*Ch4 : Introduction to probability: الاحتمالات

- **Probability**: is anumerical measure of the likelihood that an event will occure.
  إمكانية

• Sec 4.1 : Experiments, Counting Rules and Assigning probabilities
التجارب العشوائية ومبادئ العد

- **Experiment**: is a process that generates well-defined outcomes.
  هي التجربة التي يمكن ملاحظتها وتحديد النتائج الممكنة لها.

- **Sample Space**: the set of all possible outcomes, denoted by «S»
  الفضاء العيني: جميع النتائج الممكنة للتجربة العشوائية

- **Sample point** («experimental outcomes»): An element of the sample space.
  مخرجات الفضاء العيني.

Ex:

Experiment	outcomes	sample space «S»
① Toss a coin	Head, Tail	$S=\{H,T\}$
② Roll a die	1, 2, 3, 4, 5, 6	$S=\{1,2,3,4,5,6\}$
③ Select a part for inspection	Defective, nondefective	$S=\{D,ND\}$

* Counting Rules, Combinations, and Permutations.

① Multiple-step experiments:

Experiment with more than one step.

- Ex: Consider the experiment of tossing two coins.

$\longrightarrow S = \{(H,H)\ (H,T)\ (T,H)\ (T,T)\}$

- Counting rule for multiple-step experiments.

If an experiment can be described as a sequence of K stepe with n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_K$.

← العدد الكلي للتجربة

- Ex: ① In a tossing 2 coins. Find the total number of outcomes.

$$\longrightarrow \underset{H\ T}{(2)}\,\underset{H\ T}{(2)} = 4$$

② Tossing 3 dice and 4 coins, what is the total number of outcomes.

$$(6)(6)(6)(2)(2)(2)(2) = 3456$$

حجر النرد الأول له 6 مخرجات

حجر النرد الثاني له 6

4 قطع نقود وكل قطعة لها مخرجان

③ If we want to create a password of five numbers, how many password can we form, ∵ if digits can be repeated?

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$

10 10 10 10 10

كل خانة من خانات password عبارة عن رقم ونعرف أن الأرقام من: 0,1,2,3,4,5,6,7,8,9 وعددها 10 . وبما أن كل رقم من الممكن أن يتكرر .

∴ we have $10 \times 10 \times 10 \times 10 \times 10 = 10^5$ passwords.

- **Tree diagram**: is a graphical representation that helps in visualizing a multiple step experiment.

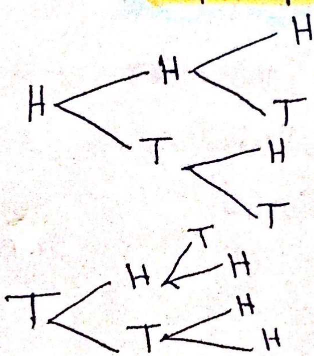
مخطط الشجرة يساعدنا في معرفة جميع مخرجات الـ multiple step experiment

- **Ex:** In tossing a coin 3 times.

a) Find the total number of outcomes:

$\rightarrow (2)(2)(2) = \underline{\underline{8}}$

b) Find the sample space (Use tree diagram).



∴ $S = \{(H,H,H)\ (H,H,T)\ (H,T,H)\ (H,T,T)\ (T,H,T)\ (T,H,H)\ (T,T,H)\ (T,T,T)\}$

② Combinations:

التوافيق

The number of combinations of $\underline{N}$ objects taken n at a time is $C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$, $N! = N(N-1)\cdots(2)(1)$.

$n! = n(n-1)\cdots(2)(1)$.

$0! = 1$ «by definition»

«! factorial» مضروب

→ Selecting $\underline{n}$ objects from a set of N object, where the order isn't important and repetition isn't allowed.

عند اختيار n من العناصر من مجموعة تحتوي على N عنصر دون مراعاة الترتيب (وهنا تلقائياً يكون التكرار ممنوع).

-Ex: In how many ways can a teacher choose $\underline{3}$ students from among $\underline{5}$ students?

