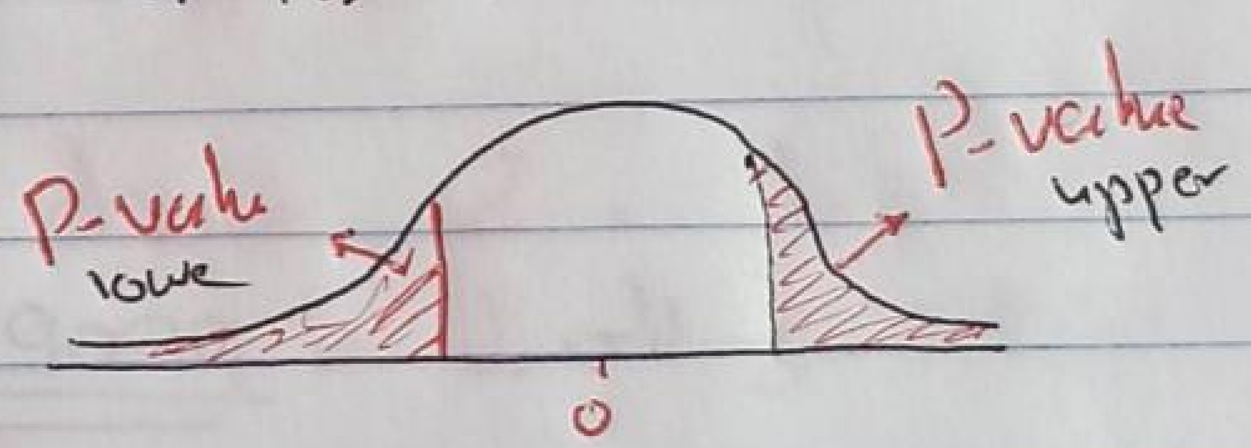
① test statistic
② نوع الاختبار

↳ in any test : if P-value $\leq \alpha$ Reject H_0
في كل الاختبارات   if P-value $> \alpha$ Don't Reject H_0

How to find P-value ?

① find test statistic ⇝ $Z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

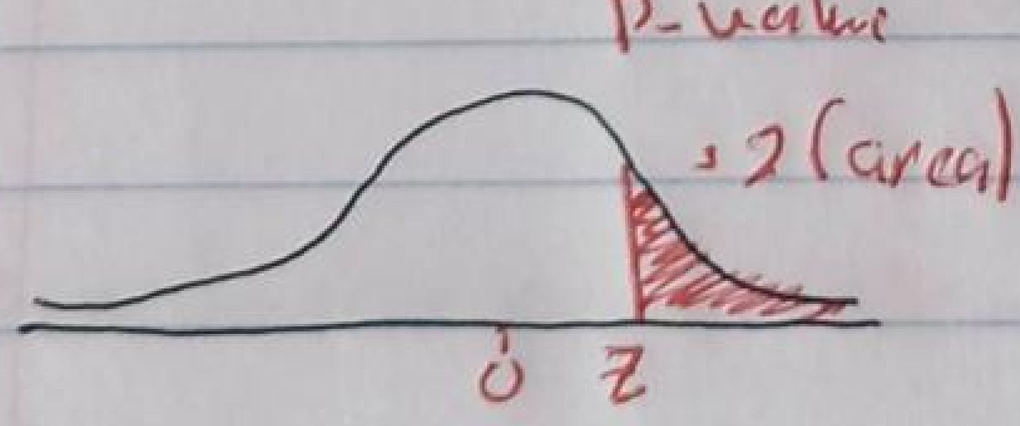
② upper tail test ⇝ P-value = area above Z

