CHAPTER 4

INTRODUCTION TO PROBABILITY

4.1 Experiments, Outcomes and Sample Space

- ✓ Probability is a numerical measures of the l**ikelihood** (chance) that an (uncertain) event will occur.

- ✓ A probability is a value between zero and one, inclusive, describing the relative possibility (chance or likelihood) an event will occur.

- ✓ An **experiment** is a **process** that, when performed, results in one and only one of many observations. These observations are called the **outcomes** of the experiment.

- ✓ The **collection** (**set**) of all outcomes for an experiment is called a **sample space**. It is denoted by **S.**

Example 1

- ✓ **Toss a coin**
  S = {Head, Tail}
  = {H, T}.

- ✓ **Roll a die**
  S= {1, 2, …, 6}.

- ✓ **Play a football game**
  S = {win, lose, tie}.

- ✓ **Select a part from production line for inspection.**
  S = {defective, non-defective}

✓ **One-step experiment**

Write the sample space of a random experiment by listing all possible outcomes of the experiment.

If $S = \{s_1, s_2, \ldots, s_n\}$ is a sample space of a random experiment, count the number of sample points $s_1, s_2, \ldots, s_n$ to find #(S) = n.

✓ **Multi-steps experiment.**

To find the number of outcomes of a random experiment with at least two steps, we need to introduce counting rules: multiplication rule, combination and permutation.

✓ **Multiplication Rule**

If an experiment consists of k steps, and if the first step can result in n_1 outcomes, the second step in n_2 outcomes, and the last step in n_k outcomes, then Total outcomes for the experiment

$$n_1 \times n_2 \times \ldots \times n_k$$

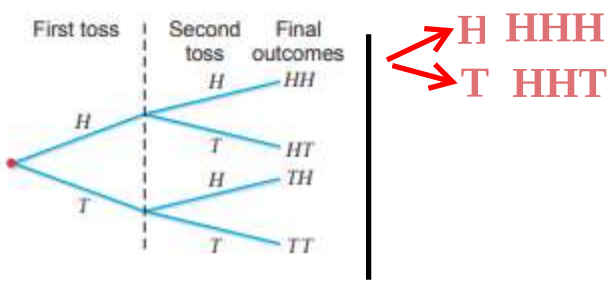
Example 2

Suppose we toss a coin **two times**. This experiment has two steps: the first toss, and the second toss. Each step has two outcomes: a head and a tail.

Total outcomes: (2) (2) = 4

Using the tree diagram the sample space for this experiment is

S = {HH, HT, TH, TT}



Suppose we toss a coin **three times**. This experiment has three steps: the first toss, the second toss, and the third toss. Each step has two outcomes: a head and a tail.

Total outcomes: (2) (2) (2) = 8

Using the sample space for this experiment is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT}

Example 3

Rolling two dice

#

of outcomes = (6) (6) = 36 ordered pairs

S = {(1, 1), (1, 2), … , (6,6)}

Example 4

Suppose we randomly select two workers from a company and observe whether the worker selected each time is a male M or a female F. Write all the outcomes for this experiment.

Number of outcomes = (2) (2) = 4

S = {MM. MF, FM, FF}.

Example 5

A local telephone number is given by a sequence of 7 digits. How many telephone numbers are there?

1. If there is no restrictions.

   We have 7 places (digits) each can be filled by 10 numbers {0, 1 … 9} . So, number of outcomes is

   10.10.10.10.10.10.10 = 10000000

2. If the first and the last digits can’t be zero?

We have 7 places (digits) each can be filled by 10 numbers {0, 1 … 9} except the first and last digit (only nine). So, number of outcomes is

9.10.10.10.10.10.9 = 8100000

The factorial of n: n!

$n! = (n)\times(n-1)(\times n-2).....\times 3\times 2\times 1$

5! = (5) (4) (3) (2) (1) = 60

CALCULATOR : Shift x^{-1}

The Combination

If the order of the selected objects is **not important**, any selection is called a combination. The formula to count the number of r object combinations from a set of n objects is:

$$nCr = \frac{n!}{(n-r)!r!} \quad r \le n$$

Example 6

12C 3 = 12! / (9!)(3!) = (12) (11) (10) / (3) (2) (1) = 220.

