

# Chapter 38 -

## Discriptive Statistics

### Numerical measures

**3.1**  $\Rightarrow$  measure of location

Example 8 -

Sample 1: 4, 7, 12, 11, 9

Sample 2: 18, 17, 20, 19, 21, 22

• Find the Sample mean  $\bar{x}$

Sample mean:  $\bar{x} = \frac{\sum X_i}{n}$

Sample  $\Rightarrow x_1, x_2, \dots, x_n$

Sample size  $\Rightarrow n$

$$\text{Sample mean} = \frac{4 + 7 + 12 + 11 + 9}{5} = \frac{43}{5}$$

(1)

$$\bar{x} = 8.6$$

$$\text{Sample mean} = \frac{18 + 17 + 20 + 19 + 21 + 22}{6}$$

(2)

$$\bar{x} = \frac{117}{6} \Rightarrow \bar{x} = 19.5$$

- Find the population means -
- population  $\Rightarrow X_1, X_2, \dots, X_n$   
 population size  $\Rightarrow n$   
population mean ( $\mu$ ) =  $\frac{\sum X_i}{N}$

$\mu(1) \approx 8.6$     $\mu(2) \approx 19.5$     $N$

\* Note

$\bar{X}$  is called Sample Statistics

$\mu$  is called population parameter

$\bar{X}$  is a point estimator for  $\mu$

إذا القيمة افترت كقر تقبل الـ population بصير معوج  
 Statistical Inference  $\leftarrow$  Max  $\bar{X}$

- Find the median:

median is the value in the middle after sort data

هي القيمة التي في الوسط بعد ترتيب البيانات  
 هي القيمة التي في الوسط بعد ترتيب البيانات

Solution:

Sample (1)  $\Rightarrow 4, 7, 9, 11, 12 \Rightarrow$  median  $\Rightarrow 9$   
 Sample (2)  $\Rightarrow 17, 18, 19, 20, 21, 22 \Rightarrow \frac{19+20}{2} = 19.5$

• طایفوں کی (median) قیمت پر جمع و تقسیم کے لیے:  
وہاں ایسا ترتیب دیا جائے گا، کہ اس کے بعد لاکھوں

— Sample 1

4, 7, 11, 12, 9

$$\text{mean} \Rightarrow \bar{X} = 8.6$$

$$\text{median} = 9$$

— Sample 2 \*

18, 19, 17, 20, 21, 22

$$\text{mean} \Rightarrow \bar{X} = 19.5$$

$$\text{median} = 19.5$$

— Sample 3

4, 7, 11, 12, 9, 10

$$\text{mean} \Rightarrow \bar{X} = \frac{4+7+11+12+9}{5} = \frac{53}{5} = 10.6$$

$$\text{median} \Rightarrow \text{ترتیب } 4, 7, 9, 11, 12, 20$$

$$\Rightarrow \frac{9+11}{2} = \frac{20}{2} = 10$$

— Sample 4

18, 19, 17, 20, 21, 22, 50

$$\text{mean} \Rightarrow \bar{X} = \frac{18+19+17+20+21+22+50}{7} = \frac{167}{7} = 23.86$$

$$\text{median} = 17, 18, 19, 20, 21, 22, 50 \Rightarrow 20$$

• ما عني ال mean اعني ان median  
 تكون انا هفت قيمه كبيره رفعت افترج (50)  
 انما ال median ما بتغير كثير لو ابنا نترج

\*Note =>

The mean is sensitive to extreme values  
 on the other hand the median is robust  
 to extreme values

mode :

the most frequent value (س) القيم المتكررات الأكثر

Sample 1 => No mode

Sample 2 => No mode

Sample 3 => No mode

Sample 4 => No mode

Sample 5 => 4, 7, 12, 11, 9, 7, 8, 9, 9  
 mode => 9 (uni-modal data)

Sample 6 => 4, 7, 11, 12, 9, 8, 7, 9, 9, 7  
 mode => 7, 9 (multi-modal data)

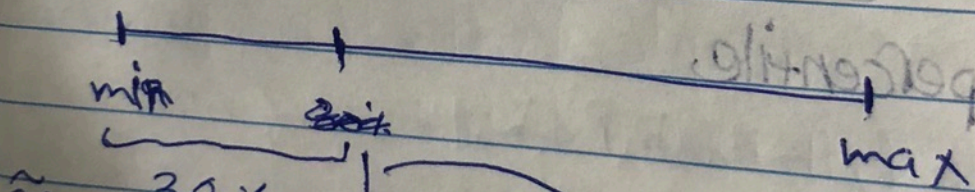
في  $mode$  ستوف اكثر رقم تكرر عندي اذا  
 كانت الرقم اكرر رقم واحد فسيكون (uni-model data)  
 اذا ادا كانت (mode) اكثر من رقم سيكون  
 (multi-model data)

\* median  
 \* mean  
 \* mode } measures of central location

Percentiles

min, ..., max (order)

Data Set (not ordered)  $\updownarrow$



30th percentile

(Roughly Speaking)

