3.5 ⇒

- measures of association between 2 variables:-

. Def. Sample covariance :-

Sample: $(x_1, y_1), \ldots, (x_n, y_n)$

$$S_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1} \Leftarrow \text{sample}\text{ال}$$

. Def population covariance :-

population: $(x_1, y_1), \ldots, (X_N, Y_N)$

$$\sigma_{xy} = \frac{\sum (X_i - \mu_x)(y_i - \mu_y)}{N} \Leftarrow \text{population}\text{ال}$$

covariance :

يعني أنا بدي أعمل اشي شبه الـ variance بس

يكون تشاركي (co) ← معناها تشاركي

→ S_{xy} (sample covariance)
  n (sample size)

→ σ_{xy} (population covariance)
  N (population size)

* S_{xy} its a point estimater of σ_{xy}

Example :-

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x-\bar{x})(y-\bar{y})$
1	2	-1	-1	1
2	3	0	0	0
3	4	1	1	1

→ total $\boxed{2}$

$\bar{X} = \frac{6}{3} = 2$, $\bar{y} = \frac{9}{3} = 3$

$n = 3$

$S_{xy} = \frac{2}{(3-1)} = \frac{2}{2} = 1$

→ $S_{xy} > 0$ → there is a positive linear relationship X and Y

$S_{xy} = 0$ → there is no linear relationship between X & Y

$S_{xy} < 0$ → there is a negative linear relationship between X & Y

Def. Sample corelation coeffeciant

(sample statistics)

$r_{xy} = \cancel{\frac{S_{xy}}{\text{Standered}}}$

$$\frac{S_{xy}}{S_x S_y}$$

population corelation coeffeciant

سو (رو) ← $\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$ ~~(sample~~ , (population parameter

r_{xy} is a point estimater of ρ_{xy}

thesame examples

$\leftarrow$ $\bar{x} = 2$

$\bar{y} = 3$

$n = 3$

$S_{xy} = 1$

$S_x = 1 \qquad S_y = 1$

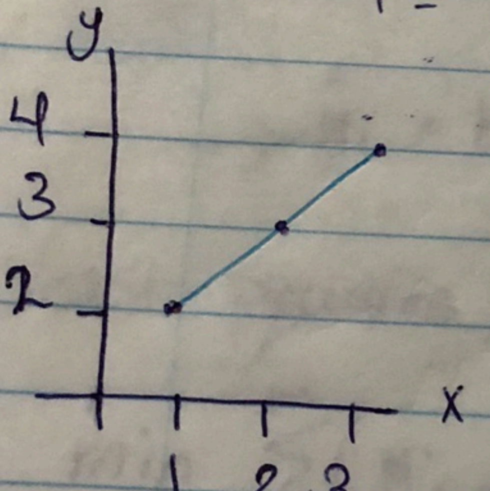
(stanred diviation) لـ على الآلة الحاسبة

