8.4 Interval Estimation for population Proportion

* The population and sample proportions, denoted by \mathbf{p} and $\overline{\mathbf{p}}$, respectively, are calculated as

$$p = \frac{x}{N} \qquad \text{and} \qquad \overline{p} = \frac{x}{n}$$

N = the population size

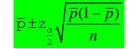
n = the sample size

 \mathbf{x} = the number of elements in the population or sample that

- processes a specific characteristic.
- * \overline{p} is a point estimate for p.

Confidence Interval for the Population Proportion

The $(100 - \alpha)100\%$ confidence interval for the population proportion p is given by:



Example

A simple random sample of 400 individuals provides 100 Yes responses.

- 1. What is the point estimate of the proportion of the population that would provide **Yes** responses?
- **2.** What is your estimate of the standard error of the proportion?

3. Compute the 95% confidence interval for the population proportion.

2

Example

A simple random sample of 800 elements generates a sample proportion p = 0.70.

1. Provide a 90% confidence interval for the population proportion.

2. Provide a 95% confidence interval for the population proportion.

Example

A company is planning to market a new offer. However, before marketing this offer, the company wants to find what percentage of customers will like it. The company's research department select a random of 500 customers and asked them to evaluate this offer. Of these 500 customers, 290 said they liked it. Find with a 99% confidence level, what percentage of customers will like this offer? **Solution**

$$\overline{\mathbf{p}} = \frac{\mathbf{x}}{\mathbf{n}} = \frac{290}{500} = 0.58$$
$$\overline{\mathbf{p}} \pm \mathbf{z} \sqrt{\frac{\overline{\mathbf{p}}(1 - \overline{\mathbf{p}})}{\mathbf{n}}}$$
$$\Rightarrow 0.58 \pm (2.575)(0.0221)$$
$$\Rightarrow 0.58 \pm .057$$
$$\Rightarrow 0.523 \text{ to } 0.637$$

Thus, we can state with 99% confidence that 52.3% to 63.7% all customers will like this offer.

SAMPLE SIZE (Interval estimation for population proportion)

SAMPLE SIZE FOR AN INTERVAL ESTIMATE OF A POPULATION PROPORTION

$$n = \frac{(z_{\alpha/2})^2 p^* (1 - p^*)}{E^2}$$
(8.7)

In practice, the planning value p* can be chosen by one of the following procedures.

- 1. Use the sample proportion from a previous sample of the same or similar units.
- Use a pilot study to select a preliminary sample. The sample proportion from this sample can be used as the planning value, p*.
- Use judgment or a "best guess" for the value of p*.
- 4. If none of the preceding alternatives apply, use a planning value of $p^* = .50$.

Example

In a survey, the planning value for the population proportion is $p^* = 0.35$. How large a sample should be taken to provide a 95% confidence interval with a margin of error of 0.05?

Example

At 95% confidence, how large a sample should be taken to obtain a margin of error of 0.03 for the estimation of a population proportion? Assume that past data are not available for developing a planning value for p*.

4

Hypothesis testing about the population proportion p:

Let p_0 be a given value for p in H_0 . Let α be a given level of significant.

Critical Value(s) Approach

Hypothesis	Statistic Test	Rejection Rule	Critical value(s) $\alpha = 0.05$
1.	$z = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$	Reject H ₀ if $z > z_{\frac{\alpha}{2}}$ or $z < -z_{\frac{\alpha}{2}}$ <i>two tailed test</i>	$\frac{z_{\alpha}}{z_{\alpha}^{2}} = \pm 1.96$
2.	$z = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$	Reject H_0 if $z < -z_{\alpha}$ <i>left tailed test</i> (one tailed test)	$-z_{\alpha} = -1.645$
3. $H_0: p \le p_0$ $H_a: p > p_0$	$z = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$	Reject H_0 if $z > z_{\alpha}$ <i>right tailed test</i> (one tailed)	$z_{\alpha} = 1.645$

 $\mathbf{P} - \mathbf{value approach}$ Reject H_0 if $p - value < \alpha$

Example

According to a survey, 20 % of all school teachers have a second job to supplement their incomes. A random sample of 400 teachers taken this year showed that 100 of them hold a second job. Testing at 1% significance level, can you conclude that the current percentage of all teachers who hold a second job to supplement their income is higher than 20%? Explain your conclusion

The sample proportion

 $\overline{p} = \frac{x}{n} = \frac{100}{400} = 0.25$ **HYPOTHESIS**

 $H_0: p \le 20\%$ $H_a: p > 20\%$

Test Statistics

 $z = \frac{0.25 - 0.2}{0.02} = 2.5$

CONCLUSION

p-value = P(z > 2.5)= 1-0.9938 = 0.0062

Since p -value < 0.01, reject H₀

Yes, the current percentage is greater than 20%.

Example

A Birzeit University professor claims that 75% or more of students attend his online lectures. A student council is suspicious of the claim and thinks that the proportion is lower than 75%. A random sample of 120 students show that only 85 students have ever done so. Is there enough evidence to show that the true proportion is lower than 75%? Conduct the test at 1% significance level.