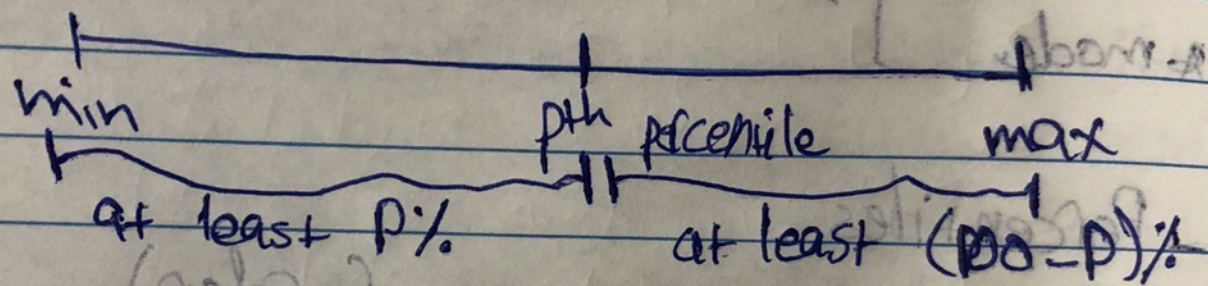
Sorted data  $\rightarrow$

min max

$P$ th percentile

$p^{\text{th}}$

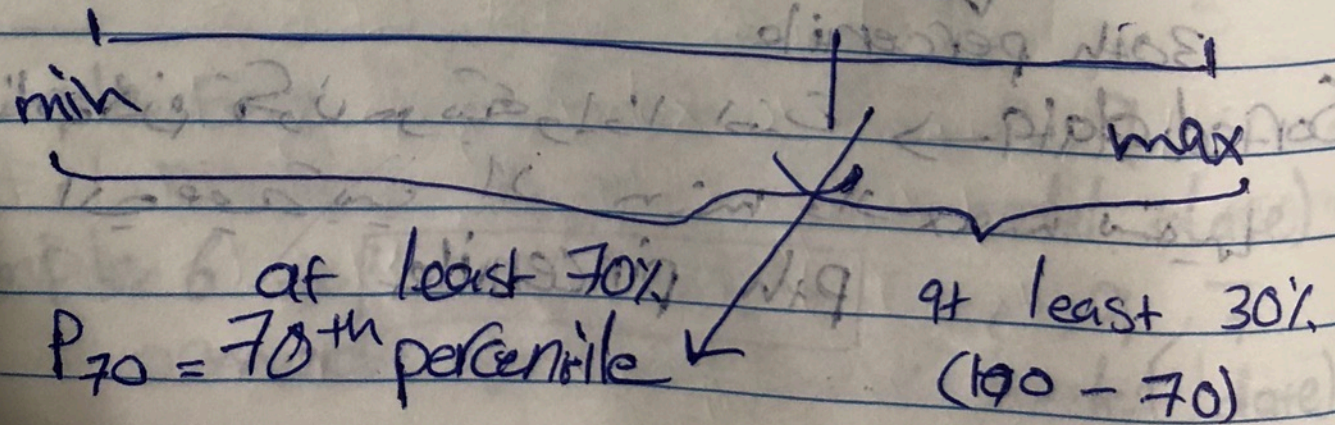
percentile  $\Rightarrow$  إذا أخذت العينة وكل أكبر منها  $P\%$  تكون  
 وإذا أخذت العينة التي أكبر منها  $(100 - P)\%$



$P_p = p^{\text{th}}$  percentile  $0 \leq P \leq 100$

Example

$P^{\text{th}}$  percentile  $\Rightarrow 70$  (According to the def...)



at least و at least  $\approx$  at least  $\approx$  at least

### Example 8-

Sample 1:

4, 7, 11, 12, 9  $\Rightarrow$  Find  $\downarrow$  85<sup>th</sup> percentile

Solution :-

Sorted data  $\Rightarrow$  4, 7, 9, 11, 12

$$4 \rightarrow \frac{1}{5} = 20\%$$

$$7 \rightarrow \frac{2}{5} = 40\%$$

$$9 \rightarrow \frac{3}{5} = 60\%$$

$$11 \rightarrow \frac{4}{5} = 80\%$$

$$12 \rightarrow \frac{5}{5} = 100\%$$

↑ الطريقة كانت حسب النسبة أو طبقاً لخطيب

~~الطريقة كانت حسب النسبة أو طبقاً لخطيب~~

① جوبت 2م بيانات

② نسبة الدول index

$$\text{index} \Rightarrow i = \frac{\text{النسبة المئوية لل سوال}}{100}$$

round up

$$= \frac{85}{100} \times 5 = 4.25$$

(take 5)

\* ال index وادنا طلع معي كسر باض لرقم  
 ايه بعدو دعيني هو عبارة عن location  
 مظهرين وبت موقع ارنه 85 اي طراليا معي  
 السؤال

$\Rightarrow P_{85} = 12$

طالبتني اخذ الرقم ايه بعد ال index من يطبع  
 كسر عاد الرقم بكون ترتيبه الرقم ايه بالجران  
 يعني 508 = 1 ← 4, 7, 9, 11, 12  
 الرقم ايه ترتيبه 5 هو 12  
 P 85 = 12

the same sample Find 40<sup>th</sup> percent

$\Rightarrow 4, 7, 9, 11, 12$

index =  $\frac{40}{100} \times 5$

$\Rightarrow 2 \rightarrow$  take 2 and 3

طالبتني معي ال index رقم صحيح باخذ  
 جاعاً الرقم ايه طلع معي واي بعدو  
 معي طلع 2 = 1 باض اننا  $P_{40} \leftarrow (2, 3)$   
 دعيني الارقام ايه ترتيبهم 2, 3  $\leftarrow (2, 7, 9)$



$$P_{40} = \frac{7+9}{2} = 8 = 2 \times \frac{0.2}{0.01} = 2 \times 20 = 40$$

average الـ  $\frac{0.2}{0.01}$  وبتبسطه

### \* $P^{\text{th}}$ percentile ( $P_p$ )

- Sort the data set from Smallest value to largest value.
- Find index:  $i = \frac{P}{100} \times n$ ,  $n$ : Sample size
- $i$ : not integer  $\Rightarrow P_p$  the data value in the position  $i$  after rounding up
- $i$ : integer  $\Rightarrow P_p$  the average of data value in positions  $i$  and next positions

Example 3

Sample 2: 18, 17, 19, 20, 21, 22

Find 70<sup>th</sup> percentile ( $P_{70}$ ), ( $P_{50}$ ) 50<sup>th</sup> perc-

70<sup>th</sup> percentile  $\Rightarrow$  17, 18, 19, 20, 21, 22

$$i = \frac{70}{100} \times 6 = 4.2 \quad (5)$$

$$P_{70} = 21$$

$$P_{50} = \frac{50}{100} \times 6 = 3$$

$$P_{40} = 19$$

$$P_{50} = \frac{19 + 20}{2} = 19.5$$

Sample 1

Sample 2

$$P_{85} = 12$$

$$P_{70} = 21$$

$$P_{40} = 80$$

$$P_{50} = 19.5$$

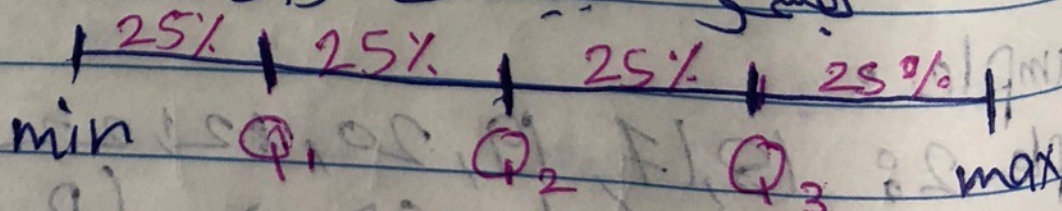
Quartiles of

1st Quartile:  $Q_1 = P_{25}$

2nd Quartile:  $Q_2 = P_{50}$  (median)

3rd Quartile:  $Q_3 = P_{75}$

تقسيم البيانات لارباع

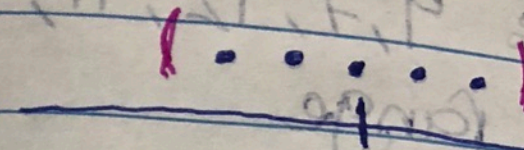


$$12 = 0.9$$

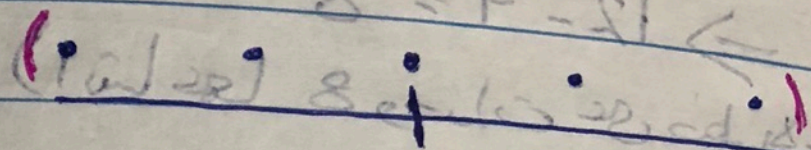
100

### 3.2 ⇒ measures of variability

Data Set 1 ⇒



Data Set 2 ⇒



• Data Set 2 has more variability than Data Set 1.