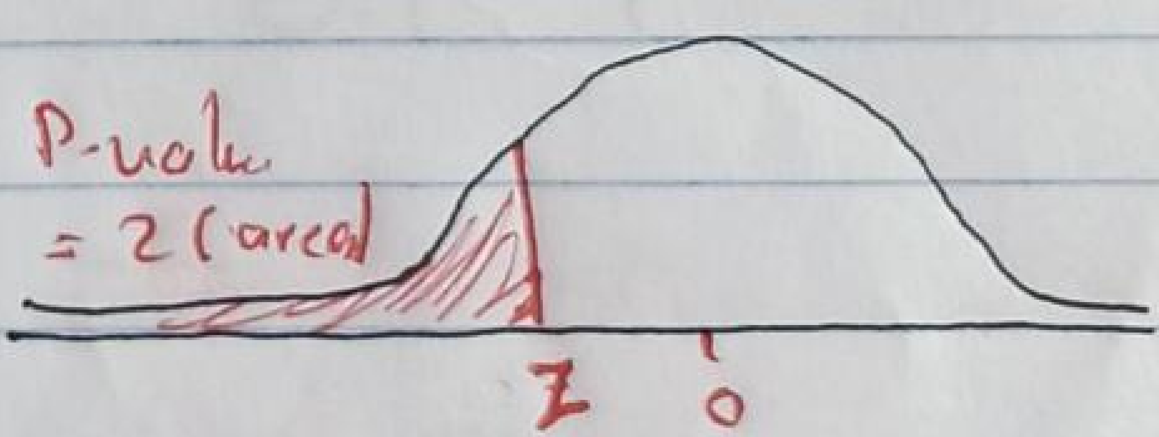


③ lower tail test ⇝ P-value = area below Z

④ two-tailed test ⇝ P-value = 2 (area above Z) if $Z > 0$
or P-value = 2 (area below Z) if $Z < 0$

+Z





Example ⇝ $n = 80$ / $\bar{x} = 41.5$ / $\sigma = 10$ / $\alpha = 10\%$

① $H_0: \mu \leq 40$ ⇝ upper tail test
$H_a: \mu > 40$

② Find test statistic = $Z = \frac{41.5 - 40}{\frac{10}{\sqrt{80}}} = 1.342$

③ write rejection Rule

الطريقة الأولى □ Critical value approach ⇝ reject H_0 if $Z \geq Z_\alpha$

$Z_\alpha = Z_{0.1} = 1 - 0.1 = 0.9000$
1.28 تقريبا بالرجوع للجدول ↲

↳ $Z_{0.1} = 1.28$

→ $1.342 \geq 1.28$? yes ⇝ reject H_0

الطريقة الثانية □ P-value approach

P-value = area above $Z = 1.342$
0.9099 ← بالرجوع للجدول ↲

↳ $1 - 0.9099 = 0.0901$

For P-value
القاعدة ↳ reject H_0 if P-value $\leq \alpha$

$0.0901 \leq 0.1$? yes ⇝ reject H_0

↳ conclusion is ⇝ $\alpha = 5\%$ ⇝ $\frac{5}{100} = 0.05$ ⇝ $0.0901 \leq 0.05$?
Dont reject H_0 ← لا ↲

Example ⇝ The averge of starting salaries for Engineering graduates is known to be 4200 LEs monthly, with standard dev. of 450 LEs

Due to demand increase on Engineering graduate, it's thought that the average is greater than 4200 LEs.

So sample of 10 Enginerring graduate was selected, if the sample mean was 4500 LE

① Set hypothesis

$H_0: \mu \le 4200$ ⇝ upper tail test
$H_a: \mu > 4200$

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② test statistic ⇝ $Z = \dfrac{4500 - 4200}{\frac{450}{\sqrt{10}}} = 0.00222$

③ ▷ Critical value approach ($\alpha = 0.05$)

reject H_0 if $Z \ge Z_\alpha$

$1 - 0.05 = 0.95$ ⇝ 1.645 = بالجدول

↳ reject H_0 if $0.00222 \ge 1.645$? No

↳ we Dont reject H_0

Example → Consider the following hypothesis test

$H_0: \mu = 2250$ → two-tailed test
$H_a: \mu \neq 2250$

A sample size of 100 is uses, $\bar{X} = 2400$ and $\sigma = 550$

1. Calculate the test statistic

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2400 - 2250}{\frac{550}{\sqrt{100}}} = \frac{150}{\frac{550}{10}} \approx 2.73 \text{ or } \underline{\underline{2.727}}$$