sample لـ تمام الأرقام كل ادخل ما بعد بجيب ← (3) (2) Shift ← $\boxed{Sx}$

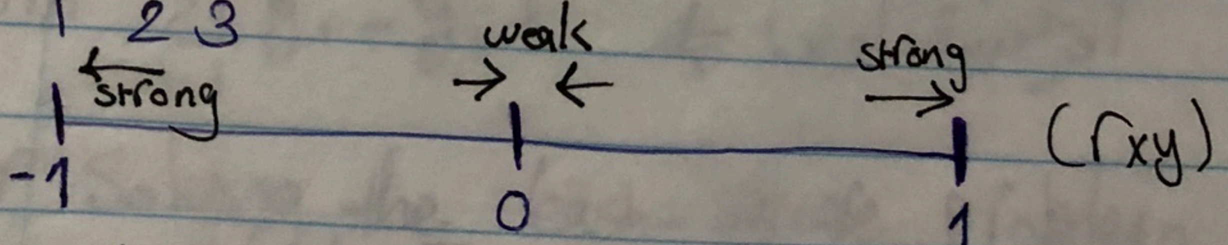
$$r_{xy} = \frac{1}{(1)(1)} = 1$$

اذا كانت r_{xy} (+) موجب بتكون العلاقة طردية بين X و Y ←

خذ المثال



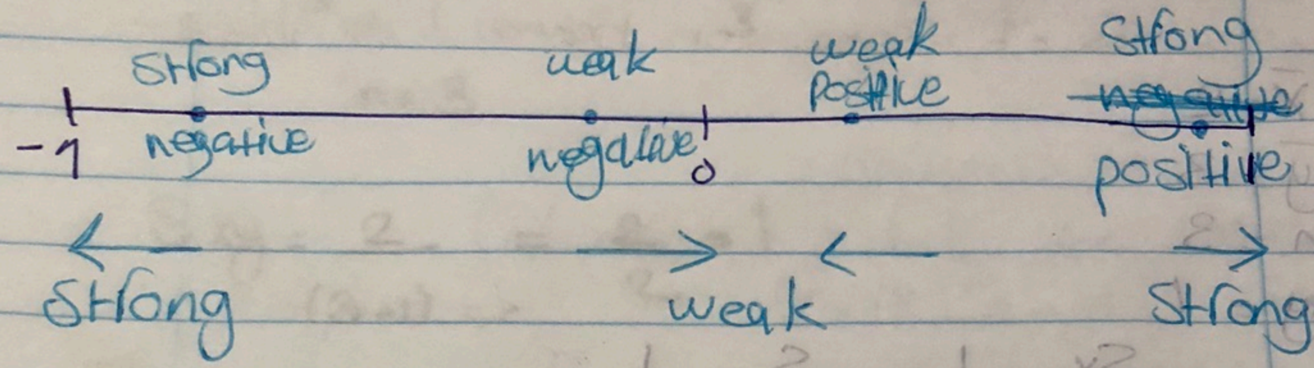
X	Y
1	2
2	3
3	4



Perfect negative linear relationship

no linear relationship

Perfect positive linear relationship



The same examples

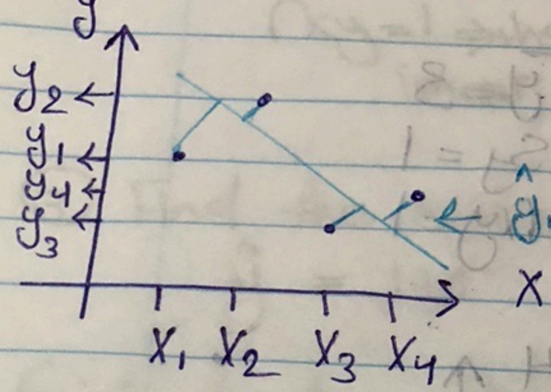
$S_{xy} = 1 \Rightarrow$ there is a positive relationship between X a Y (طلع موجب)

$r_{xy} = 1 \Rightarrow$ there is a perfect positive linear relationship between X and Y

Chapter 12:-

$\boxed{12.2}$ least Squares method:

$\Rightarrow (x_1, y_1), \ldots, (x_n, y_n)$



$\hat{y}_4$ و y_4 الفرق بين

y_i : observed value at x_i

$\hat{y}_i$: estimated value at x_i

(hat)

$\longrightarrow \hat{y}_i = b_0 + b_1 x_i$

b_0 : y-intercept

b_1 : Slope

- least squares problem:-

$$\min \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$$ ← افتراضية غاوس

- After Solving the least-square problem

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{S_{xy}}{S^2_x}$$

$\hat{y} = b_0 + b_1 x \Rightarrow$ estimated Regression Equation.

Example:-

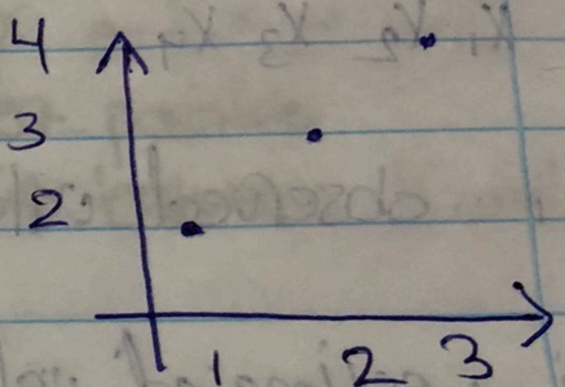
x	y	$\hat{y}$
1	2	2
2	3	3
3	4	4

$\bar{x} = 2$, $\bar{y} = 3$

$S_x = 1$, $S_y = 1$

$S_{xy} = 1$, $r_{xy} = 1$

$\Rightarrow b_1 = \frac{1}{(1)^2} = 1$



$b_0 = 3 - 1(2)$

$= 1$

$\hat{y} = b_0 + b_1 x$

$= 1 + 1 \cdot x$

$\boxed{\hat{y} = 1 + x}$

① Find the covariance, comment

$S_{xy} = 1 \Rightarrow$ positive linear relationship

② Find the corelation coeffeciant comment

$r_{xy} = 1 \Rightarrow$ there is perfect positive linear relationship.

