Chapter 5 → Discreate probability Distribution
توزيع احتمالي منفصل

5.1 Random variables متغيرات عشوائية

Random variables ⇝ is numerical Discription of th outcomes of the experiment

لارقام تصف نتيجة التجارب

هناك نوعان

1. Discrete Random variables ⇝ الذي يقبل العشار
2. Contianous Random variables ⇝ الذي يقبل العشار

Example ⇝ two Coins

$\Omega = \{(H,H)\ (H,T)\ (T,H)\ (T,T)\}$

ex → Random variables ⇝ # of Head

$(\Omega, V) \rightarrow X = 2, 1, 0$

يعني نخلي الـ R.V يساوي X

$X = 2 \rightsquigarrow (H,H)$
$X = 1 \rightsquigarrow (H,T)\ (T,H)$
$X = 0 \rightsquigarrow (T,T)$

① Discreate Random variable

Ex ~> if you have an exam Composed of 10 part

X = # of the correct answer ~> R.V

X = 8 : that mean you have 8 true answer

Ex ~> if you through 10 Coins

X = # of Head ~> R.V

X = 10, 8, 9, 7, 6, 5, ... 0

Maximum

يعني لا يقل العشار

② Continuous Random variables

Ex ~> Time need to build a House

X = Time needed to build a Home

$0 \leq X$ (days)

Ex ~> Temprature of Human

X = Temprature of Human

$37 \leq X \leq 37.5$

يعني يقبل العشار

2 ⇝ Consider the experiment of a worker assembling a product

[a] Define a random variable that represent the time in minutes required to assemble the product

[b] what value may the random variable assume?

X ⇝ time in minutes to assemble a product

$X \geq 0$

[c] is the random variable discrete or Continoues?

↳ Continuos

Example ⇝ in the experiment of rolling a die twice

The sample space = $\{(1,1) \ldots\ldots (1,6) \ldots (6,1) \ldots\ldots (6,6)\}$

X → the sum of the two faces observer مجموع الوجهين

↳ Can assume the values 2, 3, 4, 5, ... 12

$X = \{2, 3, 4, 5, 6, 7 \ldots\ldots 12\}$

Y = absolute difference of the two observer faces العرف

$Y = \{0, 1, 2, 3, 4, 5\}$

5.2 → Discrete probability Distribution

The Probability Distribution For a random variable describe → How probabilities are distribut over the values of the variable

لـ يصف توزيع الاحتمالات للمتغير العشوائي كيفية توزيع الاحتمالات على قيم المتغير

For discrete Random variable, the prob. distri. is defined by Functions

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For example → in the experiment in Tossing two Coins

X = # of Heads

X	P(X) = F(X)
2	$\frac{1}{4}$
1	$\frac{2}{4}$
0	$\frac{1}{4}$

S = [(H,H) (H,T) (T,H) (T,T)]

X	P(X) = F(X)	
0	$\frac{1}{8}$	0.125
1	$\frac{3}{8}$	0.375
2	$\frac{3}{8}$	0.375
3	$\frac{1}{8}$	0.125
		1

→ tossing three Coins

X = # of Tails

S = [(H,H,H) (T,T,T) (H,T,H) (H,H,T) (T,T,H) (T,H,T) (H,T,T) (T,H,H)]