نلاحظ أن ← الأستاذ يريد اختيار 3 طلاب من بين 5 والترتيب بينهما غير مهم

$$\rightarrow C_3^5 = \binom{5}{3} = \frac{5!}{3!(2!)} = \boxed{10}$$

توضيح ✓: لنفرض أن الطلاب A, B, C, D, E لاختيار 3 طلاب من هؤلاء

(A, B, C)	(A, C, D)	(B, C, D)	(C, D, E)
(A, B, D)	(A, C, E)	(B, C, E)	
(A, B, E)	(A, D, E)	(B, D, E)	

نلاحظ أنه لا يهم أن اخترنا (A, B, C) أو (B, C, A) نفس النتيجة

-Ex: How many ways can 3 items be selected from a group of 6 items?

$$\longrightarrow C_3^6 = \frac{6!}{3!(3!)} = 20.$$

-Ex: How many 3-letter combinations can be formed from the English apabetical?

$$\longrightarrow C_3^{26} = \frac{26!}{3!(23)!} = 2600 \text{ combinations.}$$

*** Using Calculator «combinations»**

N $\boxed{nCr}$ n $\boxed{=}$

③ Permutations: التباديل

The number of experimental outcomes when n objects are to be selected from a set of N objects where the order of selection is important.

اختيار n من العناصر من مجموعة تحتوي على N من العناصر ولكن الترتيب مهم والتكرار ممنوع.

$\longrightarrow$ The number of permutations of N objects taken n at a time is:

$$P_n^N = \frac{N!}{(N-n)!}$$

« Using calculator
N $\boxed{shift}$ $\boxed{nCr}$ n $\boxed{=}$ »

-Ex: In how many ways can a teacher form a group of $\underline{2}$ students (one of them is president and the other is vice president) from $\underline{3}$ students?

نلاحظ أن الترتيب مهم لأن الطالب
ممكن أن يمثل دور الرئيس أو النائب لذا تباديل

$$\rightarrow P_2^3 = \frac{3!}{(3-2)!} = \underline{\underline{6}}$$

لو كان الطلاب A , B , C

A الرئيس	B النائب	,	B الرئيس	A النائب
A الرئيس	C النائب	,	C الرئيس	A النائب
B الرئيس	C النائب	,	C الرئيس	B النائب

- ملاحظة: لو كان السؤال أن كلا الطالبين أعضاء فإن الترتيب يكون غير مهم وبالتالي combination

$$\frac{3!}{2!\,(3-2)!} = \underline{\underline{3}}$$ AB , AC , BC

-Ex: How many $\underline{3}$ letters permutation can be formed from the English alphabetical?

$$\longrightarrow P_3^{26} = \frac{26!}{(26-3)!} = 15600 \text{ permutations.}$$

← كلمات السر المكونة من أحرف معينة ولا يُسمح فيها التكرار تكون تباديل permutation.

*Assigning Probabilities:-

→ The 3 approaches most frequently used are:-

1. the classical method.
2. relative frequency method.
3. subjective method.

→ Basic requirements for assigning probabilities:-

let $S = \{E_1, E_2, \dots, E_n\}$

1. $0 \leq P(E_i) \leq 1$; $\forall i = 1, \dots, n$
2. $\sum_{i=1}^{n} P(E_i) = P(E_1) + P(E_2) + \dots + P(E_n) = 1$.

- The classical method:-

is appropriate when all the experimental outcomes are equally likely.

If n outcomes are possible, a probability of $1/n$ is assigned to each outcome, « that is, $P(E_1) = P(E_2) = \dots = \frac{1}{n}$ »

- Ex: Consider the experiment of tossing a fair coin.

→ $S = \{H, T\}$.

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$.

We notice that:-

$0 \leq P(H) \leq 1$ and $0 \leq P(T) \leq 1$, $P(H) + P(T) = 1$.

- **Ex:** Consider the experiment of rolling a die.

$S = \{1, 2, 3, 4, 5, 6\}$

$P(1) = P(2) = \cdots = P(6) = \frac{1}{6}$.

- **The relative frequency method:**

is appropriate when data are available to estimate the proportion of the time the experimental outcome will occure if the experiment is repeated a large number of times.

$\longrightarrow$ Probability $= r.f = \frac{f}{n}$.

- **Ex:**

Blood type	F
A	3
B	7
O	6
AB	4

a) What is the probability of blood type A?

$\longrightarrow P(A) = \frac{3}{20} = 0.15$

b) If we select a student randomly, what is the probability that his blood type is AB?

$\longrightarrow P(AB) = \frac{4}{20} = 0.2$

- **The subjective method:-**
is most appropriate when one can't assume the experimental outcomes are equally likely and when little data are available, such as experience or intuition.

- Ex: The probability that it will rain tomorrow is 0.2 .
مبني على التخمين والاعتقاد.

• Sec 4.2 :- Events and their probabilities.