\* Range

\* IQR

\* Variance

\* Standard deviation

\* CV

measure of variability

• Range:

largest value - Smallest value

• IQR:

interquartile Range

$$IQR = Q_3 - Q_1$$

Example 2 -  $n = 5$

Sample 1: 4, 7, 12, 11, 9

Find the range

$$\Rightarrow 12 - 4 = 8$$

(المدى 8 وهو الفرق بين أكبر وأصغر)

Sample 3: 4, 7, 12, 11, 9, 20

Find the range

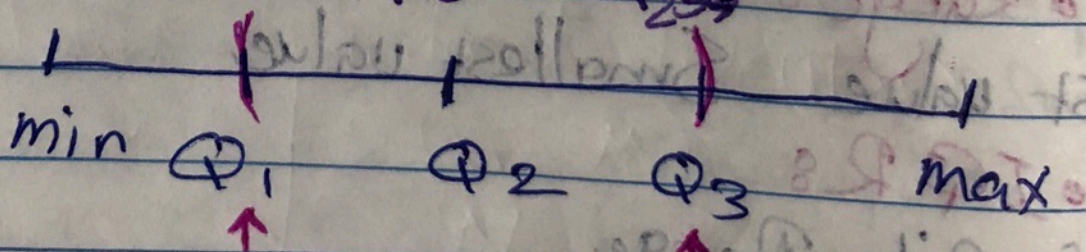
$$\Rightarrow 20 - 4 = 16$$

Note:

Range is sensitive to extreme values

المدى حساس للقيم المتطرفة (extreme values)

$$IQR = Q_{75} - Q_{25}$$



IQR

Sample 1: 4, 7, 11, 12, 9

- 4, 7, 9, 11, 12

$$- P_{25} = \frac{25}{100} \times 5 = 1.25 \Rightarrow 7$$

$$- P_{75} = \frac{75}{100} \times 5 = 3.75 \Rightarrow 11$$

$$- IQR = 11 - 7 = 4$$

$$IQR = 4$$

Sample 3: 4, 7, 11, 12, 9, 20

- 4, 7, 9, 11, 12, 20

$$- P_{25} = \frac{25}{100} \times 6 = 1.5 \Rightarrow 7$$

$$- P_{75} = \frac{75}{100} \times 6 = 4.5 \Rightarrow 12$$

$$- IQR = 12 - 7 = 5$$

⇒ • Sort Data

• Find  $P_{25}$

• Find  $P_{75}$

• Find IQR ( $P_{75} - P_{25}$ )

Sample 1

$$\text{Range} = 8$$

$$IQR = 4$$

Sample 3

$$\text{Range} = 16$$

$$IQR = 5$$

• Sample variance and Sample Standard deviation  
Data Set (sample):  $X_1, X_2, \dots, X_n$

Sample size:  $n$

$$\text{Sample variance} = S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$$

Sample standard deviation

$$S = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}}$$

• population variance and population standard deviation

Data Set (population):  $x_1, x_2, \dots, x_N$

Population size =  $N$

Population variance =

$$\sigma^2 = \frac{\sum (X_i - \mu)^2}{N}$$

Population standard deviation

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}}$$

## Notes

$S$ : Sample standard deviation  $\rightarrow$  Sample Statistics  
 $\sigma$ : population standard deviation  $\rightarrow$  population parameter

$\underline{S}$  is a point estimator of  $\underline{\sigma}$

$S^2$  is a point estimator of  $\sigma^2$



• CV (Coefficient of variation)

$$CV \Rightarrow \left( \frac{\text{Standard deviation}}{\text{mean}} \right) \times 100\%$$

• Example:

Sample 1: 4, 7, 9, 11, 12.  $R(8)$ ,  $IQR(4)$

Sample 3: 4, 7, 9, 11, 12, 20.  $R(16)$ ,  $IQR(9)$

Find variance, Stan. dev., CV for Sample 1.

$$\text{Variance} = \frac{\sum (x_i - \bar{x})^2}{n-1} \leftarrow S^2$$

$$\text{Stan. dev.} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \leftarrow S$$

$$CV = \frac{S}{\bar{x}} \times 100\%$$

$$\sum X \Rightarrow$$

Sample 1: 4, 7, 9, 11, 12

$$\bar{X} = \frac{4+7+9+11+12}{5} = \frac{43}{5} \Rightarrow 8.6$$

Sample variance  $\Rightarrow$

$$S^2 = \frac{(4-8.6)^2 + (7-8.6)^2 + (9-8.6)^2 + (11-8.6)^2 + (12-8.6)^2}{(5-1)} = \frac{41.2}{4}$$

$$= 10.3$$

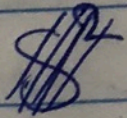
$$\text{St. dev.} = \sqrt{S^2} = \sqrt{10.3} \\ = 3.21$$

$$CV = \frac{S}{\bar{X}} \times 100\% = \frac{3.21}{8.6} \times 100\% \\ = 37.33\%$$



Sample 3: 4, 7, 9, 11, 12, 20

$$\bar{X} = \frac{4+7+9+11+12+20}{6} = 10.5$$



• على الألة كاسية

① ON → Mode 2

② الرقم الذي يدي / يا → (M+)

• جعل آد على الأرقام

ما أخيرهم كالم

$$\Rightarrow 4M+, 7M+, 9M+, 11M+, 12M+, 20M+$$

③ AC

④ Shift 2

( $\bar{X}$  = mean sample)

population varstion ← ( $\sigma^2$  = standered divirgation)

( $S^2$  = sample varstion)

$\bar{X} \Rightarrow$  Shif 2  $\Rightarrow$  1  $\Rightarrow$  سا وس

$$\bar{X} = 10.5$$

⑤ Shift + 2  $\rightarrow$  3 (sx)  $\rightarrow$   $\bar{x}$   
 Standard deviation

$$S = 5.47$$

⑥  $S^2$   $\rightarrow$   $\bar{x}$   $\rightarrow$   $\bar{x}^2$   $\rightarrow$   $\bar{x}^2$   
 اي طرح عندي  $(\bar{x}^2)$   $\rightarrow$   $\bar{x}^2$

$$S^2 = 29.9$$

$$CV = \frac{S}{\bar{x}} = \frac{5.47}{10.5} \times 100\% = 52.10\%$$

• Sample 1

median 9  
 Range 8  
 IQR 4  
 $\bar{x}$  8.6  
 S 3.21  
 $S^2$  10.3  
 CV 37.33%