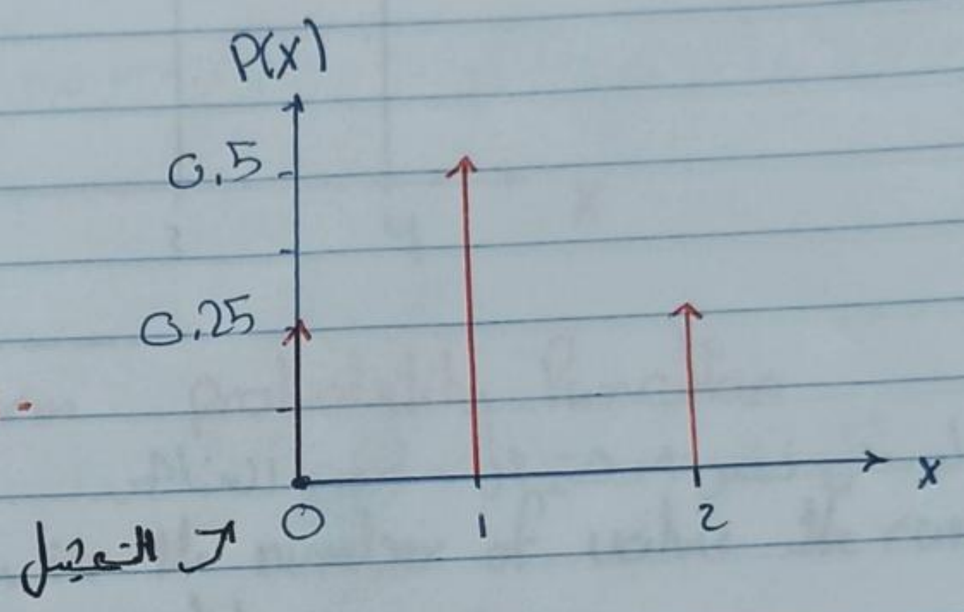
Note that ⇝ $P(x) \geq 0$ ، $\sum P(x) = 1$

↳ Required Conditions for a discrete probability function

Ex⇝ (حسب المثال السابق) ⇝ $\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8} = 1$

x	P(x)
0	1/4
1	2/4 = 1/2
2	1/4

represent the prob. Dis.



Example ⇝ عدد السيارات المباعة في 100 يوم

X = '# of Cars Sold

* over 20 days ⇝ no Car sold
* over 15 days ⇝ 1 Car sold
* over 5 days ⇝ 2 Car sold
* over 25 days ⇝ 3 Car sold
* over 35 days ⇝ 4 Car sold

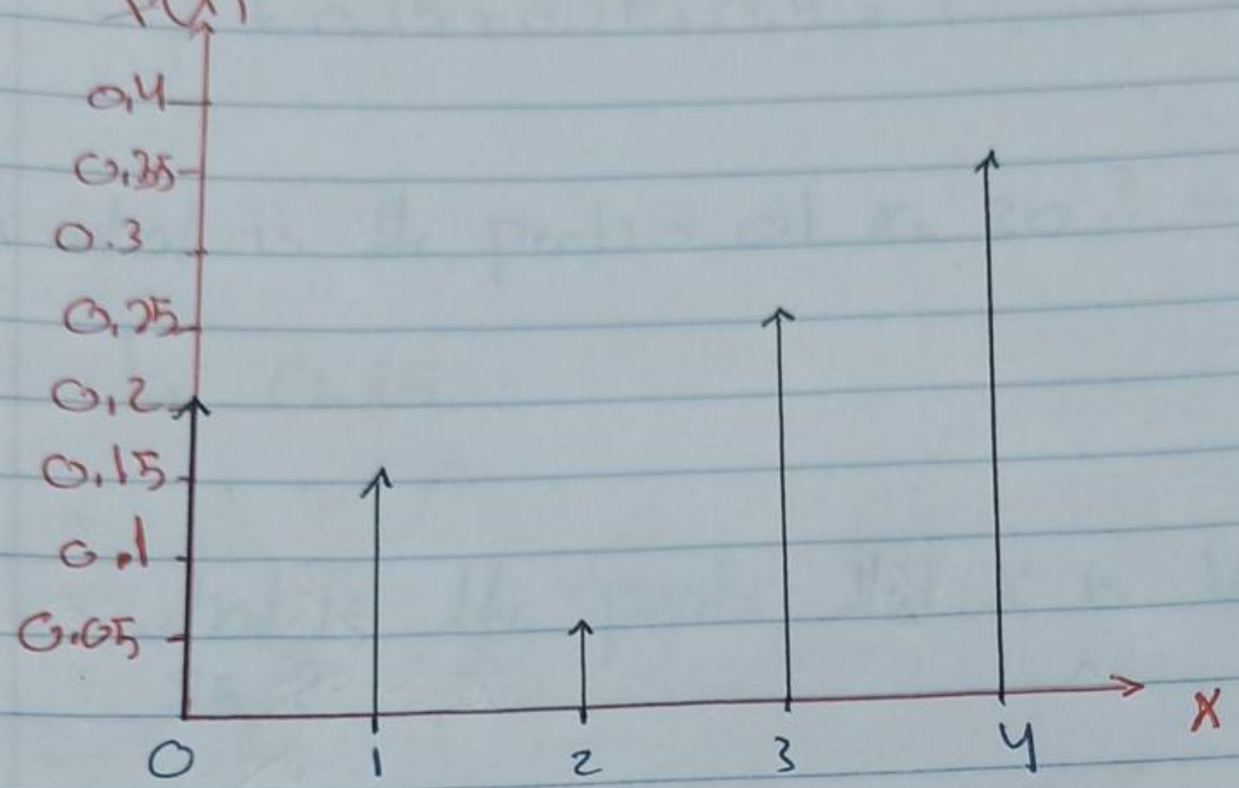
x	F(x) = P(x)
0	20/100 = 0.2
1	15/100 = 0.15
2	5/100 = 0.05
3	25/100 = 0.25
4	35/100 = 0.35

