CHAPTER 9
Hypothesis Testing

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Definitions and concepts

Hypothesis: A **statement** about a population parameter for the purpose of testing.

Hypothesis Testing:

- A procedure based on sample evidence and probability theory to determine whether the hypothesis is reasonable statement.
- **Hypothesis testing** can be used to determine whether a statement about the value of a population parameter should or should not be **rejected.**

Two types of hypothesis

- **Null Hypothesis (**H_0**)**: A statement (assumption /a claim) about the value of a population parameter that is assumed to be true until it is declared false.
- **Alternative Hypothesis (**H_a**)**: A statement that is accepted if the sample data provide enough evidence that the null hypothesis is false. The alternative hypothesis is called the researcher hypothesis

Steps of Hypothesis Testing

1. State null and alternative hypothesis

The null and alternative hypotheses are competing statements about the population. Either H_0 is true or H_a is true, but not both.

2. Select level of significance

Level of significance: The probability of rejecting the null hypothesis when it is true.

Since hypothesis testing based on sample results, we must allow for the probability of errors. The two types of errors that can be made in hypothesis testing are illustrated in the following table:

		Researcher	
		Do not reject H_0 (Accept)	Reject H_0
Conclusion about H_0	H_0 is True	Correct Conclusion	**Type I Error**
	H_0 is False	**Type II Error**	Correct Conclusion

Type I Error:

- A *Type I error* occurs when a true null hypothesis is rejected.
  Type I error: Rejecting the null hypothesis when it is true

  Level of significance = P (Type I error) = α

- The *most frequently used* significance levels are 0.01, 0.05, and 0.1, but any value between 0 and 1 is possible.

Type II Error:

- A *Type II error* occurs when a false null hypothesis is not rejected (accepted).
  Type II error: Accepting the null hypothesis when it is false

- The probability of committing this type of error is denoted by β
  P (Type II error) = β

- The power of the test = 1 – β

3. Identify the test statistic.

Statistic Test: A value, determined from sample information, used to determine whether to reject the null hypothesis.
In hypothesis testing about the mean μ for σ **known case**, the test statistic is computed by:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

μ_0 is the given value for the population mean (in H_0)
If σ **is unknown**, the test statistic is computed by:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

4. Formulate a decision rule.

One -Tailed and Two Tailed Tests of Significant

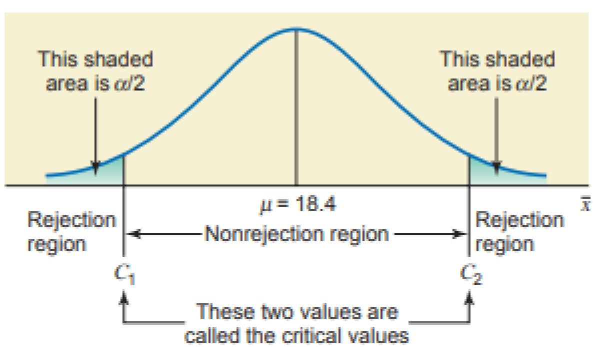
Let μ_0 be a given value for the population mean (in H_0). Let α be a given level of significant.

***Critical value(s) approach:** (σ known case)

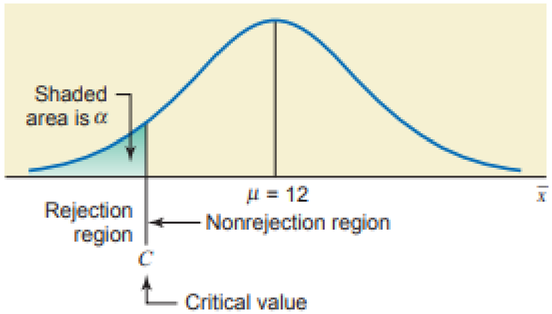
Hypothesis	**Statistic Test**	**Rejection Rule**	**Critical value(s)** $\alpha = 0.05$
1. $H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	Reject H_0 if $z > z_{\frac{\alpha}{2}}$ or $z < -z_{\frac{\alpha}{2}}$ ***two tailed test***	$z_{\frac{\alpha}{2}} = \pm 1.96$
2. $H_0 : \mu \geq \mu_0$ $H_a : \mu < \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	Reject H_0 if $z < -z_{\alpha}$ ***left tailed test*** (one tailed test)	$-z_{\alpha} =$ -1.645
3. $H_0 : \mu \leq \mu_0$ $H_a : \mu > \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	Reject H_0 if $z > z_{\alpha}$ ***right tailed test*** (one tailed)	$z_{\alpha} = 1.645$

Case 1 is called ***two tailed test***

A two-tailed test.