③ Find the estimated regression equation

$\hat{y} = 1 + x$

④ Find the y-intercept of the estimated

$b_0 = 1$

⑤ Find the slope of the estimated

$b_1 = 1$

⑥ Estimat y when $x = 4$

$\hat{y} = 1 + 4 = 5$

Example:-

X	2	6	9	13	30
y	7	18	9	26	23

① Find $\bar{x}$ = 12

② Find S_x = 10.48

③ Find $\bar{y}$ = 16.6

④ Find S_y = 8.38

⑤ Find S_{xy} = 59.05

⑥ Find r_{xy} = 0.65

$$* \; r_{xy} = \frac{S_{xy}}{S_x \cdot S_y}$$

← على الآلة الحاسبة :-

عشان أحل على شابتر 4.5 وشابتر 12 على الآلة الحاسبة بعمل هاي الخطوات :-

① mode ← 3 (REG) ← 1 (Lin)

② بدخل X,y كالآتي 2 $\boxed{,}$ 7

يعني بتصير قيمة X بعدها فاصلة بعدها Y

$\boxed{,}$ ← على يسار $\boxed{M+}$

③ بعد ما دخلت أول قيمة لـ X,y بدخل باقي القيم بحط $\boxed{M+}$ و بعمل قيمة قيمة

→ 2,7 $\boxed{M+}$ 6,18 $\boxed{M+}$ 9,9 $\boxed{M+}$ 13,26 $\boxed{M+}$ 30,23 $\boxed{M+}$

(1) ← (2) ← Shift ← $\bar{x}$ عشان أجيب ⇐

⇒ $\bar{x} = 12$

(3) ← (2) ← Shift ← S_x

⇒ $S_x = 10.84$

* عشان أجيب معلومات y اللي بدي ياها بعد

ما أحط (Shift) ← (2) رح يكون

في سهم عالشاشة بضغط على الكبسة

اليمين في الآلة الحاسبة

وبظهر عندي كل معلومات y .

بعد ما ضغطت عالسهم اليمين بجيب $\bar{y}$ ⇐

(Shift) ← (2) ← (1)

⇒ $\bar{y} = 16.6$

S_y ⇐ (Shift) ← (2) ⇐ اليمين ⇐

⇒ $S_y = 8.38$

* عشان أجيب r_{xy} ⇐

(Shift) ⇐ (2) ⇐ عاليمين مرتين لحد ما

يظهر عندي (A B r) ⇐ (3)

⇒ $r_{xy} = 0.65$

$$\Rightarrow S_{xy} = r_{xy} \cdot S_x \cdot S_y$$

$$0.65 \times 10.48 \times 8.38$$

$$\Rightarrow S_{xy} = 59.05$$

إذا بدي أجيب Sxy بطريقة أدوه على الآلة الحاسبة ⇐

(3) ⇐ (2) ⇐ Shift

ضرب x

(3) ⇐ مرة لليمين ⇐ (2) ⇐ Shift

ضرب x

(3) ⇐ مرتين لليمين ⇐ (2) ⇐ Shift

يساوي ⇐ بطلع عندي الناتج

Sxy ⇒ في مثال السابق

$S_{xy} = 58.75$

⑦ Comment on part 5

⇒ there is a positive linear relationship between X and Y

⑧ Comment on part 6

⇒ $r_{xy} = 0.65$ → there is a moderate positive linear relationship between X and Y