CALCULATOR **nCr**

Example 7

In a group of 5 students, 3 are to be selected at random. Find all possible selections. Assume that the five students are A, B, C, D, and E. list all possible selections

Number of selections 5C2 = 10

Selections:

ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, and CDE

PERMUTATION

Any arrangement of r objects selected from a group of n possible objects. The formula to count the total number of different permutations is:

$$n\,Pr = \frac{n!}{(n-r)!} \quad r \le n$$

Example 8

9 P 6 = 9! / 6! = 504

CALCULATOR SHIFT **nCr**

Example 9

In how many ways can you arrange 4 books on a shelve with 10 spaces?
10 P 4 = 5040

Calculating Probability

Let $S = \{s_1, s_2, \ldots, s_n\}$ be a sample space for a random experiment.

- An **event** is a collection of one or more of the outcomes of an experiment.

- An event that includes one and only one simple point is called a **simple event**.

- **Compound** event is a collection of more than one outcome for an experiment.

- **Probability** is numerical measure of the likelihood that an (uncertain) event, will occur and is denoted by P.

- The probability that an event A will occur is denoted by **P (A).**

Two assumptions

- The probability of an event E always lies in the range 0 to 1.
  $0 \le p(E) \le 1$
- The sum of the probabilities of all simple events (or final outcomes) is 1
  $\sum p(E_i) = p(E_1) + p(E_2) + \ldots + p(E_n) = 1$

Three Approaches (Methods) to assign Probability for events

(1) Classical probability method,

(2) The relative frequency method

(3) The subjective probability method

Classical Probability Method

- ❖ In many experiments, various outcomes for an experiment may have the same probability of occurrence, that is, are **equally likely** outcomes.
- ❖ The classical probability rule is applied to compute the probabilities of events for an experiment for which all outcomes are equally likely.
- ❖ Two or more outcomes (or events) that have the same probability of occurrence are said to be **equally likely** outcomes (or events).
- ❖ According to the classical probability rule, the probability of a simple event is equal to 1 divided by the total number of outcomes for the experiment.
- ❖ The sum of the probabilities of all final outcomes for an experiment is 1.
- ❖ The probability of a compound event A is equal to the number of outcomes of that event divided by the total number of outcomes for the experiment.

$$S = \{E_1, E_2, \ldots, E_n\}, \quad \text{classical} \Rightarrow P(E_1) = P(E_2) = \ldots = P(E_n) = \frac{1}{n}$$

If E event is an event of S, then

P **(E)** = # of elements of E / # number of elements of S

EXAMPLE 10

Find the probability of obtaining a head and the probability of obtaining a tail for one toss of a coin.

The two outcomes, head and tail, are equally likely outcomes. Therefore, P (H) = P (T) = 1/2

EXAMPLE 11

Find the probability of obtaining an even number in one roll of a die.

S = {1, 2, 3, 4, 5, 6}

All outcomes are equally likely

P (1) =P (2) = …. = p (6) = 1/6

E = even = {2, 4, 6}, p (E) = 3/6 = 1/2

Relative Frequency Method

If an experiment is repeated **n** times and an event E is observed **f** times, then, according to the relative frequency concept of probability,

P (E) = the relative frequency of E = **f / n**

Example 12

In a sample of 180 students, 30 are math major (M), 80 are accounting major (A), 50 are history major (H), and 20 are business major (B).

P (Math) = relative frequency of math = 30/180 = 1/6

P (Accounting) = 80 / 180 =4 /9

P (History) = 20 / 180 =1 /9

P (Business) = 20/280 = 1/14

Subjective Probability Method

Many times we face experiments that neither have equally likely outcomes nor can be repeated to generate data. In such cases, we cannot compute the probabilities of events using the classical probability rule or relative frequency approach.

Subjective probability is the probability assigned to an event based on subjective judgment, experience, information, and belief.