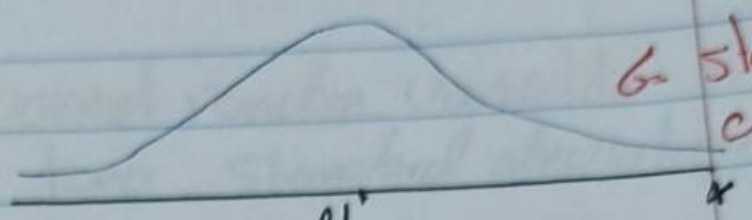
Chapter 6.2 The Normal Distribution

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

هذا القانون فقط
المعرفة ولن ننظمه



Properties خصائص

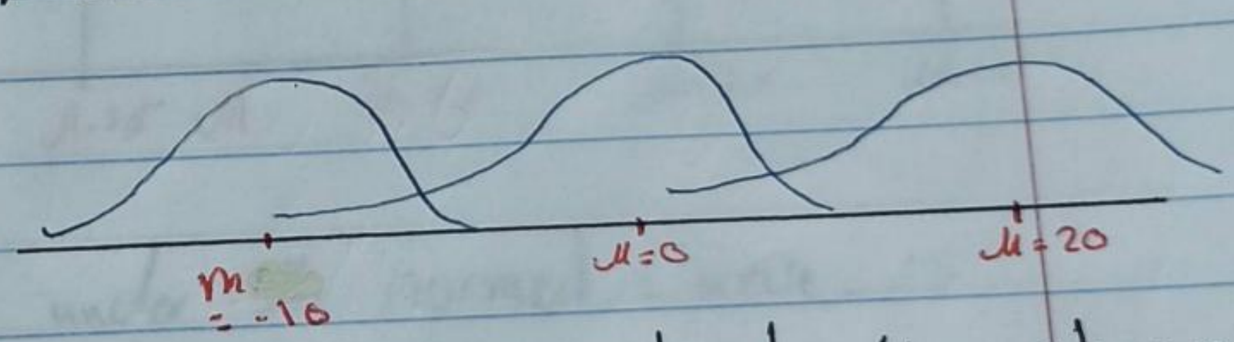
[1] The entire family of normal Distribution is differenticated by two parameter μ and σ

[2] The highest point on the normal Curve is at the mean which is also the median and the also mode of the distribution

Mean = Median = Mode

[3] The mean of the distribution can be any value positive, negative or zero

القيمة الحاصل عليك ان تكون موجب / سالب / صفر

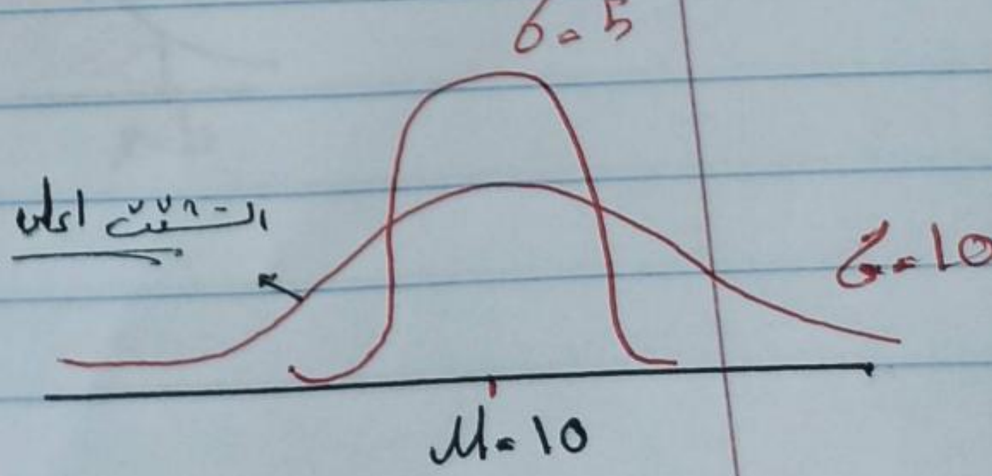


[4] The normal curve is symmetric about μ ⇝ Skewness=0

[5] The standard deviation determines how ~~flat~~ flat and wide the normal curv is