- Event: is a collection of sample point.
- Any Event is a subset of the sample space.
يرمز لأي حدث بحرف للتسهيل.

- Ex: Consider the experiment of rolling a die.
$S = \{1, 2, 3, 4, 5, 6\}$.

a) let A be the event of even numbers.
$\rightarrow A = \{2, 4, 6\}$

b) let B be the event of all numbers less than 6.
$\rightarrow B = \{1, 2, 3, 4, 5\}$.

c) let C be the event of all numbers greater than 6.
$\rightarrow C = \{\}$.

d) let M be the event of all numbers less than 7.
$\rightarrow M = \{1, 2, 3, 4, 5, 6\} = S$

* Probability of an event:-

P(E) = the sum of the probabilities of the sample point in the event.

- Ex: For the previous example:-

a) P(A):

$\longrightarrow P(A) = \frac{3}{6} = \frac{1}{2}$.

b) P(B)

$\longrightarrow P(B) = \frac{5}{6}$

c) P(C)

$\longrightarrow P(C) = 0$

d) P(M)

$\longrightarrow P(M) = \frac{6}{6} = 1$

* Note :

① $0 \leqslant P(A) \leqslant 1$ for any event A.

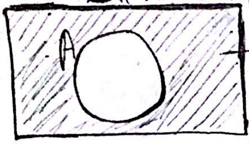
② $P(\phi) = 0$

③ $P(S) = 1$; S: sample space.

-Sec 4.3: Some Basic Relationships of Probability:-

* The complement of A « A^c »: all sample points that are not in A.

S « the sample space »



A^c: the complement of A.

- Either event A or its complement must occure, so

$$P(A) + P(A^c) = 1$$

$$\longrightarrow P(A^c) = 1 - P(A)$$

• **Ex:** let A be the event of all smokers BZU students.

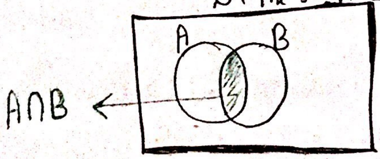
→ A^c is the event of all non smokers BZU students.

• **Ex:** let $P(A) = 0.65$, find $P(A^c)$.

→ $P(A^c) = 1 - P(A) = 1 - 0.65 = 0.45$

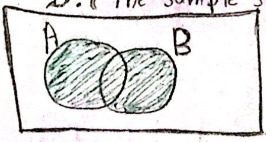
* The intersection of A and B: the event containing the the sample points belonging to both A and B. « $A \cap B$ »

S: the sample space



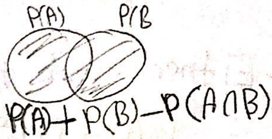
*Union of two Events A and B: all sample points belonging to A or B or both. «$A \cap B$»

S: « the sample space »



- Additive law:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$



*Note: $\cap$ « intersection » $\longrightarrow$ and

$\cup$ « union » $\longrightarrow$ or

- Mutually exclusive events:

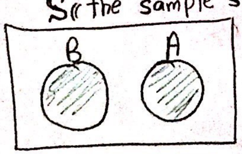
Two events are said to be mutually exclusive if the events have no sample points in common.

« A and B are mutually exclusive if when one event occurs, the other can't occure that is $A \cap B = \phi$ »

$\longrightarrow$ If A and B are mutually exclusive, $P(A \cap B) = 0$

and $P(A \cup B) = P(A) + P(B)$

S « the sample space ».



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$\boxed{22}$ Suppose that we have a sample space with five equally likely experimental outcomes E_1, E_2, E_3, E_4, E_5. let.

$$A = \{E_1, E_2\}$$

$$B = \{E_3, E_4\}$$

$$C = \{E_2, E_3, E_5\}$$

a) Find $P(A)$, $P(B)$, and $P(C)$

$\longrightarrow P(A) = \frac{2}{5}$

$P(B) = \frac{2}{5}$

$P(C) = \frac{3}{5}$

b) Find $P(A \cup B)$. Are A and B mutually exclusive.

$\longrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{2}{5} + \frac{2}{5} - 0$; $A \cap B = \{\}$

$= \frac{4}{5}$ A and B are mutually exclusive.

c) Find A^c, C^c, $P(A^c)$ and $P(C^c)$.

$\longrightarrow A^c = \{E_3, E_4, E_5\}$

$B^c = \{E_1, E_2, E_5\}$, $C^c = \{E_1, E_4\}$

$\therefore P(A^c) = \frac{3}{5}$, $P(C^c) = \frac{2}{5}$.

d) Find $A \cap B^c$ and $P(A \cap B^c)$.