• Sample 2

10  
 16  
 5  
 10.5  
 5.47  
 29.9  
 52.10%

Example :- population 1 استراتيا population 2

4, 7, 9, 11, 12 4, 7, 9, 11, 12, 20

median	9	10
Range	8	16
IQR	4	5
$\sigma^2$	8.24	24.92
$\sigma$	2.87	4.99
CV	33.37%	47.52%
$\mu$	8.6	10.5

Sample  $\leftarrow$  بدل نفس الكمية اي حلتها على ال Sample  
 Shift 2 3  $\leftarrow$  بدل 5  $\leftarrow$  5  
 Shift 2 2  $\leftarrow$

مقياس يتطلع حسي الى  $\sigma$  و  $\mu$   
 مربع مقياس اجيب الى  $\sigma^2$  و  $\mu^2$  و  $\sigma$  و  $\mu$

- ~~Median~~
- Median,  $\bar{X}$  /  $\mu$   $\leftarrow$  measure of location
  - $S$ ,  $S^2$ , CV /  $\sigma$ ,  $\sigma^2$ , CV  $\leftarrow$  measure of variability
  - Range, IQR

لازم اعتبار انهم Sample لا توفى الوال  
حاجتي Sample .

لذا ما حدد هو Sample او Population  
بفترة انو Sample .

NOTE :-

Sample 3 has mor variability than  
Sample 1 because  $CV_3 > CV_1$

طابدي أقرنا بين ال locations بسبب  $\bar{X}$

Done 😊

**3.3** ⇒ measures of disto. Shape, Relative location, and detecting outliers

Distribution Shape:-

\* Histogram

\* Compare mean vs. median

- mean > median ⇒ skewed to the right
- mean < median ⇒ skewed to the left
- mean = median ⇒ similar (symmetric)

(لازم نتعجبها) من داتا تجزیه

this doesn't work for whole cases

\* Skewness

$$= \frac{1}{N} \sum_{i=1}^N \frac{(x_i - \bar{x})^3}{s^3}$$

سواء من اليمين

- Skewness > 0 ⇒ positively skewed (right)
- Skewness = 0 ⇒ not skewed (similar)
- Skewness < 0 ⇒ negatively skewed (left)

من الطرف الأيسر تجزیه

Distribution Shape

Z-score :-

Z-score for a data value  $X$  is defined as :-  $Z = \frac{X - \text{mean}}{\text{St. dev. (s)}}$

sample  $\Rightarrow Z = \frac{X - \bar{X}}{\text{St. dev. (s)}}$

population  $\Rightarrow Z = \frac{X - \mu}{\text{St. dev. (s)}}$

Example :-

Taujiri grades

$$\mu = 78$$

$$\sigma = 11$$

• Someone Score 97 Find the ~~z~~ z-score

$$\Rightarrow Z = \frac{X - \mu}{\sigma} = \frac{97 - 78}{11} = 1.73$$

• Someone scored 65 Find the z score

$$\Rightarrow Z = \frac{65 - 78}{11} = -1.18$$

~~High school~~  
~~Some~~

High school exam. (out of 20)