Larger value of σ ⇝ wide

Smaller value of σ ⇝ narrower

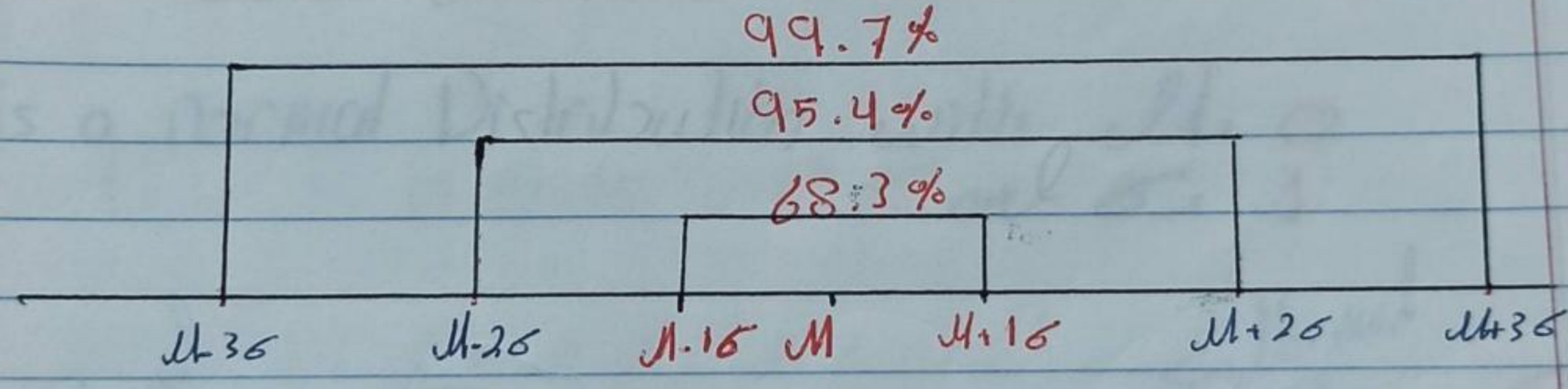


7. Percentage of values in some commonly uses interval

a. 68.3 % of the values of a normal random variable are within plus of minus one standard deviation of its mean

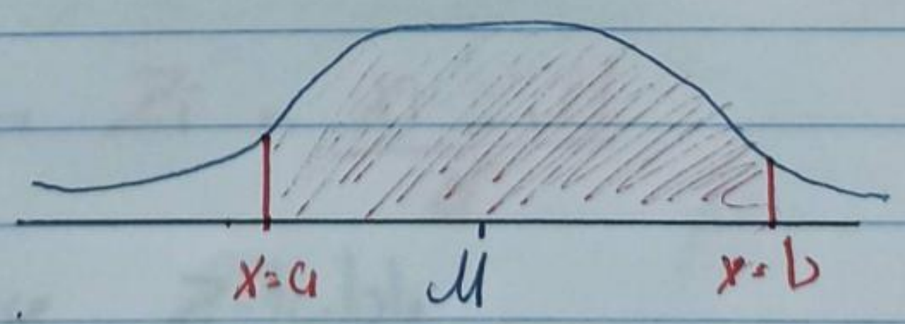
b. 95.4 % of the value of normal random variable are within plus or minus two standard deviations of it's mean

c. 99.7 % of the value of normal random variable are within plus or minus three standard deviation of it's mean



8. Probability = area under the normal Curve

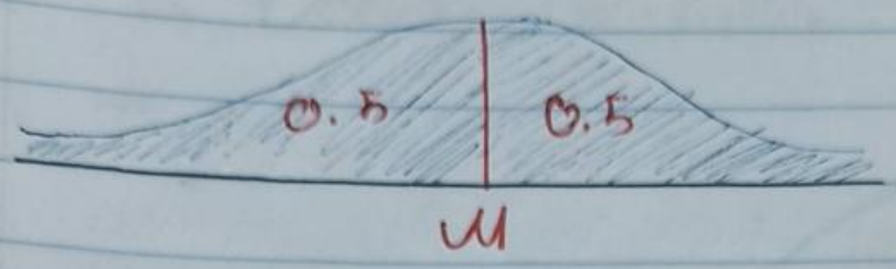
$P(a < x < b) = P(a \leq x \leq b)$ = area