⑨ What is the estimated regression equation

$\hat{y} = b_0 + b_1 X =$

A B

على الآلة الحاسبة ⇐

Shift ⇐ (5) ⇐ مرتين لليمين ⇐ A بعدها B

$A\ (b_0) = 10.6$
$B\ (b_1) = 0.5$

$\Rightarrow \hat{y} = 10.6 + 0.5x$

(10) Find the y-intercept of estimated

$b_0 = 10.6$

(11) Slope $\Rightarrow b_1 = 0.5$

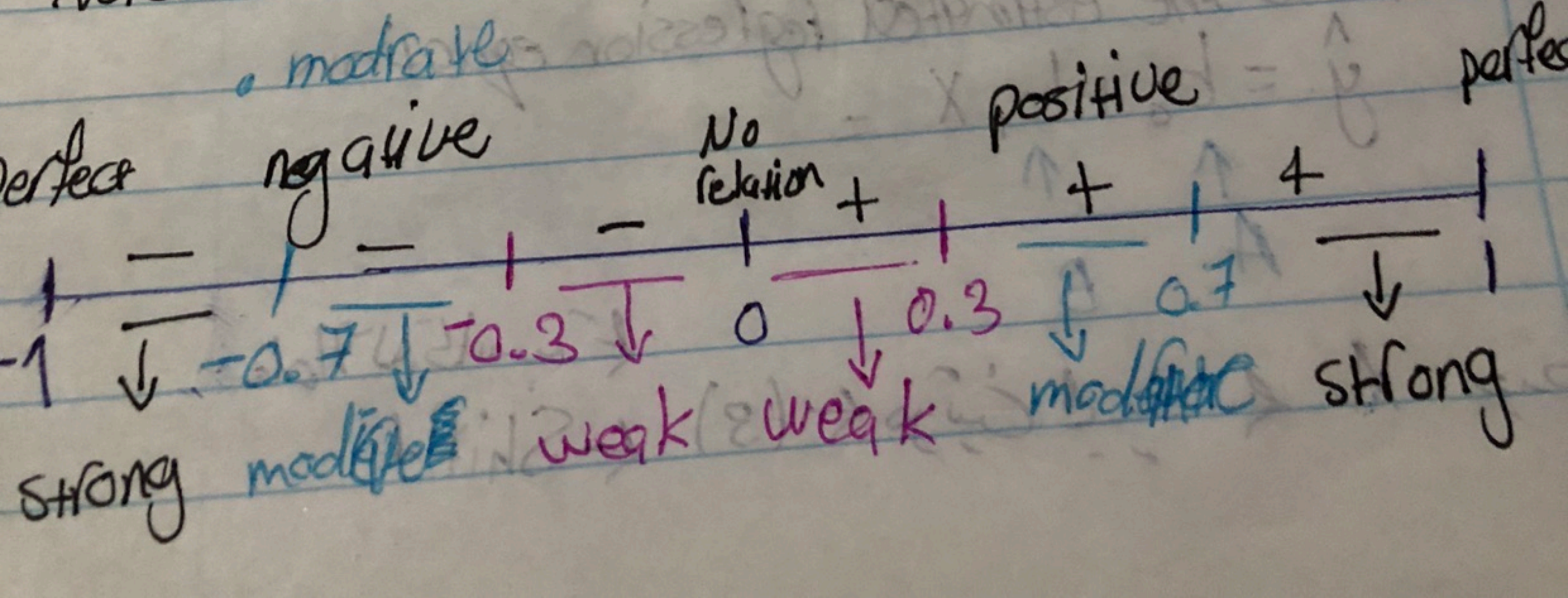
(12) estimate y when $X = 25$

$\Rightarrow \hat{y} = 10.6 + 0.5 \times 25$
$= 23.1$

$\hat{y}$ when $X = 9$

$\hat{y} = 10.6 + 0.5 \times 9$
$= 15.1$

Notes -



Chapter 4:

Introduction to probility

علم الاحتمالات

4.1 ⇒ experiments, counting rules, and assigning probabilites

Def: Experiments: process that generates well-defined outcomes.

التجربة ← أي عملية بتعطيني نواتج محددة المعالم

- probabilites experimets → Possible outcomes

↲ عمليات أنا ما بقدر أحدد نتائجها بشكل مسبق

Experiment	Experimental outcomes	
Toss a coin ⇒	Head, Tail	عملة نقدية
Roll a die ⇒	1, 2, 3, 4, 5, 6	قطعة نرد
play a game ⇒	win, loss, tie	نلعب لعبة

- Sample point: is one of possible outcomes.
- Sample space: is the set of the possible outco