Note that

= 0.2 + 0.15 + 0.05 + 0.25 + 0.35

= 1

→ Graphical representation



□ Discribe uniform prolxalzility Function توزيع منتظم

لا يعني نفس الاحتمال لجميع العناصر

$F(X) = \frac{1}{n}$ $n \rightarrow$ is the number of value the ranclom variable may assume

Ex→ Rolling a clie X = # of data on the upward face

$X = 1, 2, 3, 4, 5, 6$ $\boxed{P(x) = \frac{1}{6}}$

Example 7/192 ⇒ the prob. dist. for the ranclom variable x follow.

x	P(x)
20	0.2
25	0.15
30	0.25
35	0.4

- is this a distribution valid ?

  $0.2 + 0.15 + 0.25 + 0.4 = 1$ ⇝ are all nonnegative

- what is the prob at $X = 30$?

  ↳ 0.25

- what is the prob that X is less than or equal 25 ?

  ↳ $0.15 + 0.2 = 0.35$

- what is the prob that X is greater than 30?

  ↳ 0.4

ملاحظة مهمة ← شروط انه الاحتمال يكون uniform

① ⇝ الارقام لازم تكون بالسالب

② → مجموع الاحتمالات = 1

at least a	$X \geq a$	a ✓
more than a	$X > a$	a ✗
at most a	$X \leq a$	a ✓
less than a	$X < a$	a ✗

5.3 Expected value and variances

الأمل الرياضي = متوسط = Mean

Expected value of discrete random variable

$E(x) = \mu = \sum x \cdot f(x)$

Note that the expected value is weighted mean of the values the random variable may assume

$E(x)$ = weighted Mean

$= \sum x \cdot (P(x))$

The weights are the probabilities

Ex →

X	P(X)	x.P(X)
3	0.25	0.75 → 3x(0.25)
6	0.50	3
9	0.25	2.25

Compute the expected value of X

So $E(X) = \mu = \sum x \cdot P(x)$

$0.75 + 3 + 2.25 = 6$

$\mu = \frac{18}{3} = 6$

- variance of discrete Random variable

$$Var(x) = \sigma^2 = \Sigma (x-\mu)^2 P(x)$$

Ex → $\mu = 6$

x	P(x)	x-μ	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
3	0.25	-3	9	2.25
6	0.5	0	0	0
9	0.25	3	9	2.25
				4.5

variances = $\sigma^2 = 4.5$

Standard Dev. = $\sigma = \sqrt{4.5}$

$= 2.12$

$$E(x) = \Sigma x\, P(x) = \frac{\Sigma x\, P(x)}{\Sigma P(x)} \rightarrow 1$$

5.4 The Binomial Probability Distribution
التوزيع ذو الحدين
Discrete Probability Distribution

- The Binomial experiment is an experiment satisfies the following four properties
التجربة ذات الحدين
لديها 4 شروط

1/ The experiment Consists of a seq. of n identical trials
لـ يعني التجارب يجب ان تكون متطابقة
مثلا: رمي/ضرب السهم من مسافة معينة
نفس/ضرب السهم من مسافة معينة
وهكذا تكون التجارب "identical" أي اجراء التجارب بطريقة متطابقة

2/ Two outcomes, success and failure are possible on each trial
لـ التجارب تحتمل الحصول على احد
يجب ان يكون
النجاح او الفشل ← بشكل فعال
نتيجتين فقط
لـ يعني لكل حالة نقول ان عدم النجاح يشمل الخارج الاخر