$$\mu = 13 \quad \sigma = 1.5$$

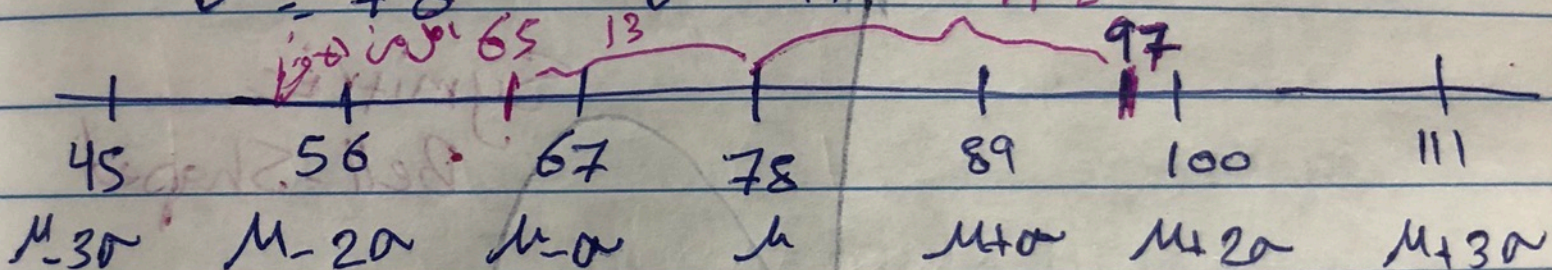
Some scored 17  
find Z score

$$Z \Rightarrow \frac{X - \mu}{\sigma} = \frac{17 - 13}{1.5} = 2.67$$

• Ex: 1  $\Rightarrow$

$$\mu = 78$$

$$\sigma = 11$$



•  $X = 97$

Z score = 1.73

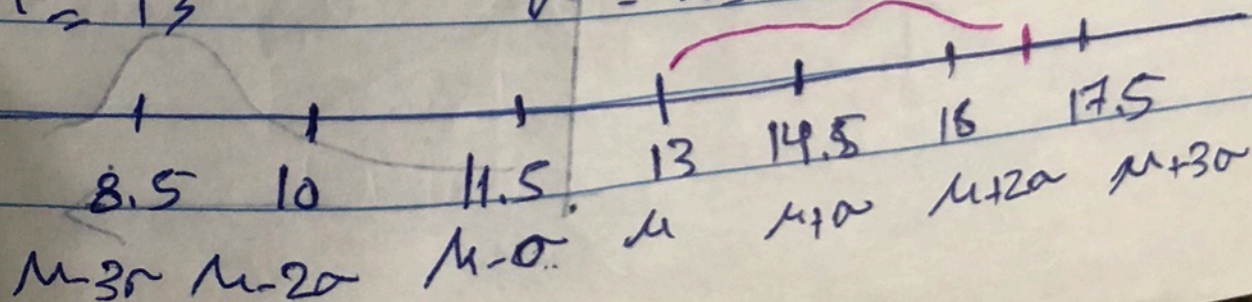
•  $X = 65$

Z score = -1.18

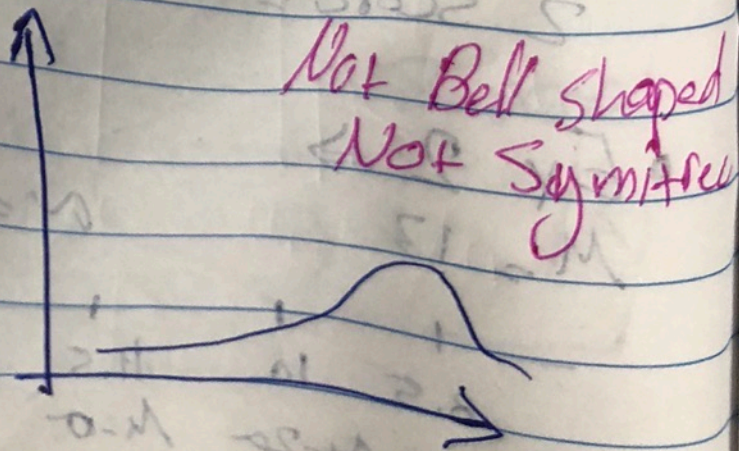
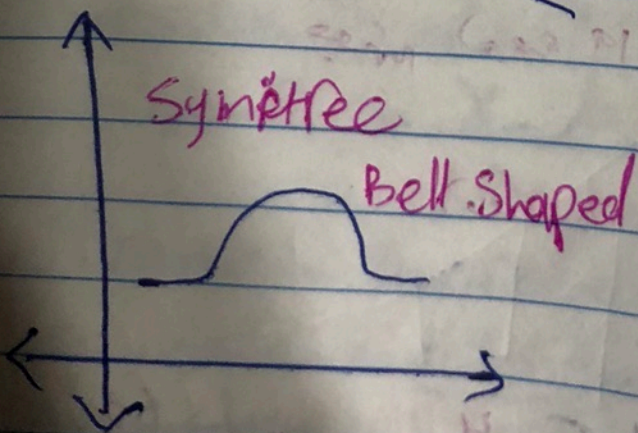
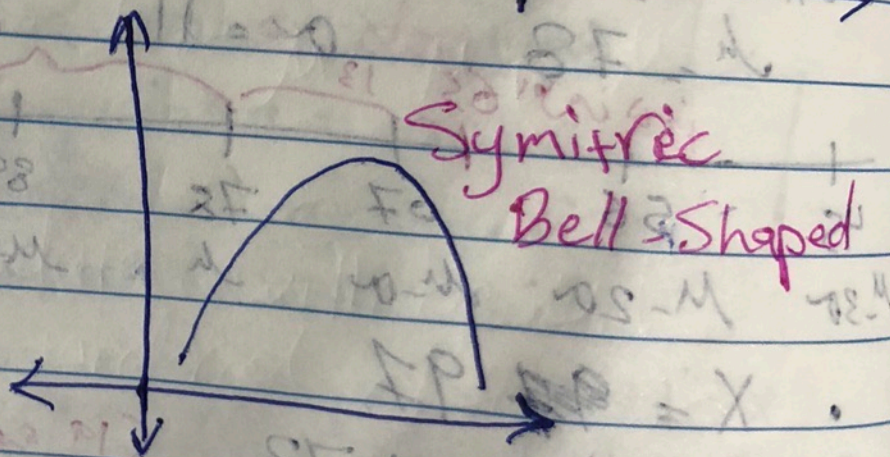
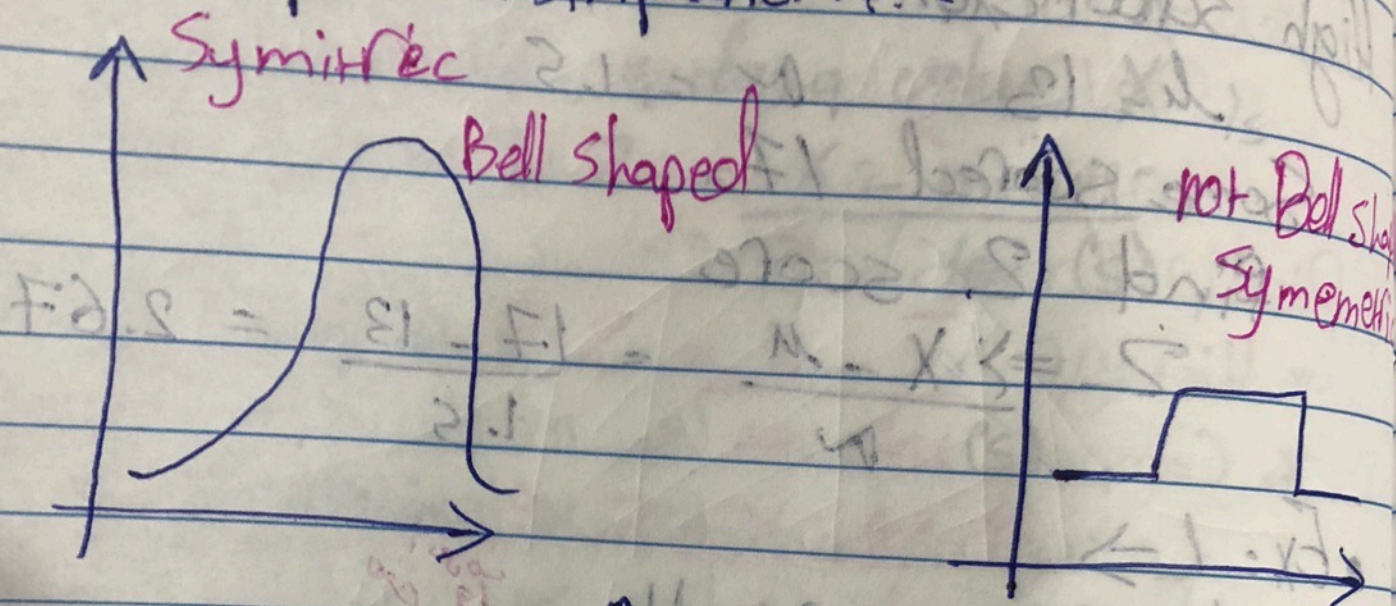
• Ex 2  $\Rightarrow$