2. using Critical value approach

reject Ho if $Z \leq -Z_{\alpha/2}$ or $Z \geq Z_{\alpha/2}$

$$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$$

$$= 1 - 0.005 = 0.9950$$ → للرجوع للجدول تكون بين رقمين

$$\frac{2.57 + 2.58}{2} = 2.575$$

↳ so $2.727 \leq -2.575$ or $2.727 \geq 2.575$

✗ ✓

مادام هذه صح ∴ نقبل Ha, ونرفض $\underline{\underline{H_0}}$

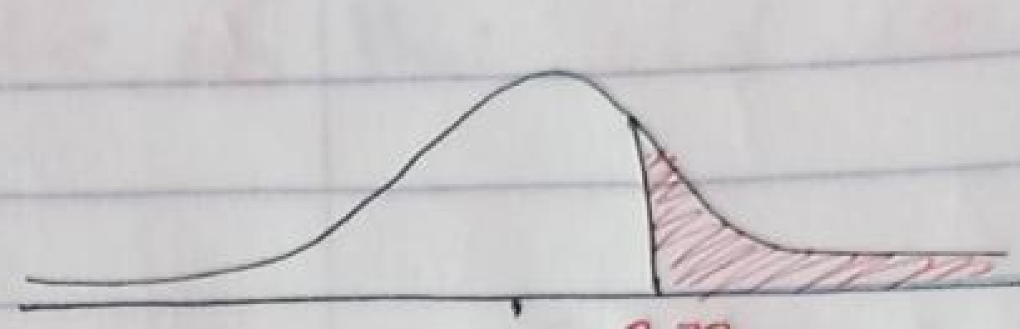
reject Ho

- using P-value approach

P-value = 2 (area above Z) if Z +
2 (area below Z) if Z -

(above) Z + = 1 - area below Z
= 1 - 0.9968

P-value = 0.0064



↳ reject Ho if P-value ≤ α

0.0064 ≤ 0.01 ✓

reject Ho

لذلك نرفض Ho

↳ at 0.1 significance. what is your Conclusion
p-value approach

reject if P-value ≤ α

0.0064 ≤ 0.1 ? → yes

so reject Ho

9.4 ~ Population mean when σ is unknown

	upper tail test	lower tail test	two tailed test
Hypothesis	$H_0: \mu \le \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu \ge \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \ne \mu_0$
test statistic	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
Rejection Rule	Reject Ho if $t \ge t_\alpha$	Reject Ho if $t \le t_\alpha$	Reject Ho if $t \le -t_{\frac{\alpha}{2}}$ $t \ge t_{\frac{\alpha}{2}}$ or
Critical value	t_α	$-t_\alpha$	$\pm t_{\frac{\alpha}{2}}$

* هنا على جدول t

* لا يوجد P-value هنا ← فقط طريقة واحدة

* هنا عندما تكون σ غير معروفة

Example ⇝ Consider the following hypothesis test

$H_0 : \mu \leq 260$ ⇝ upper tail

$H_a : \mu > 260$

A sample size of 25 provided, a mean 268 and standard deviation of 16

[1] Compute the test statistic

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{268 - 260}{\frac{16}{\sqrt{25}}} = 2.5$$

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[2] at $\alpha = 0.01$, what is your conclusion

↳ reject H_0 if $t \geq t_\alpha$

Degrees of Freedom $n-1$ [illegible]

$t_\alpha = t_{0.01} =$

$n - 1 = \underline{24}$

Critical value ← $2.492 = \underline{\underline{t}}$ [illegible]

↳ reject H_0 if $t \geq t_\alpha$ ⇝ $2.5 \geq 2.492$

so reject H_0 [illegible]

Example ⇝ the average local Cell phone was 2.27 minutes
A sample of 20 phone Calls showed a mean of 1.85 minutes, and standard deviation of 0.98 at $\alpha = 0.1$

→ Can it be Concluded that the mean differ from the pop. mean?

Solution ⇝ [1] hypothesis → $H_0: \mu = 2.27$
$H_a: \mu \neq 2.27$
↳ two tailed test

[2] ~~Crit~~. test statistic

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.85 - 2.27}{\frac{0.98}{\sqrt{20}}} = \approx -1.92 \text{ or } -1.917$$

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[3] Rejection Rule
reject H_0 if $t \leq -t_{\frac{\alpha}{2}}$ or $t \geq t_{\frac{\alpha}{2}}$

$\frac{\alpha}{2} = \frac{0.1}{2} = 0.05$
$= ↳ t_{\alpha} = t_{0.05}$

▹ Degree of Freedom $= 20 - 1 = \underline{19}$
± 1.729 ← الجدول

So $-1.917 \leq -1.729$ or $1.917 \geq 1.729$

So reject H_0
↳ Accept H_a

9.2 → Type I and Type II errors → "مهم"

الاستنتاجات إذا طلبت توضيح

Conclusion	Population Condition: Ho True	Population Condition: Ha true
Accept Ho	True Decision	Type II error "خطأ أخف"
Reject Ho	Type I error "خطأ صعب" أو خطأ قوي	True Decision

1) * اذا كانت Ho صح وانا قبلتها بسوي قرار صحيح

2) * اذا كانت Ho صح وانا رفضتها بسوي خطأ من النوع II

3) * اذا كانت Ho غلط وانا رفضتها بسوي قرار سليم / صحيح

4) * اذا كانت Ho غلط وانا قبلتها بسوي خطأ من النوع I

→ level of significance → the level of significance is the probability of making Type I error when the null hypothesis is true as an equality