↳ $S = \{\ ,\ \}$

Toss a coin ⇒

Sample point: H / T

Sample space: $\{H, T\} = S$

Counting Rules:-

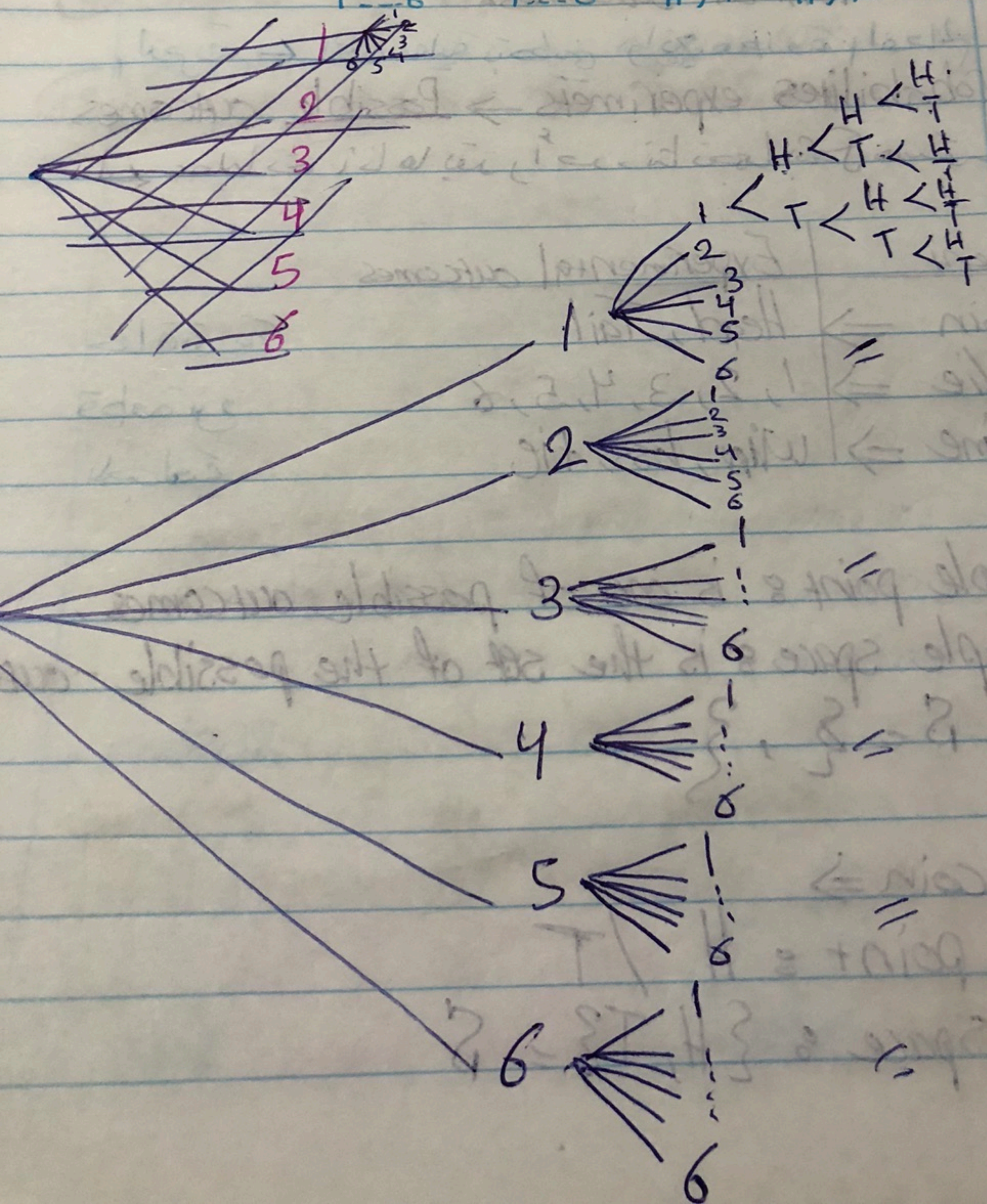
Ex:-

Experiments: roll a die twice and coin 3 times

- write the Sample space
- How many elements are ther in the Sample space.

Tree diagram ⇒ die, die, coin, coin, coin

1...6 1...6 H, T H, T H, T



$S = \{(1,1,H,H,H), (1,1,H,H,T), (1,1,H,T,H)$
$(1,1,H,T,T), \text{------}, (6,6,T,T,T)$

Multi step experiments ⇒ تجربة متعددة الخطوات

5-step experements = (6) (6) (2) (2) (2)
1...6 1-6 H.T H,T H,T

$6 \times 6 \times 2 \times 2 \times 2 = 288$

k-step experimets (Counting rule)

$\# S = (n_1)(n_2) \text{....} (n_k)$

n_1: number of possible outcomes in step (1)

n_2: no. of possible outcomes in step (2)

n_k: no. of possible outcomes in step k

Example (2) :-
we want to constract a passward, the length of the passward is 7. we are allowed to use English letters case sensitive and numbers, how many passwords can we constract.

A → Z
a → z // // // // // //
0 → 9

A B C ... Z ⟶ 26
~~A B C ... Z ⟶ 26~~
0 1 2 3 ... 9 ⟶ 10
a b c ⟶ 26

62 possible outcomes

$62 \times 62 \times 62 \times 62 \times 62 \times 62 \times 62$
$= 62^7 = 3.52 \times 10^{12}$

⇐ بحط عالآلة حاسبة 62 ← ^ ← 7

Example 3:-

You are going on a trip, you are allowed to take with you a limited number of items, You have a total of 20 items, are allowed to take only 4 items. How many possibilities do we have?