$$\mu = 13$$

$$\sigma = 1.5$$

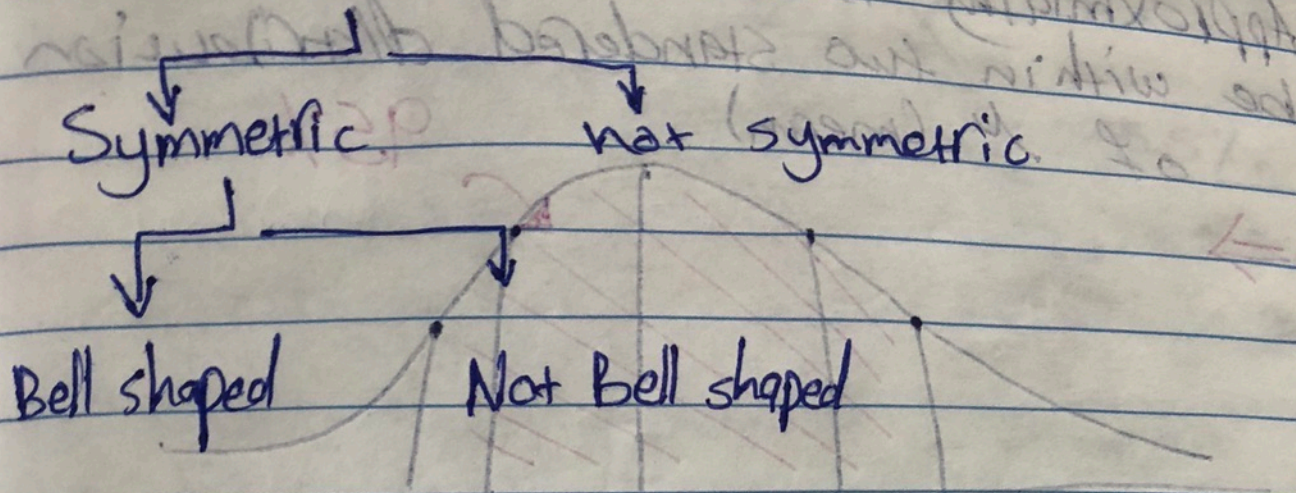


# Bell shaped distribution





# Distribution

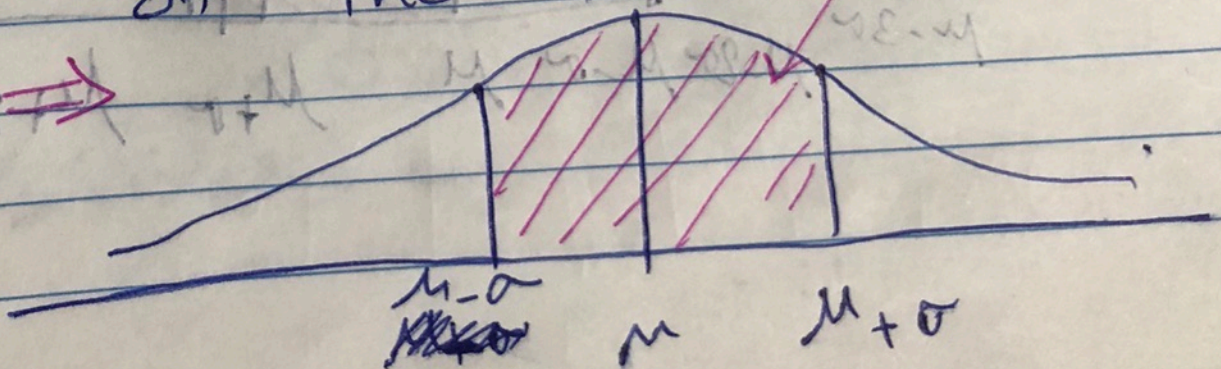


تسمى التوزيعات التي تكون على شكل Bell shaped أو على شكل الجرس  
تسمى التوزيعات التي لا تكون على شكل الجرس Not Bell Shaped

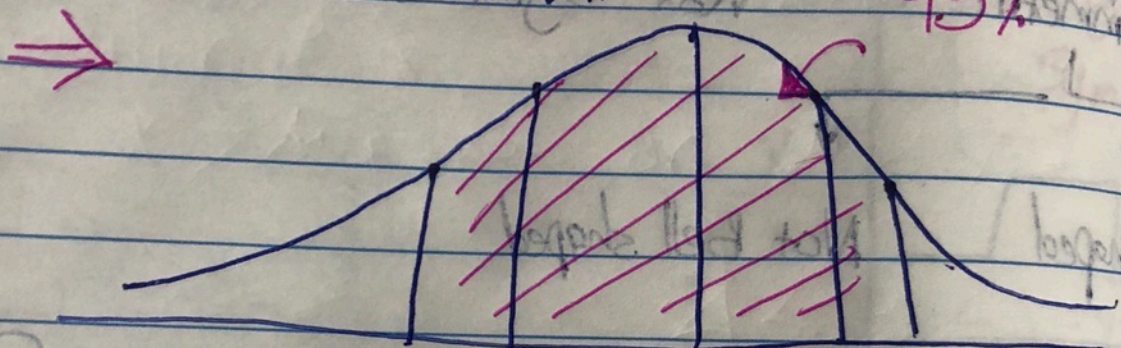
## \* Empirical Rule :-

For data having the Bell-Shape Distribution

① Approximately 68% of the data will be within one standard deviation of the mean.  $\approx 68\%$



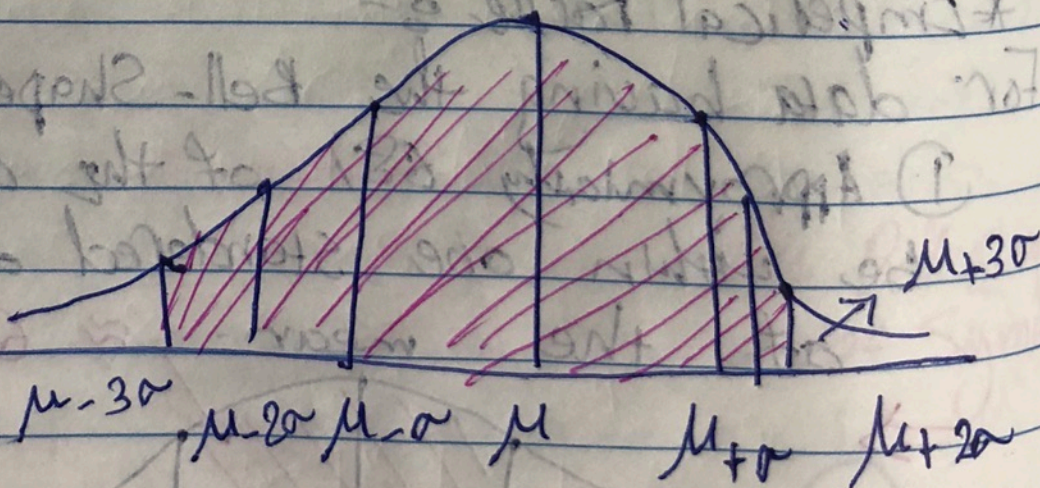
② Approximately 95% of the data will be within two standard deviations of  $\mu$  (mean)