[9] Total area = 1

موضع المنحنى ← $P(x < \mu) = P(x \leq \mu) = 0.5$

$P(x > \mu) = P(x \geq \mu) = 0.5$



المساحة = الاحتمال ← فإن المجموع بالمساحة = 1

ولإيجاد المساحة يجب استخدام جدول Z ← Z-table

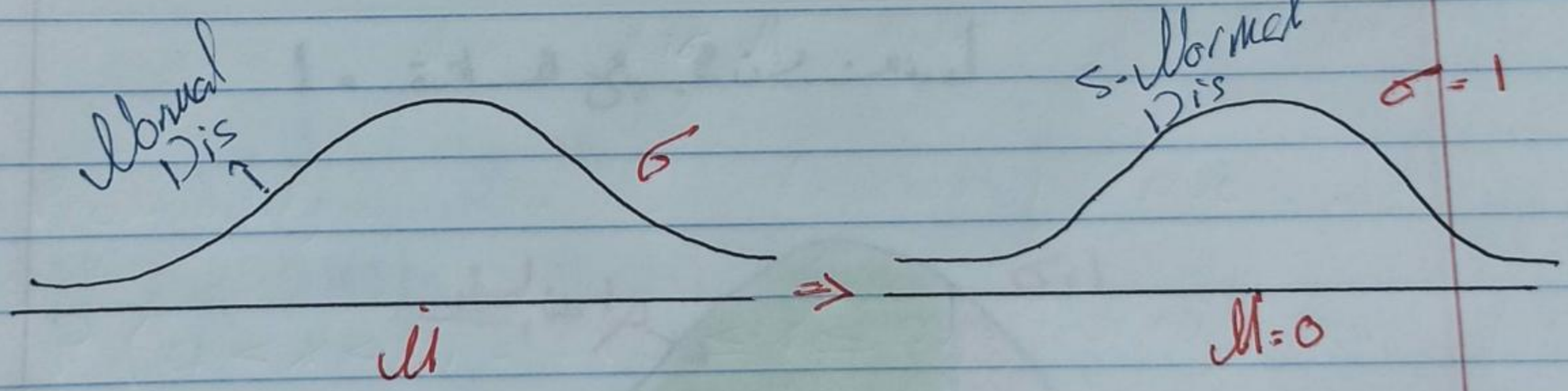
لجدول خارجي

لهذا الجدول يعمل لـ Standard normal

---

[5] Standard Normal Distribution

is a normal Distribution with $\mu = 0$ and $\sigma = 1$



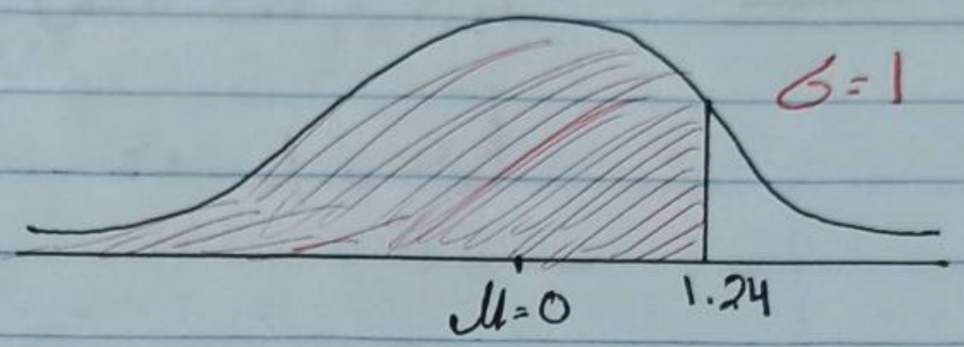
$$X_i = Z_i = \frac{X_i - \mu}{\sigma}$$

