Chapter 9

14/35

14. Consider the following hypothesis test:

$$H_0: \mu = 22$$
$$H_a: \mu \neq 22$$

A sample of 75 is used and the population standard deviation is 10. Compute the *p*-value and state your conclusion for each of the following sample results. Use $\alpha = .01$.

a. $\bar{x} = 23$
b. $\bar{x} = 25.1$
c. $\bar{x} = 20$

$\sigma = 15 \Rightarrow \sigma$ Known case $\Rightarrow z - \text{test}: z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

a. $z_{23} = 0.87$

p –value = 2 P (Z > 0.87) = 2(1 - .8078) =0.3844

p –value = 0.3844 > α = 0.01. Do not reject H_0

b. $z_{25.1} = 2.68$

p –value = 2 P (Z > 2.68) = 2(1 - .9963) = 0.0074

p –value = 0.0074 < α = 0.01. Reject H_0

c. $z_{20} = -1.73$

p –value = 2 P (Z < -1.73) = 2(1 - 0.9582) = 0.0836

p –value = 0.0836 > α = 0.01. Do not reject H_0

Past Exams

(Questions 11 and 12) Suppose that the cumulative averages of all BZU students are normally distributed with mean of 76 and standard deviation of 8.

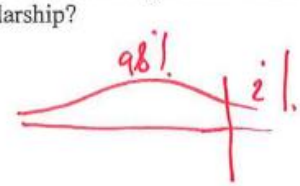
11. Find the percentage of students with averages between 68 and 86.

$$P(68 < X < 86) = P(-1 < Z < 1.25)$$

$$= P(Z < 1.25) + P(Z < 1) - 1$$

$$= 0.8944 + 0.8413 - 1 = \boxed{0.7357}$$

12. The highest 2% of averages will be given a scholarship to continue their graduate studies. What is the minimum average that will be considered for a scholarship?



$$0.98 \Rightarrow z = 2.05$$

$$2.05 = \frac{x - 76}{8}$$

$$x = (2.05)(8) + 76$$

$$= \boxed{92.4}$$

11. The z value for a 97.8% confidence interval estimation is

$0.989 \Rightarrow z = 2.29$

0.011 | 0.978 | 0.011

The manager of a grocery store has taken a random sample of 100 customers. The average length of time it took these 100 customers to check out was 3.0 minutes. It is known that the standard deviation of the population of checkout times is one minute. **Answer questions 12 – 14**

12. The standard error of the mean equals

$n = 100$
$\bar{x} = 3$
$\sigma = 1$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = \boxed{0.1}$$

13. With a .95 probability, the sample mean will provide a margin of error of

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = (1.96)(0.1) = \boxed{0.196}$$

14. The 95% confidence interval for the true average checkout time (in minutes) is

$$\bar{x} \pm E \Rightarrow 3 \pm 0.196$$

Interval $\boxed{2.804 \text{ to } 3.196}$

A simple random sample of five observations from a population containing 400 elements was taken, and the following values were obtained. **Answer questions 4 and 5**

12 18 19 20 21

4. A point estimate of the mean is
   a. 3.54
   b. 18
   c. 20
   d. 12.5
   e. 3.16
5. A point estimate of the population ~~standard~~ variance is
   a. 3.54
   b. 18
   c. 20
   d. 12.5
   e. 3.16
6. In order to determine an interval for the mean of a population with unknown standard deviation a sample of 62 items is selected. The mean of the sample is determined to be. The number of degrees of freedom for reading the t value is
   a. 23
   b. 24
   c. 60
   d. 61

$df = n - 1$

15. A random sample of 64 students at a university showed an average age of 25 years and a standard deviation of 2 years. The 99% confidence interval for the true average age of all students in the university is

$n=64, \quad \bar{x}=25, \quad s=2$

$\bar{x} \pm t_{\alpha/2}\, s/\sqrt{n} \qquad t_{0.005}=2.656 \quad df=63$

$25 \pm (2.656)\left(\frac{2}{8}\right) : 25 \pm 0.66 \; : \boxed{(24.34, 25.66)}$

16. The sample size needed to provide a margin of error of 2 or less with a .99 probability when the population standard deviation equals 11 is

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{(2.575)(11)}{2}\right)^2 = 200.57 \Rightarrow \boxed{201}$$

Question 14 (3 points). Consider the following hypothesis test:

$$H_0: \mu \leq 185$$
$$H_a: \mu > 185$$

A sample of size 49 provided a sample mean $\bar{x} = 187$ and a sample standard deviation $s = 7$. At $\alpha = 0.05$, What is your conclusion? Use the critical value approach.

$t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{187 - 185}{\frac{7}{\sqrt{49}}} = +2$

$df = n - 1 = 48$

critical value $= +t_\alpha = +t_{0.05} = +1.677$

$t > t_\alpha \Rightarrow$ Reject H_0 $(\alpha = 0.05)$

Question 15 (3 points). Consider the following hypothesis test:

$$H_0: \mu \geq 39$$
$$H_a: \mu < 39$$

A sample of size 36 provided a sample mean of 37. Assume a population standard deviation of $\sigma = 9$. At $\alpha = 0.10$, what is your conclusion? Use the p-value approach.

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{37 - 39}{\frac{9}{\sqrt{36}}} = -1.33$$

$$\text{p-value} = P(z < -1.33)$$
$$= 1 - 0.9082 = 0.0918$$

$$\text{p-value} = 0.0918 < \alpha = 0.10$$
$$\Rightarrow \text{Reject } H_0 \ (\alpha = 0.10)$$

Example:

$H_0: \mu = 12$

$H_a: \mu \neq 12$ $\qquad \alpha = 5\%$

Sample: $n = 18$, $\bar{x} = 12.4$, $S = 0.8$

$S = 0.8 \Rightarrow \sigma$ - Known case

test statistic $t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = 2.121$

$n = 18 \rightarrow df = 18 - 1 = 17$

Critical values $= \pm t_{\alpha/2} = \pm t_{0.025}$

$= 2.11$

* Critical values approach:

$t = 2.21 > t_{\alpha/2} = 2.11$

$\Rightarrow$ Reject H_0

* p-value approach

	0.1	0.05	0.025	0.01	0.005
$df = 17$	1.333	1.740	2.110	2.567	2.898

$t = 2.12$

$\Rightarrow 2(0.01) < p\text{-value} < (0.025)(2)$

$0.02 < p\text{-value} < 0.05$

$\Rightarrow p\text{-value} < \alpha = 0.05 \longrightarrow$ Reject H_0

* Confidence interval approach: Construct 95% C.I

$\bar{x} \pm t_{\alpha/2}\, S/\sqrt{n}$: $12.4 \pm 2.11\left(\frac{0.8}{\sqrt{18}}\right)$

: 12.4 ± 0.39

$\mu_0 = 12 \notin (12.01, 12.79) \Longrightarrow$ Reject H_0