8.4 Interval Estimation for population Proportion

- The population and sample proportions, denoted by **p** and $\bar{p}$, respectively, are calculated as
- $$p = \frac{x}{N} \quad \text{and} \quad \bar{p} = \frac{x}{n}$$

N = the population size
n = the sample size
x = the number of elements in the population or sample that processes a specific characteristic.

- $\bar{p}$ is a point estimate for p.

❖ Confidence Interval for the Population Proportion

The $(100-\alpha)100\%$ confidence interval for the population proportion p is given by:

$$\overline{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Example

A simple random sample of 400 individuals provides 100 Yes responses.

1. What is the point estimate of the proportion of the population that would provide **Yes** responses?

2. What is your estimate of the standard error of the proportion?

3. Compute the 95% confidence interval for the population proportion.

Example

A simple random sample of 800 elements generates a sample proportion p = 0.70.

1. Provide a 90% confidence interval for the population proportion.

2. Provide a 95% confidence interval for the population proportion.

Example

A company is planning to market a new offer. However, before marketing this offer, the company wants to find what percentage of customers will like it. The company's research department select a random of 500 customers and asked them to evaluate this offer. Of these 500 customers, 290 said they liked it. Find with a 99% confidence level, what percentage of customers will like this offer?

Solution

$$\bar{p} = \frac{x}{n} = \frac{290}{500} = 0.58$$

$$\bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\Rightarrow 0.58 \pm (2.575)(0.0221)$$

$$\Rightarrow 0.58 \pm .057$$

$$\Rightarrow 0.523 \textbf{ to } 0.637$$

Thus, we can state with 99% confidence that 52.3% to 63.7% all customers will like this offer.

SAMPLE SIZE (Interval estimation for population proportion)

SAMPLE SIZE FOR AN INTERVAL ESTIMATE OF A POPULATION PROPORTION

$$n = \frac{(z_{\alpha/2})^2 p^*(1 - p^*)}{E^2} \quad \textbf{(8.7)}$$

In practice, the planning value p^* can be chosen by one of the following procedures.

1. Use the sample proportion from a previous sample of the same or similar units.
2. Use a pilot study to select a preliminary sample. The sample proportion from this sample can be used as the planning value, p^*.
3. Use judgment or a "best guess" for the value of p^*.
4. If none of the preceding alternatives apply, use a planning value of $p^* = .50$.

Example

In a survey, the planning value for the population proportion is p* = 0.35. How large a sample should be taken to provide a 95% confidence interval with a margin of error of 0.05?

Example

At 95% confidence, how large a sample should be taken to obtain a margin of error of 0.03 for the estimation of a population proportion? Assume that past data are not available for developing a planning value for p*.

Hypothesis testing about the population proportion p:

Let p_0 be a given value for p in H_0. Let α be a given level of significant.

➢ Critical Value(s) Approach

Hypothesis	Statistic Test	Rejection Rule	Critical value(s) $\alpha = 0.05$
1. $H_0 : p = p_0$ $H_a : p \neq p_0$	$z=\frac{\bar{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	Reject H_0 if $z > z_{\frac{\alpha}{2}}$ or $z < -z_{\frac{\alpha}{2}}$ ***two tailed test***	$z_{\frac{\alpha}{2}} = \pm 1.96$
2. $H_0 : p \geq p_0$ $H_a : p < p_0$	$z=\frac{\bar{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	Reject H_0 if $z < -z_{\alpha}$ ***left tailed test*** (one tailed test)	$-z_{\alpha} = -1.645$
3. $H_0 : p \leq p_0$ $H_a : p > p_0$	$z=\frac{\bar{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	Reject H_0 if $z > z_{\alpha}$ ***right tailed test*** (one tailed)	$z_{\alpha} = 1.645$

➢ p – value approach

Reject H_0 if p – value < α

Example

According to a survey, 20 % of all school teachers have a second job to supplement their incomes. A random sample of 400 teachers taken this year showed that 100 of them hold a second job. Testing at 1% significance level, can you conclude that the current percentage of all teachers who hold a second job to supplement their income is higher than 20%? Explain your conclusion

The sample proportion

$$\bar{p} = \frac{x}{n} = \frac{100}{400} = 0.25$$

HYPOTHESIS

$H_0 : p \le 20\%$

$H_a : p > 20\%$

Test Statistics

$$z = \frac{0.25 - 0.2}{0.02} = 2.5$$

CONCLUSION

$p - \text{value} = P(z > 2.5)$

$= 1-0.9938$

$= 0.0062$

Since p -value < 0.01, reject H_0

Yes, the current percentage is greater than 20%.

Example

A Birzeit University professor claims that 75% or more of students attend his on-line lectures. A student council is suspicious of the claim and thinks that the proportion is lower than 75%. A random sample of 120 students show that only 85 students have ever done so. Is there enough evidence to show that the true proportion is lower than 75%? Conduct the test at 1% significance level.