⇒ we will use Z-table

Examples ⇐ بالوجوه الجدول

① Find the area below 1.24 . $P(z < 1.24)$

↳ useing z-table = 0.8925

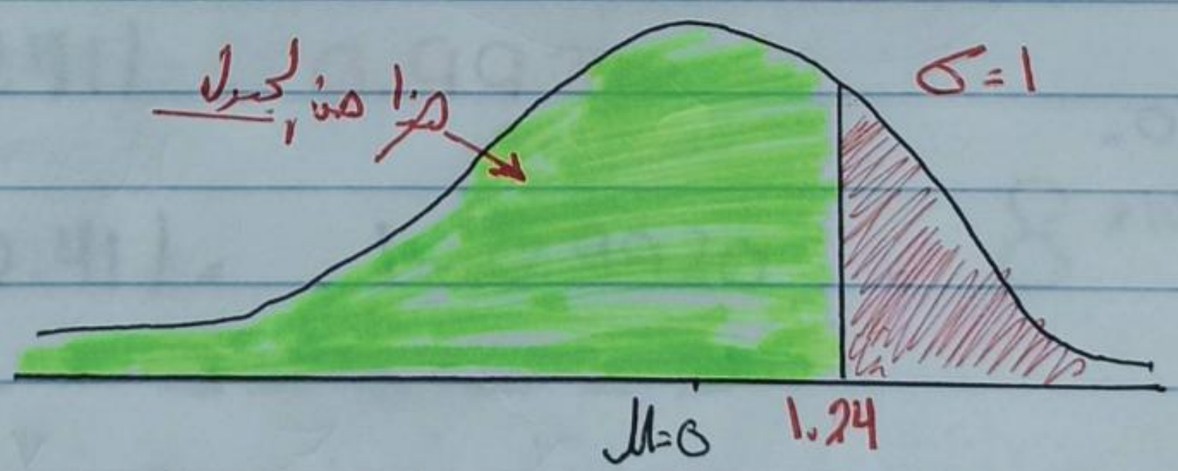


② $P(z \leq 1.24) = 0.8925$ ⇐ $(\leq / <)$ لا تفرق بين

③ $P(z \geq 1.24)$

↳ = 1 - 0.8925 = 0.1075

لأن المجموع الكلي = 1

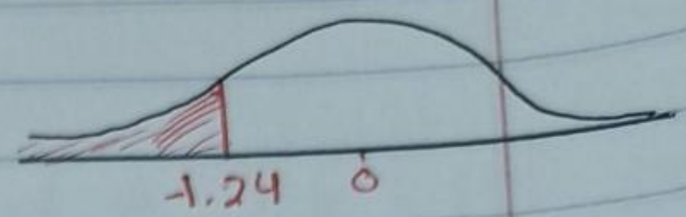


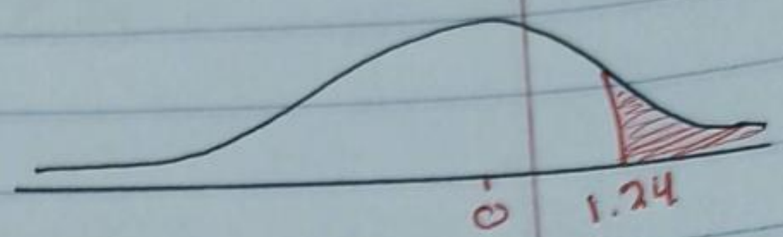
④ $P(z \leq -1.24)$

↳ $= P(z \geq 1.24)$

$= 1 - 0.8925 = 0.1075$

لانه تقوم فقط بعكس الاشارة ($\leq$)