$\longrightarrow A \cap B^c = \{E_1, E_2\}$

$\therefore P(A \cap B^c) = \frac{2}{5}$.

e) Find $P(B \cup C)$.

$\longrightarrow P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{2}{5} + \frac{3}{5} - \frac{1}{5}$

$= \frac{4}{5}$

- Ex: Roll a die.

let A = event of all even number is observed.

B = event of all numbers less than 3.

Find:

1. $P(A)$.

$A = \{2, 4, 6\}$, $B = \{1, 2\}$

$\longrightarrow P(A) = \frac{3}{6} = 0.5$

2. $P(B)$

$\longrightarrow P(B) = \frac{2}{6} = 0.33$

③ $P(A \cap B)$: $A \cap B = \{2\}$.

$\rightarrow P(A \cap B) = \frac{1}{6} = 0.1667$

④ $P(A \cup B)$. $A \cup B = \{2, 4, 6, 1\}$

$\rightarrow P(A \cup B) = \frac{4}{6} = 0.6667$

- Ex: Roll 2 dice.

let A = the sum is equal to 8.

B = an odd number is observed on both face.

Find $P(A \cup B)$.

$\rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$A = \{(2,6)(6,2)(3,5)(5,3)(4,4)\}$.

$B = \{(1,1)(1,3)(1,5)(3,1)(3,3)(3,5)(5,1)(5,3)(5,5)\}$.

$\therefore P(A \cup B) = \frac{5}{36} + \frac{9}{36} - \frac{2}{36} = \frac{12}{36} = 0.3333$

- Demorgan's law:-

① $(A \cup B)^c = A^c \cap B^c$

② $(A \cap B)^c = A^c \cup B^c$

- Ex: If $P(A \cup B)^c = 0.2$, $P(A) = 0.6$ and $P(B) = 0.5$

1. are A and B mutully exclusive.

$\rightarrow P(A \cup B)^c = 1 - P(A \cup B)$

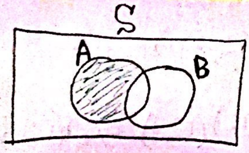
$0.2 = 1 - P(A \cup B) \longrightarrow P(A \cup B) = 0.8$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.5 - P(A \cap B)$

$\therefore P(A \cap B) = 0.3 \neq 0$; A and Ba are not mutully exclusive.

* The difference between A and B: all elements in A and not in B. « A − B »



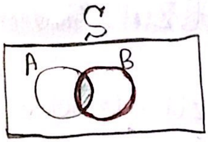
$\rightarrow P(A - B) = P(A) - P(A \cap B).$

CamScanner المسوحة ضوئيا بـ

- Sec 4.4 : Conditional probability. الاحتمال المشروط

* **Conditional probability « $P(A|B)$ »** : the probability of event A **given** that the **condition B** has occured.

→ $P(A|B)$: the probability of A given « if » B.
$P(B|A)$: the probability of B given « if » A.



A|B: حصول A علماً بأن B قد حدث

$$\therefore P(A|B) = \frac{P(A\cap B)}{P(B)}$$

احتمال التقاطع على عدد احتمال حدوث B

← أي وكأن sample space قد أصبح B.

→ $$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

- Ex: If $P(A) = 0.4$, $P(B) = 0.6$, $P(A\cap B) = 0.245$, find:-

① $P(A|B)$.

→ $$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.245}{0.6} = 0.4083$$

② $P(B|A)$.

→ $$P(B|A) = \frac{P(A\cap B)}{P(B)} = \frac{0.245}{0.4} = 0.6125$$

- Note:

In general $P(A|B) \neq P(B|A)$ لكن في التقاطع بالاتجاه متساويان.

* The probablity of 2 events both occuring, that is, the probability of the interesection of two events, is called the **Joint probability.**

* **Marginal probability** The values in the margins of a Joint probability table that provide the probabities of each event separately.

- Ex: A survey of 80 student at BZU «Gender and smoking».

	Male	Female
Smoker	18	7
Not smoker	22	33

→ Joint probability table: $P(S \cap M)$, $P(S \cap F)$, $P(N \cap M)$, $P(N \cap F)$

	M	F	T
S	$\frac{18}{80} = 0.225$	$\frac{7}{80} = 0.0875$	0.3125
N	$\frac{22}{80} = 0.275$	$\frac{33}{80} = 0.4125$	0.6875
T	0.5	0.5	1

Marginal probabilies $P(S)$, $P(N)$, $P(M)$, $P(F)$.