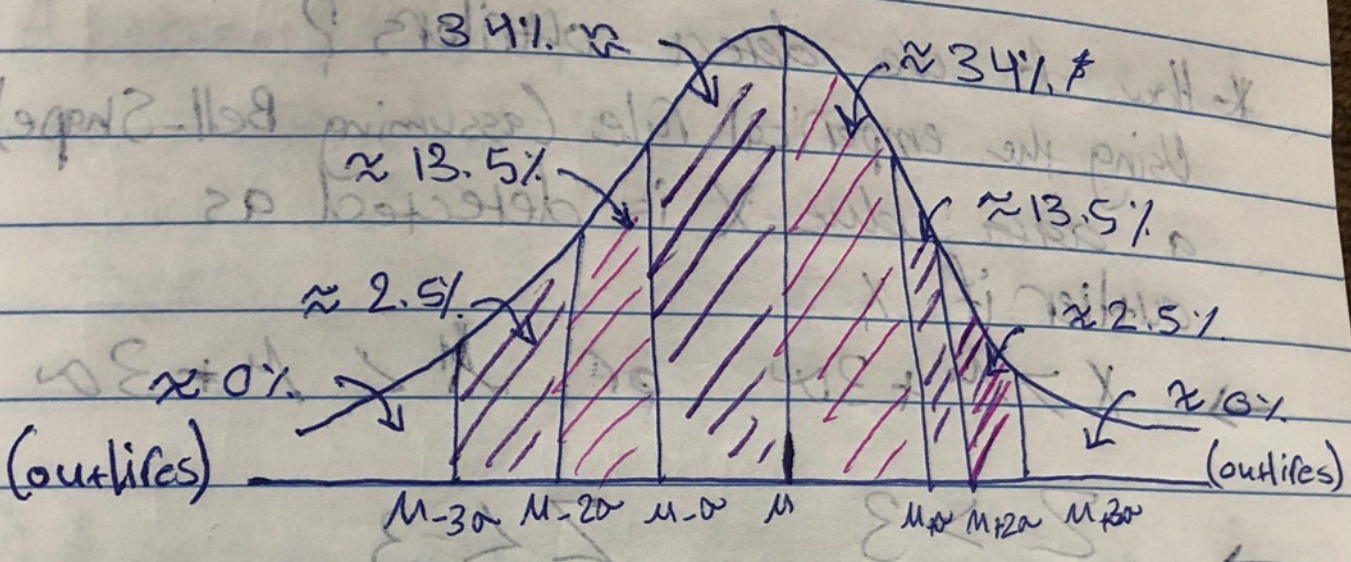
$\mu - 2\sigma$     $\mu - \sigma$     $\mu$     $\mu + \sigma$     $\mu + 2\sigma$

~~③ Approx~~

③ Almost All data will be within three 3 standard deviations



$\mu - 3\sigma$     $\mu - 2\sigma$     $\mu - \sigma$     $\mu$     $\mu + \sigma$     $\mu + 2\sigma$



← أنا هون حكاية انو  $\mu - \sigma$  ←  $\mu + \sigma$  68%  
 \* فاذا بيدك اقسيم عالم ربعين بيطرح عندي 34%, 34%  
 وفض  $X + 2\sigma$  ←  $X - 2\sigma$  ← 95%  
 فقطح از 68 من اذ 95 لان توصيف هافس  
 وبقية الاعداد لانه ربعين بيطرح عندي  
 13.5%, 13.5% فضل عندي آخر ربعين  
 حكاية في قاعدة (3) انها  $\mu - 3\sigma$  ←  $\mu + 3\sigma$   
 سبقتها 100% واننا اخذت قبلها 100% ← 95%  
 بطرح 95% من اذ 100 بطرح هسي 5  
 عنان ربعين 2.5%, 2.5%  
 ← الباقى ان طرف 0%  
 ← وكونت حكاية

1) A person Sleep 9 hours find the Z score

$$Z(9) = \frac{X - \mu}{\sigma} = \frac{9 - 6.9}{1.2}$$

$$Z(9) = \frac{X - \mu}{\sigma} = \frac{9 - 6.9}{1.2} = 1.75$$

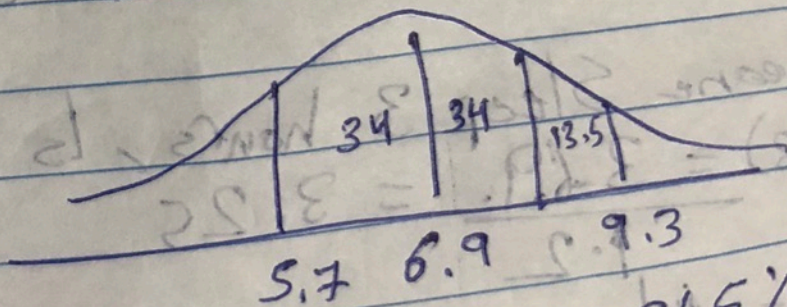
2) A person Sleep 4 hours find the Z score

$$Z(4) = \frac{4 - 6.9}{1.2} = -2.42$$

3) What Percentage of the people Sleep between 5.7 hours to 9.3 hours

$$Z(5.7) = \frac{5.7 - 6.9}{1.2} = -1$$

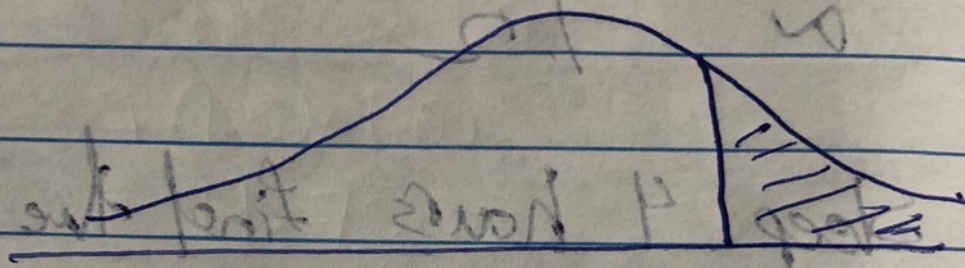
$$Z(9.3) = \frac{9.3 - 6.9}{1.2} = +2$$



$$34 + 34 + 13.5 = 81.5\%$$

4) What Percentage of people Sleep more than 9.3 hours)

$$Z(9.3) = \frac{9.3 - 6.9}{1.2} = 2$$



2.5% ← percentage

5) assume someone Sleeps 9.7 hours  
Is that value an outlier?

$$\Rightarrow Z(9.7) = \frac{9.7 - 6.9}{1.2} = 2.22$$

the value 9.7 is not an outlier

6) Someone Sleep 3 hours, Is that

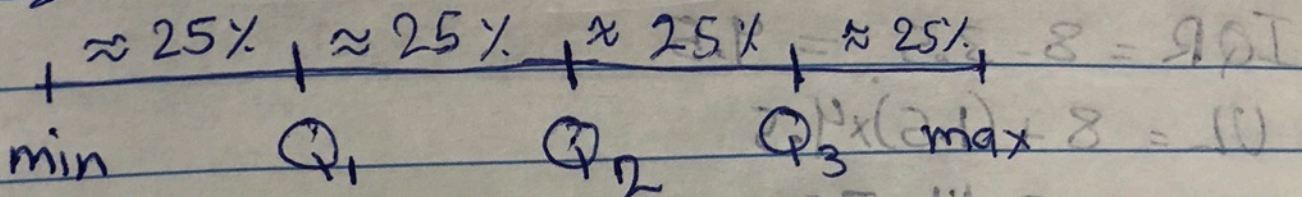
$$\Rightarrow Z(3) = \frac{3 - 6.9}{1.2} = -3.25$$

this value is an outlier

### 3.4 $\Rightarrow$ Exploratory Data Analysis

- Five number Summary.