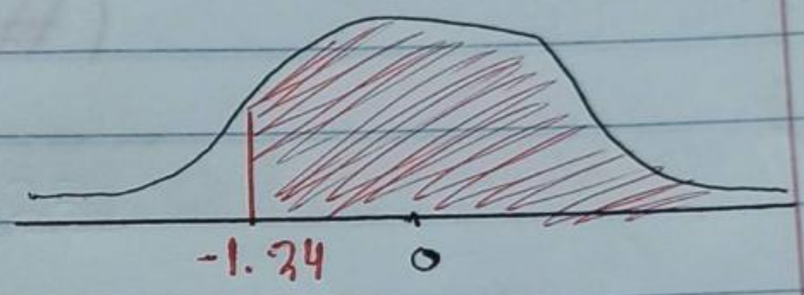
Symmatric لانه

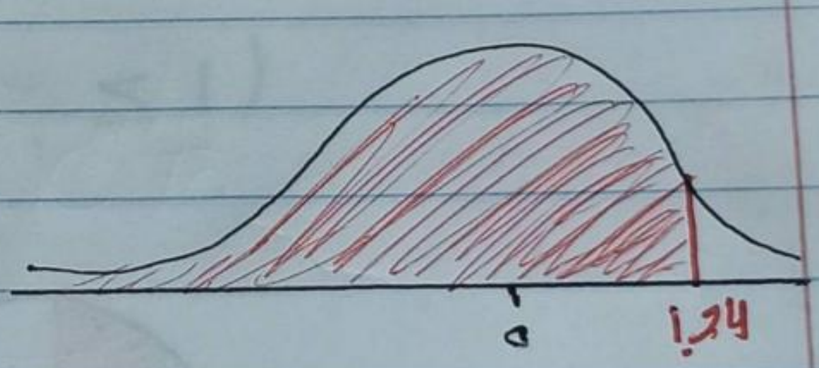
---

⑤ $P(z \geq -1.24)$

↳ $P(z \leq 1.24)$

$= 0.8925$





Example ← بالرجوع إلى الجدول

$P(z \leq 2.41) = 0.9920$

$P(z \geq 2.41) = 1 - 0.9920 = 8^{-03} \times 10$

Notes ~>

→ $P(z \leq \#) = P(z < \#)$

→ $P(z \geq \#) = P(z > \#)$

→ $P(z < \#)$ نجدها من الجدول مباشرة

→ $P(z < -\#) = P(z > \#) = 1 - P(z < \#)$

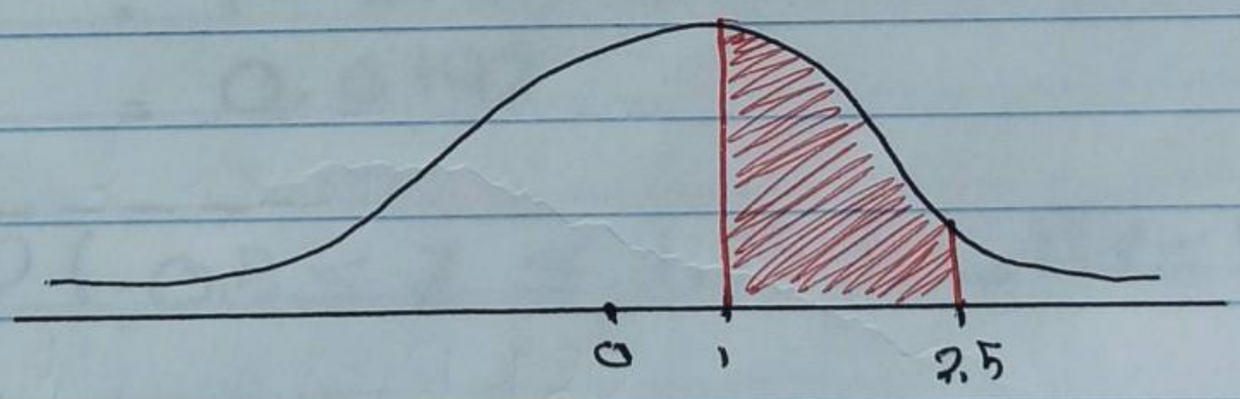
→ $P(z > -\#) = P(z < \#)$

Exampl ~> مثال

$P(1 \leq z \leq 2.5)$

$= P(z \leq 2.5) - P(z \leq 1)$

من الجدول مباشرة نجد



$= 0.9938 - 0.8413$

$= 0.1525$

لانه نوجد المنطقة ماتحت الـ 1 — 2.5

لو شطبنا ما تحت الـ 1 لذلك قمنا بطرح 1 من الكل

Q→ مثال بدون حل

↳ The grades of BZU student in Stat 2361 are usually normally Distributed with average of 71 and standard deviation 11.6

1) what is the probability that a student grade will exceed 95 ?

$\mu = 71 \quad \sigma = 11.6$

$Z = \frac{95 - 71}{11.6} = \frac{24}{11.6} \approx 2.07$

$P(x > 95) = P(Z > 2.07)$

$= 1 - P(Z > 2.07)$

$= 1 - 0.9808$

$= 0.0192$

Example → $P(-0.5 \leq Z \leq 1.35) \rightsquigarrow P(Z \leq 1.35) - P(Z \leq -0.5)$

$P(Z \leq 1.35) = 0.9115$

$P(Z \leq -0.5) = P(Z \geq 0.5)$

$= 1 - P(Z \leq 0.5)$

$= 0.3085$

↳ $0.9115 - 0.3085 = 0.603$

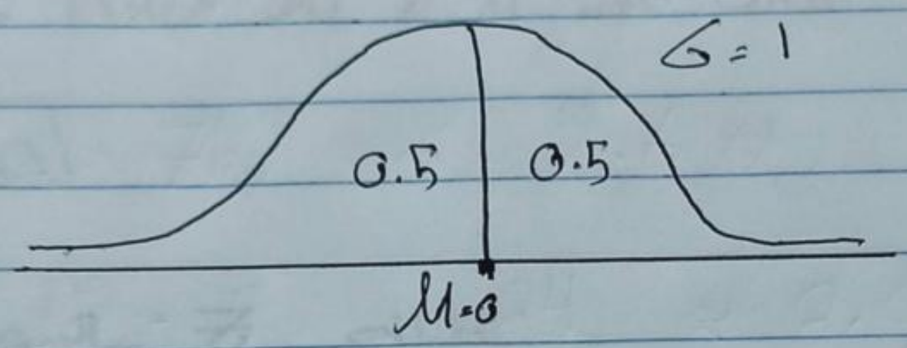
First we were given Z → and we found area probability

Now we have the opposite process

↳ we will be given the area of probability

→ and we should found Z

لما بيني نعكس العملية



حالة ١ ←

□ اذا طلب بالسؤال رقم "كتلة" اقل من 0.5 ← معناها الرقم سالب

□ اذا طلب بالسؤال رقم "كتلة" اكثر من 0.5 ← معناها الرقم موجب

□ اذا طلب بالسؤال رقم فوقه اقل من 0.5 ← معناها الرقم موجب

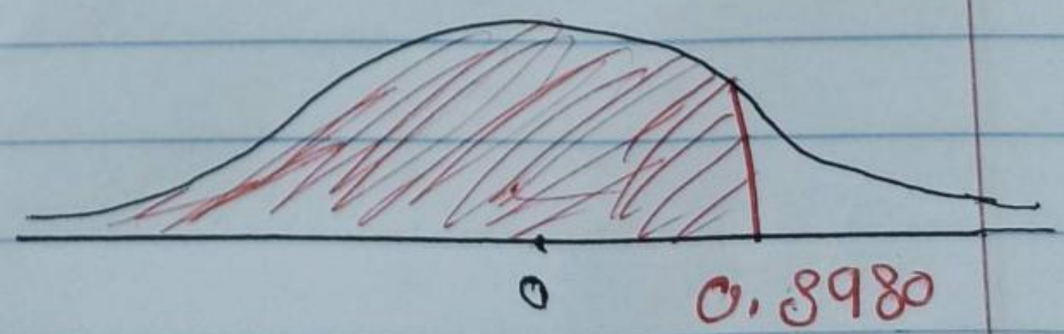
□ اذا طلب بالسؤال رقم فوقه اكثر من 0.5 ← معناها الرقم سالب

Example → Find z if the area to the left of z is 0.8980?