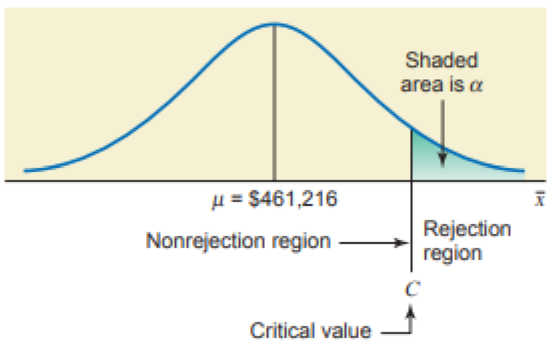


Case 2 is called ***left tailed test*** (one tailed test)



Case 3 is called ***right tailed test*** (one tailed)

right-tailed test.



The values z_{α} ($-z_{\alpha}$) or $z_{\frac{\alpha}{2}}$ ($-z_{\frac{\alpha}{2}}$) are called **critical values** (The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected).

***Critical value(s) approach:** (σ unknown case)

TABLE 9.3 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ UNKNOWN CASE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \ge \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \le \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \ne \mu_0$
Test Statistic	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\le \alpha$	Reject H_0 if p-value $\le \alpha$	Reject H_0 if p-value $\le \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $t \le -t_\alpha$	Reject H_0 if $t \ge t_\alpha$	Reject H_0 if $t \le -t_{\alpha/2}$ or if $t \ge t_{\alpha/2}$

5. Take a sample, arrive at a decision

The final step in hypothesis testing is computing the test statistic, comparing it to the critical value, and making a decision to **reject** or **do not reject** the null hypothesis.

*The p- value approach

p-VALUE

A *p*-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample. Smaller *p*-values indicate more evidence against H_0.

One tailed (left): P (Z < z)
One tailed (right): P (Z > z)
z = test statistic
Two tailed = 2 p (Z > |z|)
Rejection rule: Reject H_0 if the p-value $< \alpha$

1. Determine the null and alternative hypotheses.
2. Select the level of significance α.

(Critical value(s) approach)

3. Use α to determine the critical value(s) for the test statistic and state the rejection rule for H_0.
4. Collect the sample data and compute the value of the test statistic.
5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0.

OR Using the *p*-Value

6. Collect the sample data and compute the value of the test statistic.
7. Use the value of the test statistic to compute the *p*-value. (Reject H_0 if *p*-value $< \alpha$.)

Examples

9. Consider the following hypothesis test:

$$H_0: \mu \geq 20$$
$$H_a: \mu < 20$$

A sample of 50 provided a sample mean of 19.4. The population standard deviation is 2.

a. Compute the value of the test statistic.
b. What is the *p*-value?
c. Using $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

σ is known, use z - test

a. $z = \dfrac{\bar{x} - \mu_0}{\sigma}\sqrt{n} = \dfrac{19.4 - 20}{2}\sqrt{50} = -2.12$

b. p – value = P (Z > 2.12) = 1 – P (Z < 2.12) = 1 – 0.9830 = 0. 017

c. p – value = 0.017 < $\alpha = 0.05 \Rightarrow$ Reject H_0

d. Reject H_0 if $z < -z_\alpha = -z_{0.05} = -1.645$

Since $z = -2.12 < -z_\alpha = -1.645$, reject H_0

11. Consider the following hypothesis test:

$$H_0: \mu = 15$$
$$H_a: \mu \neq 15$$

A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3.

a. Compute the value of the test statistic.
b. What is the *p*-value?
c. At $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