min,  $Q_1$ ,  $Q_2$ ,  $Q_3$ , max



### Box plot

- Five number summary

-  $IQR = Q_3 - Q_2$

- Upper limit of box plot  $\Rightarrow Q_3 + 1.5 IQR$

- lower limit box plot  $\Rightarrow Q_1 - 1.5 IQR$

- Outlier  $X > UL$  or  $X < LL$

### • Example 8

Samples 5, 7, 10, 7, 3, 1, 4, 9

- Find Five number summary

- Construct a box plot

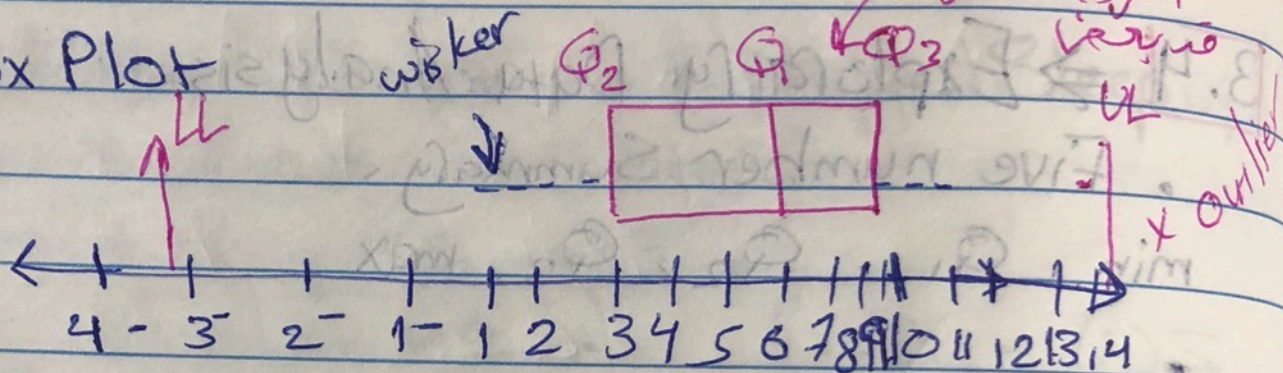
$\Rightarrow$  Five number summary

Sort  $\Rightarrow$  1, 3, 4, 5, 7, 7, 9, 10

FNS = 1, 3.5, 6, 8, 10

~~IQR~~

# \* Box Plot



$$IQR = 8 - 3.5 = 4.5$$

$$UL = 8 + (1.5) \times 4.5$$

$$= 14.75$$

$$LL = 3.5 - (1.5 \times 4.5)$$

$$= -3.25$$

**3.6** ⇒ The weighted mean working with group data

• Weighted mean

Ex 8 (weighted data)

$X_i$	$w_i$
90	4
80	3
70	2
60	1

$X_i = \text{data value}$   
 $w_i = \text{weighted}$

Sample mean (without weighted)

$$\bar{X} = \frac{90 + 80 + 70 + 60}{4}$$

⇒ the sample mean of weighted data

$$\bar{X} = \frac{\sum X_i \cdot w_i}{\sum w_i}$$

1) Find the sample mean assuming the values  $X_i$  has no weights

$$\bar{X} = \frac{90 + 80 + 70 + 60}{4} = 75$$



② Find the Sample mean of the data  
 Showed

$$\bar{X} = \frac{\sum w_i x_i}{\sum w_i}$$

$$\Rightarrow \frac{(90 \times 4) + (80 \times 3) + (70 \times 2) + (60 \times 1)}{10} = \frac{800}{10} = 80$$

\* Example:  
 grouped data

waiting time	freq.	$x_i$ $\downarrow$ $M_i$	$n_i$ $\leftarrow$ $F_i$
0-4	4	2	4
5-9	8	7	8
10-14	5	12	5
15-19	2	17	2
20-24	1	22	1
total	20	total	20

$M_i \Rightarrow$  mid point of class  $i$   

$$\frac{U_i + L_i}{2}$$

Sample mean of group data

$$\bar{X} = \frac{\sum M_i F_i}{\sum F_i (n)}$$

$$2.88 = \frac{180}{62.5}$$

$$= \frac{(2 \times 4) + (7 \times 8) + (12 \times 5) + (17 \times 2) + (22 \times 1)}{20}$$

$$20$$

$$= \frac{180}{20} = 9$$

⇒ Sample variance of grouped data

$$S^2 = \frac{\sum (M_i - \bar{X})^2 \cdot f_i}{(\sum f_i) - 1}$$

$$(\sum f_i) - 1$$

⇒ Sample standard dev. of grouped data

$$S = \sqrt{\frac{\sum (M_i - \bar{X})^2 \cdot f_i}{(\sum f_i) - 1}}$$

$$\sqrt{\frac{\sum (M_i - \bar{X})^2 \cdot f_i}{(\sum f_i) - 1}}$$

Population mean of group data:

$$\mu = \frac{\sum M_i f_i}{\sum f_i (N)}$$

$$\sigma^2 = \frac{\sum (M_i - \mu)^2 \cdot f_i}{\sum f_i (N)}$$

$$\sigma = \sqrt{\frac{\sum (M_i - \mu)^2 \cdot f_i}{\sum f_i (N)}}$$

إذا اعتبرنا ما pop.

$M_i$	$f_i$	$\bar{X} = 9$	$\hat{\mu} = 9$
2	4	$S = 6.48$	$\sigma^2 = 5.34$
7	8	$S^2 = 30$	$\sigma^2 = 28.5$
12		$(1 \times 5) + (5 \times 1) + (2 \times 5) + (8 \times 1) + (4 \times 5) =$	
17	2		
22	1		$P = \frac{0.81}{0.5} =$
total	20		

عند دراسة اعداد في  
 بقول مثل اظهرت اي قبل بين تغير اد freq  
 مثل ما هو عدد طيني لال في ال بصير انزل  
 وانخير كل رقم واحد ياروي بالآخر بصير  
 بعد ما اكل عادي عادي  
 يوجد  $\bar{X}, S, S^2$

(17) - 1  
 1-1

direction mean of group data

$\frac{1}{n} \sum (m_i - \bar{m})^2 = \sigma^2$