$Z > 0$

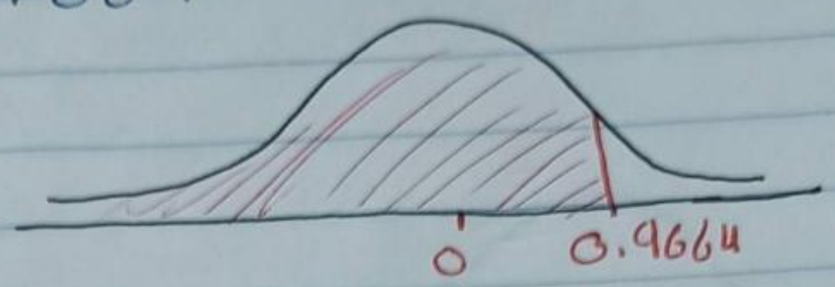
موجب لأن الجدول

$Z = 1.27$



Example → Find the z if the area to the left of z is 0.9664

$z > 0$

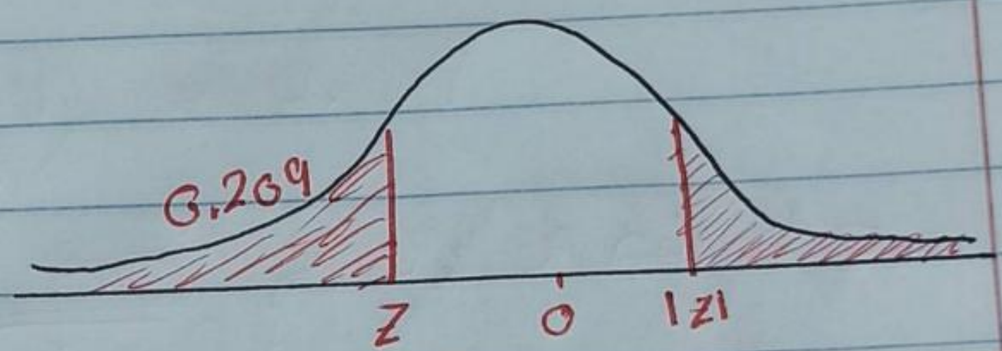


$z = 1.83$ ← بالجدول الجدول

Example → Find the z if the area to the left of z is 0.209

$z < 0$

بما انه z السالب فاحنا بدنا
نحولها لـ موجب



لـ موجب فاتلناه سابقاً

↳ ~~////////~~ $1 - 0.209$
$= 0.7910$

↳ $z = 0.81$ ← جدول

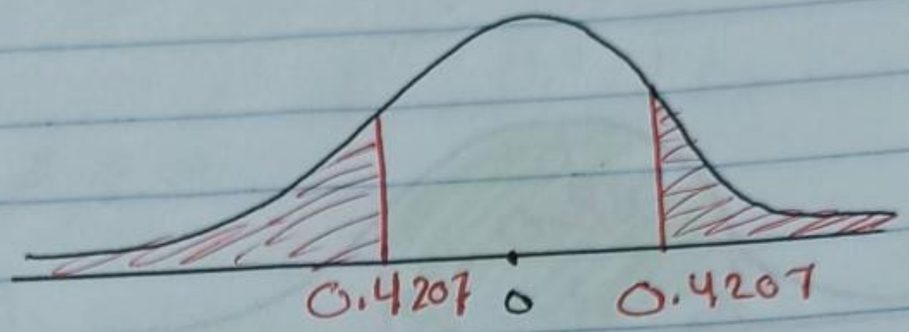
لـ لكن اذا كان بالخيارات $z = -0.81$ فنقوم باختيارها
لانه بالاصل z سالب لكن هنا بالتحويل لـ
موجب لكي نستطيع استخدام الجدول