20 19 18 17

ما بصير أضربهم لأنو لو ضربتهم بكون اخطأت الترتيب
وأهميته بس بالمثال الترتيب مش مهم

$$S = \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} = 4845$$

$4! \leftarrow$

→ Combinations counting rule

You Select n items from N items

- The item is selected only one.
- The order is not important.

Number of combinations =

$$C_n^N = \binom{N}{n} = \frac{N!}{(N-n)!\,n!}$$

المثال (3)

$$C_4^{20} = \frac{20!}{(20-4)!\,4!} = \frac{20!}{16!\,4!}$$

$$= \frac{20 \times 19 \times 18 \times 17 \times \cancel{(16 \times 15 \ldots \times 1)}}{\cancel{(16 \times 15 \times 14 \times 13 \times \ldots \times 1)}(4 \times 3 \times 2 \times 1)}$$

$$= \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} = 4845$$

على الآلة الحاسبة :-
بوجبها هنا شرحنا خلال اني احط ال N
بعدها nCr بعدها بحط ال n
20 nCr 4 = 4845

Example (4) :-
We are intrested in making a team of 3 members out of 25 members
(a) how many teams can we constract the team member can assign their roles later :

← الترتيب غير مهم.

$$C_{25}^{3} = \binom{25}{3} = 2300 \leftarrow \text{على الآلة الحاسبة}$$

(b) how many teams can we constract ? the team should have a team leader, a financial offeciar and and PR offeciar

← الترتيب مهم !!

25 24 23

$$\times S = 25 \times 24 \times 23 = 13800$$

* Permetation counting Rules:-

Select small n items from N items

- The item is selected only one time
- The order is important

Number of Permetations =

$$P_n^N = \left(\frac{N!}{(N-n)!}\right)$$

$$\frac{25!}{22!} = \frac{25 \times 24 \times 23 \times \cancel{(22 \times 21 \dots \times 1)}}{\cancel{(22 \times 21 \times 20 \times 19 \dots \times 1)}}$$

$$P_3^{25} = 25 \times 24 \times 23$$
$$= 13800$$

$3 \leftarrow (nCr)$ Shift $\leftarrow 25 \leftarrow$ على الآلة الحاسبة

$(n\ 3 \leftarrow$ Shift nCr $\leftarrow$ N

$\boxed{\text{Shift nCr} = \text{nPr}}$

Rules for assigning probilities

Example:

Toss a coin $S = H, T$

Assign probailities for the sample points

$P(H) = \frac{1}{2}, 0.5$

$P(T) = \frac{1}{2}, 0.5$

H,T احتمالية رمي قطعة نقد، ويطلع شي

The classical method for assigning probailities

→ Equaly likely outcomes

Example (2):-

Toss a coin $S = (H, T)$ assign Prob. for the sample points.

outcome	freq.
H	7
T	13
Total	20

$P(H) = \frac{7}{20} = 0.35$

$P(T) = \frac{13}{20} = 0.65$

→ Relative freq. method

Example (3)

Experemints: Toss a coin $S = \{H, T\}$ Assign prob. for the sample points.

$P(H) = 0.49$

$P(T) = 0.51$

رأيي هيدي فانوي وام لا

→ The subjective method

Rules of assigning probalities :-

$S = \{E_1, E_2, E_3, \dots E_n\}$

S = Sample space
E_i = Sample point

① We want to assign prob. $P(E_i)$ for the sample point E_i

1. $0 \leq P(E_i) \leq 1$
2. $\sum_{i=1}^{n} P(E_i) = 1$

4.2 ⇒ events and their probabilities

Subset from the sample space is called ⇒ Event

P(Event) = sum of the ~~the~~ prob. of the Sample points that are in the event.

Example :-

$S = \{E_1, E_2, E_3\}$

$P(E_1) = 0.20, \; P(E_2) = 0.65, \; P(E_3) = ?$

$P(E_3) = 0.15$ ← لأنو بنعرف انو مجموعهم (1)

* $A = \{E_1, E_3\}$ ← Event

$P(A) = P(E_1) + P(E_3)$

$= 0.20 + 0.15$

$= 0.35$

* $B = \{E_1, E_2\}$ ← Event

$= 0.20 + 0.65$

$= 0.85$

* $C = \{E_1\}$ ← Event

$= 0.20$

* $D = \{E_1, E_2, E_3\}$ ← Event

$P(D) = P(S)$

$= 1$

* $M = \{\}$ ← Event

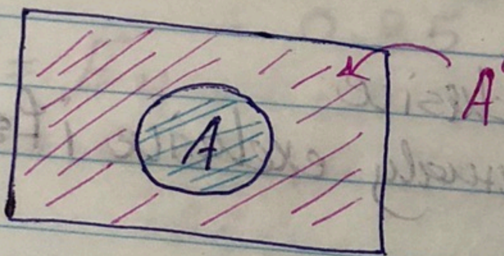
$= \emptyset$

$P(M) = P(\emptyset) = 0$

4.3 ⇒ Some basic Probabilities to relationships:-

* Complement Law

A^c = complement of event A (المتممة)