3/ The probability of success is P and the prob. of failure is 1-P do not change from trial to trial
Q

مرادفات
success ← يعني الفوز/الطرف
failure ← يعني الخسارة/الطرف

4/ all trial are independent
لـ جميع التجارب يجب ان تكون مستقلة

Example → ① Toss a Coin → Head or Tails

↳ assume that success is Head $\Big\} H = \frac{1}{2}$

② get the score of an exam
Fail or pass

The Binomial Distribution is used to obtain the probability of observing
يعني احتمال النجاح في عدة تجارب

X → success in a trial

↳ The probability function for X is give by

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

n = # of trials
X = # of successes
P = Probability of success
1-P = probability of failure

and some time
1-P = q

$\binom{n}{x}$ → التوافيق

✓ → X = 0, 1, 2, 3 ... n

X → ~~0~~ $0 \leq X \leq n$
يعني ممكن يبقى اكتر

X → Maximum = n

X → Minimum = 0

Example → 20% of BZU student are smokers, In a sample of 10 student

[1] what is the probability Distribution?

↳ Binomial

Identical / 3 التجارب مستقلة

success 0.2 / 4 بوجد نتيجتين فقط

Failure 1-0.2 = 0.8 / 2 =

[2] Is it discreate or Continoues?

↳ Discrete

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[3] what is the probability that in this sample 2 students are smokers?

x = # of smoker

p = 0.2 ← هاي دسبة السؤال

q = 0.8 (1-0.2)

n = 10

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(2) = \binom{10}{2} (0.2)^2 (0.8)^{10-2}$$

$\binom{10}{2}$ = هاي عن الة الكاسة ↓ $10C_2$

$$P(2) = \underline{0.3019}$$

4/ what is the probability that all of them are smoker?

$$P(10) = \binom{10}{10} * (0.2)^{10} * (0.8)^{10-10}$$

يعني بترك الفاصلة العشرية ٧ مراتب
حيث (-7) = عشرية سالبة / يوضع بالعكس

$$= 1.024 * 10$$

$$= 0.0000001024$$

5/ what is the probability that 6 students are non smoker

6 ⟹ nonsmoker
4 ⟹ smoker ← success

$$P(4) = \binom{10}{4} (0.2)^4 (0.8)^6 = 0.088$$

6/ what the probability that at least 8 student are smoker

at least 8 → 8, 9, 10 smokers

$$P(X \geq 8) = P(8) + P(9) + P(10)$$

$$= \binom{10}{8} (0.2)^8 (0.8)^2 + \binom{10}{9} (0.2)^9 (0.8)^1$$

$$+ \binom{10}{10} (0.2)^{10} (0.8)^0$$

$$0.0000073728 + 0.0000040960 + 0.0000001024$$

$$= 0.0000779784$$

7/ what is the probability that at most 9 students are smokers?

at most 9 → 9, 8, 7, 6, 5, 4, 3, 2, 1, 0

$P(x \leq 9) = P(0) + P(1) \ldots + P(9)$

لان هذه العملية طويلة نذهب لاستظام بالتمة

$1 - P(10)$ = كما بين نفسها

$P(A') = 1 - P(A)$

↳ $= 1 - \binom{10}{10} (0.2)^{10} (0.8)^{0}$

$= 1 - 1.024 \times 10^{-7} = 0.9999999897$

8/ what is the expected number of smoker in this sample?

Expected number / value of X ← قانون مهم

$E(x) = n_x p$

$E(x) = (0.2)(10) = 8$

9/ what is the expected number of nonsmokers in this sample ?

8

10/ what is the variances of this distribution

variances = $V(x) = np(1-p)$ ← قانون حساب تباين توزيعات ذات الحدين Binomial

$= (10)(0.2)(0.8)$

$= 1.6$

11/ what is the standard Deviation ?

standard Deviation = $\sigma = \sqrt{np(1-p)}$

$= \sqrt{1.6}$

$= 1.264$

$B(n/p)$ توزيعات ذات الحدين

$B(10/0.2) \rightsquigarrow n = 10 \quad p = 0.2$

B → Binomial