Example → Find z if the area to the left of z is 0.4207

$\boxed{z < 0}$

$1 - 0.4207$

$= 0.5793$



$|z| = 0.2 \rightarrow z = -0.2$

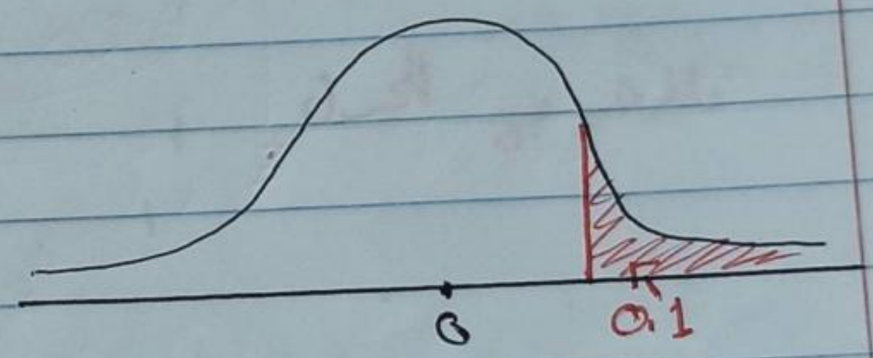
Example → area to the <u>right</u> of z is 0.1

$\boxed{z > 0}$

$1 - 0.1 = 0.9$

$z \approx 1.28$

حسب الجدول



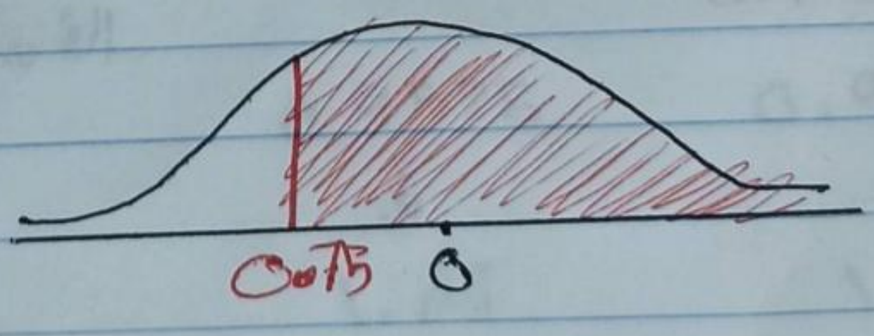
Example → area to the <u>right</u> of z is 0.75