• A
• A^c

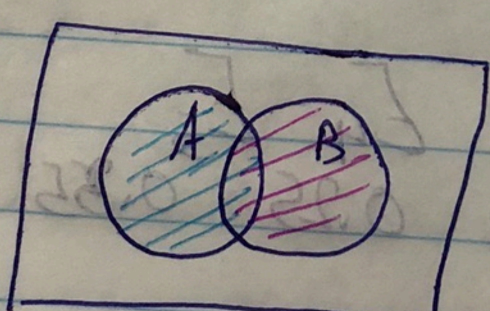
$A^c = \{x \in S : x \notin A\}$

$P(A) + P(A^c) = 1$ ← Complent law

• $P(A) = 1 - P(A^c)$

• $P(A^c) = 1 - P(A)$

* the addition law :



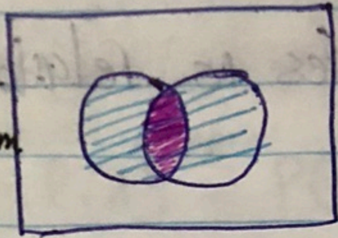
$A \cup B$: A unian B (اتحاد)

$A \cap B$: A interception B (تقاطع)

$A \cup B = \{X \in S : X \in A \text{ or } X \in B\}$

$A \cap B = \{X \in S : X \in A \text{ and } X \in B\}$

vemn diagram



$A \cup B$ (كل العناصر)

$A \cap B$ (العناصر المشتركة)

• بدون تكرار

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

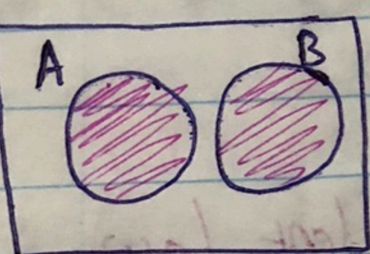
addition law

* Mutually exclusive

• A and B are mutualy exclusive if:

$A \cap B = \{\}$

S



$A \cap B = \emptyset$

Addition law for mutualy exclusive events

$$P(A \cup B) = P(A) + P(B)$$

Example (1):-

$S = \{E_1, E_2, E_3, E_4, E_5\}$

E_i	E_1	E_2	E_3	E_4	E_5
$P(E_i)$	0.05		0.10	0.25	0.35

$A = \{E_2, E_3\}$

$B = \{E_4, E_5\}$

$C = \{E_2, E_3, E_4\}$

① $P(E_i) = 1 - (0.05 + 0.10 + 0.25 + 0.35)$

$= 1 - 0.75$

$= 0.25$

② $P(A) = E_2 + E_3$

$= 0.25 + 0.10$

$= 0.35$

③ $P(B) = E_4 + E_5$

$= 0.25 + 0.35$

$= 0.60$

④ $P(C) = E_2 + E_3 + E_4$

$= 0.25 + 0.10 + 0.25$

$= 0.60$

⑤ $P(A^c) = E_1, E_4, E_5$

$P(A^c) = 1 - P(A)$

$= 1 - 0.35$

$= 0.65$

⑥ $P(B^c) = 1 - 0.60$

$= 0.40$

⑦ $P(C^c) = 1 - 0.60$

$= 0.40$

⑧ $P(A \cap C) = P(E_2, E_3)$

$= 0.25 + 0.10$

$= 0.35$

⑨ Are A and C mutually exclusive

$\Rightarrow$ A and C <u>aren't</u> mutualy exclusive

⑩ $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= 0.35 + 0.60 - 0.30$

$= 0.60$

⑪ Are A and B muetually exclusive event?

Yes, A and B are mutually exclusive

$A \cap B = \emptyset$

⑫ $P(A \cap B) = 0$ ← التقاطع = صفر

⑬ $P(A \cup B) = P(A) + P(B)$

$= 0.35 + 0.60$

$= 0.95$

4.4 ⇒ conditional probability الاحتمال الشرطي

Let A and B be events from a sample spaces S. the conditional probability of the event A given the event B is difined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

(B ↑ condition)

- the conditional probability of the event B given the event A is defind as:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0$$

(A ↑ condition)

- independent ما في اثر لأي واحد على الثاني

A and B are independent if:

$P(A|B) = P(A)$

$P(B|A) = P(B)$

$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$

$\Rightarrow P(A \cap B) = P(B|A) \cdot P(A)$

← multiplication law of conditional prob.