Suppose that a student was selected at random

1. what is the probability that the student is male.

$\rightarrow P(M) = 0.5$

② What is a probability that a student is smoker.

$\longrightarrow P(S) = 0.3125.$

③ What is the probability that a student is male and not smoker.

$\longrightarrow P(M \cap N) = 0.275$

④ What is a probability that a student is female or not smoker.

$\longrightarrow P(F \cup N) = P(F) + P(N) - P(F \cap N).$

$= 0.5 + 0.6875 - 0.4125$

$= 0.775$

⑤ What is the probability that a student is male given that he is smoker.

$\longrightarrow P(M \mid S) = \frac{P(M \cap S)}{P(S)} = \frac{0.225}{0.3125} = 0.72$

⑥ If the person is female, what is the probability that she is smoker?

$\longrightarrow P(S \mid F) = \frac{P(S \cap F)}{P(F)} = \frac{0.0875}{0.5} = 0.175$

* Independent Events:

A and B are independent if حدوث احداهما لا يؤثر على الاخر

$$P(A|B) = P(A)$$

or

$$P(B|A) = P(B)$$

$$\rightarrow P(A \cap B) = P(A) \cdot P(B)$$

∴ A and B don't affect each other.

* Multiplication law:-

$$P(A \cap B) = P(B)\, P(A|B)$$

or $$P(A \cap B) = P(A) \cdot P(B|A).$$

-Ex: If A and B are independent, $P(A) = 0.6$, $P(B) = 0.4$. Find:-

1. $P(A|B) = P(A) = 0.6$
2. $P(B|A) = P(B) = 0.4$
3. $P(A \cap B) = P(A) \cdot P(B) = 0.24$
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   $= 0.76$

- Ex: related to the previous example. (Gender, Smoking).

(7) Are the variable $\therefore$ Gender and Smoking mutually exclusive?

→ check any Joint probability :- (حتى تكون منفصلة يجب أن تكون جميع التقاطعات (بين المتغيرين) = 0)

$P(M \cap S) \neq 0$

$\therefore$ Gender and Smoking aren't mutually exclusive.

(8) Are the variables: Gender and Smoking independent?

→ check any conditional probability. ويجوز فحصها عن طريق التقاطع $P(A \cap B) \stackrel{??}{=} P(A).P(B)$

$P(M | S) = 0.72 \neq 0.625$

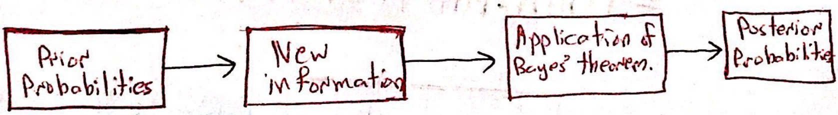
$\therefore$ Gender and Smoking aren't independent.

– Sec 4.5 : Bayes' Theorem:

نظرية بيز

- We begin the analysis with initial or prior probability estimates for specific events of interest. Then from sources such as a sample, special report, ... we obtain additional information about the event. Given this information, we update the prior probability values by calculating revised probabilities, reffered to as posterior probabilities.

Bayes' theorem provides a mean for making these probability calculations.



التحليل الذي تستند فيه الاحتمالات المسبقة لحالات الطبيعة يُعرف بالتحليل القبلي أو المسبق وبناء عليه نتخذ القرار الأمثل. إذا قمنا بالبحث عن عن معلومات إضافية من خلال دراسة المعاينة (العينة) واستخدام هذه المعلومات في تنقيح الاحتمالات المسبقة بدمج الاثنين معاً من خلال قانون بيز

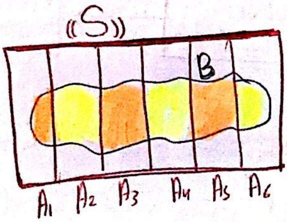
* Bayes' theorem:

Let S be a sample space, and let $A_1, A_2, \dots, A_n$ be the n-mutually exclusive events of S whose union is the entire sample space, that is $A_i \cap A_J = \phi \ \forall i, J = 1, \dots, n$ and $A_1 \cup A_2 \cup \dots \cup A_n = S$. With prior probabilities $P(A_1), P(A_2), \dots, P(A_n)$ and the appropriate conditional probabilities $P(B|A_1), P(B|A_2), \dots, P(B|A_n)$ where B is any event of S. Then:-

① $P(B) = P(\underline{A_1 \cap B} \cup \underline{A_2 \cap B} \cup \dots \cup \underline{A_n \cap B})$

$= P(A_1 \cap B) + P(A_2 \cap B) + \dots + P(A_n \cap B)$

$= P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \dots + P(B|A_n) \cdot P(A_n)$.