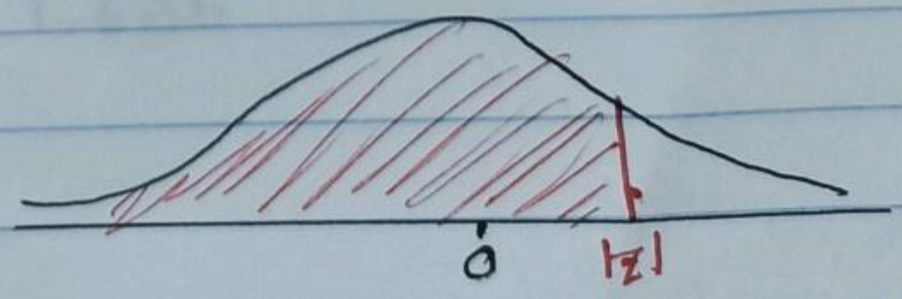
$\boxed{z < 0}$

هذه من الجدول مباشرة

$|z| = 0.67$

$z = -0.67$



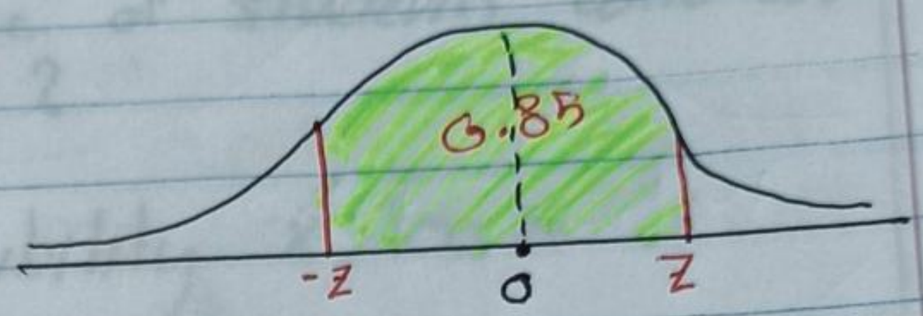


Example → Area between Z and -Z is 0.85

$\frac{0.85}{2} = 0.425$

$1 - 0.85 = 0.15$

$\frac{0.15}{2} = 0.075$



So area under Z is $0.85 + 0.075$

$= 0.925$

or $0.5 + 0.425 = 0.925$

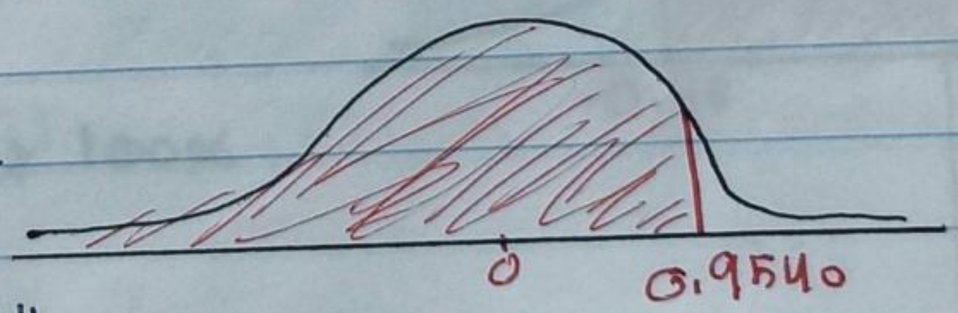
$Z = 1.44$ ← بالرصيد الجدول

$-Z = -1.44$ ←

Example → area to the ~~Right~~ left of Z is 0.9540

$Z > 0$

عشان ما لقينا الرقم
لما ناخد الاقرب القيم ← للرقمين
القريبين



0.9535 ↓ 1.68

0.9545 ↓ 1.69

$\frac{1.68 + 1.69}{2} = 1.685$

$Z = 1.685$

Example → The grades of BZU students in Stat 2361 are usually normally distributed with average of 71 and standard deviation of 11.6

$\mu = 71 \quad \sigma = 11.6$

Q → what is the percentage of students who will pass this Course ?

Percentage = Probability × 100%

$P(x \geq 60)$

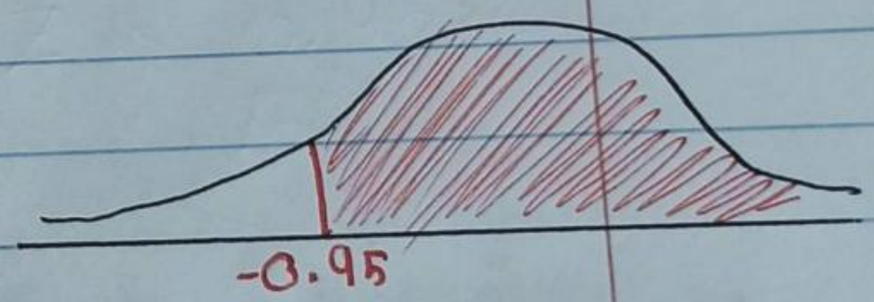
$$Z = \frac{60 - 71}{11.6} = \frac{-11}{11.6} = -0.9483$$

≈ -0.95

$P(x \geq 6) = P(Z \geq -0.95)$

$= P(Z \leq 0.95)$ ← بالرجوع الجدول

$= 0.8289$



Percentage = $0.8289 \times 100\%$

$= 8289\%$

Q → "حسب البيانات" ⇝ Find the 90th percentile

P_{90} → the value where 90% of the Data are less than or equal it

90% = 0.9 below P_{90}

0.08 or 0.09

1.2 → 0.3997 or 0.3995

↓

هذه القيمة

$Z = 1.28$

$$Z = \frac{X - \mu}{\sigma}$$

$$1.28 = \frac{X - 71}{11.6}$$

$$1.28 \times 11.6 = X - 71$$

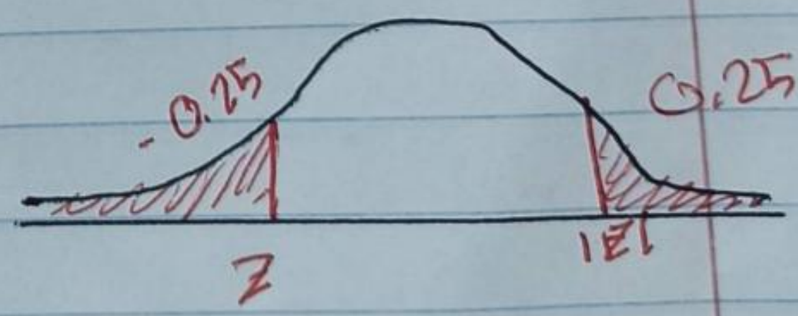
$$14.848 = X - 71$$

$$+71 \qquad +71$$

$$X = 85.848$$

Q → A grade in the bottom 25% is considered as a hopeless

what is the hopeless grade?



1 - 0.25 = 0.75

0.07 or 0.08

0.6 → هذا الرقم ← .7486 or .7517

|Z| = 0.67 → Z = -0.67

→ نأخذه

$\rightarrow Z = \frac{x - \mu}{\sigma}$ $\rightsquigarrow$ $-0.67 = \frac{x - 71}{11.6}$

$11.6 \times 0.67 = x - 71$

$+71 \qquad +71$

$x = (11.6) \times (0.67) + 71$

$= 63.23$

↳ so the hopless grade is 63.23 or less

" باقي الاسئلة على الصور " " افكار فدعة "