$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

$\Rightarrow P(B \cap A) = P(B) \cdot P(A)$

← mulipkation law of independent

Example:-

$S = \{E_1, E_2, E_3, E_4, E_5\}$

$A = \{E_1, E_3\}$, $P(A) = 0.35$

$B = \{E_4 - E_5\}$, $P(B) = 0.60$

$C = \{E_2, E_3, E_4\}$, $P(C) = 0.60$

14. 4.3 ③ $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{B(\phi)}{0.60} = \dfrac{0}{0.60} = 0$

15. Are A and B independent

$P(A|B)$ $P(A)$

$0 \neq 0.35$

$\Rightarrow$ A and B are not independent

16. $P(B|C) = \dfrac{B(B \cap C)}{P(C)} = \dfrac{0.25}{0.60} = 0.42 \approx 0.4167$

$B \cap C = E_4 = 0.60 \leftarrow$ (1) مثال في سكشن 4.3

17. Are B and C independent

$P(B|C)$ $P(B)$

$0.4167 \neq 0.60$

B and C are not independent

Examples:- Data from Q 32, p168

Health insurance :-

	Yes	No	Total
18-34	750	170	920
35+	950	130	1080
Total	1700	300	2000

(cross-tabulation)

① Ddevelop the joint probility table :-

Age / H.I	Yes	No	Total
18-34	0.375	0.085	0.46
35+	0.475	0.065	0.54
Total	0.85	0.15	1.00

← marginal probabilites

(Relative freq. method) تقسم على Sample size (2000)

② Find the joint probibilites (تقاطع حدثين)

$P(18\text{-}34 \cap Yes) = 0.375$

$P(18\text{-}34 \cap No) = 0.085$

$P(35+ \cap Yes) = 0.475$

$P(35+ \cap No) = 0.065$

1 ← دايماً مجموعهم واحد

③ Find the marginal probabilites :- كل وحدة لحال

$P(18-34) = 0.46$ ⎫ 1 ← كل variable
$P(35+) = 0.54$ ⎭ لحال بطلع مجموعو 1

$P(Yes) = 0.85$ ⎫ 1
$P(No) = 0.15$ ⎭

④ What percentage of the population go without health insurance.

$P(NO) = 0.15 \Rightarrow$ percentage $\approx 15\%$

⑤ What percentage of people who are 18-34 Years old in the population

$P(18-34) = 0.46 \approx 46\%$

⑥ What percentage of people who are 18-34 Y.O and without H.I

$P(18-34 \cap No) = 0.085$

$\approx 8.5\%$ percantege

⑦ A person was selected at random what is probability that a person 35+ Y.O and has health insurance

$P(35+ \cap Yes) = 0.475$

⑧ what is the prob. that a randomly selected person 35+ or doesn't have a H.I

$P(35+ \cup No)$

$= P(35+) + P(No) - P(35+ \cap No)$

$= 0.54 + 0.15 - 0.065$

$= 0.625$

⑨ A person with H.I was selected at random whats probability that she/he is 18-34 Years old?

~~$P(18\text{-}34 \mid Yes)$~~ =

$P(18\text{-}34 \mid Yes) = \frac{P(18\text{-}34 \cap Yes)}{P(Yes)}$

$= \frac{0.375}{0.85} = 0.4412$

⑩ If a person who are 35+ selected whats prob. that he/she has H.I?

$P(Yes \mid 35+) = \frac{P(Yes \cap 35+)}{P(35+)}$

$= \frac{0.475}{0.54} = 0.8796$

① are the events Yes and 35+ independent

$P(Yes \mid 35+) = 0.8796$

$P(Yes) = 0.85$

$P(Yes \mid 35+) \neq P(Yes)$

∴ Yes and 35+ are not independent

- Another solution

$P(Yes) = 0.85$

$P(35+) = 0.54$

$P(Yes) \cdot P(35+) = 0.459$

$P(Yes \cap 35+) = 0.475$

Yes and 35+ aren't independent

أكثر من طريقة للحل →
① $P(A \mid B) = P(A)$
② $P(B \mid A) = P(B)$
③ $P(A \cap B) = P(A) \cdot P(B)$

② Are the variables Age and H.I independent?

18-34 / Yes	✓	✓
18-34 / No	✓	✓
35+ / Yes	✓	✗
35+ / No	✓	✓
	independent	not independent

بمجرد وجود ✗ يعطي