σ Known case, use z – test (two tailed test)

a. $z = \dfrac{\bar{x} - \mu_0}{\sigma}\sqrt{n} = \dfrac{14.15 - 15}{3}\sqrt{50} = -2.00$

b. p – value = 2P (Z > 2.00) = 2(1 – P (Z < 2.00)) = 2(1 – 0.9772) = 0. 0456

c. p – value = 0.0456 < $\alpha = 0.05 \Rightarrow$ Reject H_0

d. Reject H_0 if $z < -z_{\frac{\alpha}{2}} = -z_{0.025} = -1.96$ or $z > z_{\frac{\alpha}{2}} = 1.96$

Since $z = -2.00 < -z_{\frac{\alpha}{2}} = -1.96$, reject H_0

At $\alpha = 0.01$, what is your conclusion?
p – value = 0.0456 > $\alpha = 0.01 \Rightarrow$ Do Not Reject H_0

23. Consider the following hypothesis test:

$$H_0: \mu \leq 12$$
$$H_a: \mu > 12$$

A sample of 25 provided a sample mean $\bar{x} = 14$ and a sample standard deviation $s = 4.32$.

a. Compute the value of the test statistic.

b. Use the t distribution table (Table 2 in Appendix B) to compute a range for the p-value.

c. At $\alpha = .05$, what is your conclusion?

d. What is the rejection rule using the critical value? What is your conclusion?

σ unknown case, use t – test (upper tailed test)

a)

$$t = \frac{\bar{x} - \mu_0}{s}\sqrt{n} = \frac{14 - 12}{4.32}\sqrt{25} = 2.31$$

d)

Rejection rule: Reject H_0 if $t > t_\alpha$

$t_\alpha = t_{0.05} = 1.711$ with df $= 25 - 1 = 24$

Since $t = 2.31 > t_\alpha = 1.711$, reject H_0.

Using $\alpha = 0.01$, what is your conclusion?

Rejection rule: Reject H_0 if $t > t_\alpha$

$t_\alpha = t_{0.01} = 2.492$ with df $= 25 - 1 = 24$

Since $t = 2.31 < t_\alpha = 2.492$, do not reject (accept) H_0.

Example

1. You are given the following information obtained from a random sample of four observations.

   25 47 32 56

You want to determine whether the mean of the population from which this sample was taken is significantly different from 48. (Assume the population is normally distributed.)

a. State the null and the alternative hypotheses.

b. Determine the test statistic.

c. At the 5% level of significance. Determine whether the mean of the population is significantly different from 48.

σ unknown case, use t – test (two tailed test)

$\bar{x} = 40$

$s = 14.07$

a.

$$H_0: \mu = 48$$
$$H_a: \mu \neq 48$$

b. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{40 - 48}{14.07/4} = -1.14$

c. rejection rule: Reject H_0 if $t < -t_{\frac{\alpha}{2}}$ or $t > t_{\frac{\alpha}{2}}$

$t_{\frac{\alpha}{2}} = 3.182$ with df = 3 – 1 = 2

Conclusion: since $-t_{\frac{\alpha}{2}} = -3.182 < t = -114 < t_{\frac{\alpha}{2}} = 3.182$, do not reject H_0

That is, the mean population is not significantly different from 48.

Example

A restaurant company has a policy of opening new restaurants only in those areas that have a mean household income of at least \$35000 per year. The company is currently considering an area to open a new restaurant. The company's research department took a sample of 100 households from this area and found that the mean income of these households s \$33250. Assume that the population standard deviation is \$7400. Using the 1% significant level, would you conclude that the company should not open a restaurant in this area?

Solution

1. State the null and alternative hypothesis

   $H_o : \mu \geq 35000$

   $H_a : \mu < 35000$

2. Select a distribution to use.
   Since σ is known use the z- distribution
3. Determine the rejection and non-rejection regions. One tailed test (lower test):

Rejection rule: Reject H_0 if $z < -z_\alpha \Rightarrow$ reject H_0 if $z < -2.33$)

4. Calculate the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{33250 - 35000}{\frac{7400}{\sqrt{100}}} = -2.36$$

5. Make a decision

Since z = - 2.36 < z_α = -2.33 reject H_0

Conclusion: The Company should not open a restaurant in this area.

P – Value Approach

P value = p (z > 2.36) = 1 -.9909 = 0.0091