$P(B) = P(A_1 \cap B) + \dots + P(A_n \cap B)$

Now we Know:-

$\cdot \ P(B|A_1) = \dfrac{P(A_1 \cap B)}{P(A_1)}$

$\rightarrow P(A_1 \cap B) = P(B|A_1) P(A_1)$

$\vdots$

$P(A_n \cap B) = P(B|A_n) P(A_n)$

② $P(A_i|B) = \dfrac{P(A_i \cap B)}{P(B)}, \ \forall i$

$= \dfrac{P(B|A_i) P(A_i)}{P(B|A_1)P(A_1) + \dots + P(B|A_n)P(A_n)}, \ \forall i = 1, \dots, n$

- Ex: The prior probabilities for events A_1 and A_2 are $P(A_1) = 0.4$ and $P(A_2) = 0.6$. It is also Known that $P(A_1 \cap A_2) = 0$. Suppose $P(B|A_1) = 0.2$ and $P(B|A_2) = 0.05$.

a) Are A_1 and A_2 mutually exclusive?

$\longrightarrow$ $P(A_1 \cap A_2) = 0$ so A_1 and A_2 are mutually exclusive

b) Compute $P(A_1 \cap B)$ and $P(A_2 \cap B)$.

$$\longrightarrow P(A_1 \cap B) = P(B|A_1) \cdot P(A_1)$$
$$= (0.2)(0.4)$$
$$= 0.08$$

$$\longrightarrow P(A_2 \cap B) = P(B|A_2)\, P(A_2)$$
$$= (0.05)(0.6)$$
$$= 0.03$$

c) Compute $P(B)$.

$$\longrightarrow P(B) = P(B \cap A_1) + P(B \cap A_2)$$
$$= 0.08 + 0.03$$
$$= 0.11$$

d) Apply Bayes' theorem to compute $P(A_1|B)$ and $P(A_2|B)$.

$\longrightarrow P(A_1|B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{0.08}{0.11} = 0.7273$

$\longrightarrow P(A_2|B) = \dfrac{P(A_2 \cap B)}{P(B)} = \dfrac{0.03}{0.11} = 0.2727$

— Ex: In a certain assemply plant, 3 machines B_1, B_2, and B_3 make 30%, 45%, and 25% respectively, of the product. It's Known from past experience that 2%, 3%, and 2% of products made by each machine respictively, are deffictive. What is the probability that a finished product is defective.

B_1 : 30% → D 2% « $P(D|B_1) = 0.02$ »
→ ND

B_2 : 45% → D 3% « $P(D|B_2) = 0.03$ »
→ ND

B_3 : 25% → D 2% « $P(D|B_3) = 0.02$ »
→ ND

$$\rightarrow P(D) = P(D \cap B_1) + P(D \cap B_2) + P(D \cap B_3)$$
$$= P(D|B_1)P(B_1) + P(D|B_2)P(B_2) + P(D|B_3)P(B_3)$$
$$= (0.02)(0.3) + (0.03)(0.45) + (0.02)(0.25)$$
$$= 0.0245$$

-Ex: With reference to previous example, if a product was chosen randomly and found to be deffective, what is the probability that it was made by machine B_3?

$$\rightarrow P(B_3|D) = \frac{P(B_3 \cap D)}{P(D)}$$

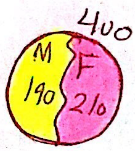
$$= \frac{P(D|B_3)\,P(B_3)}{P(D)}$$

$$= \frac{(0.02)\,(0.25)}{0.0245} = 0.2041$$

-Ex: A sample of 400 students was taken from some university. In the sample of 190 students were males. Among the male students 40 were science students, 70 were information technology and 50 were engineering students. Among the female students, 30 were engineering, 50 students were information technology, and 70 were education students.

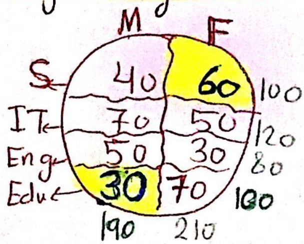
a) A student was selected at random, what is the probability the student is a female?

$\longrightarrow P(F) = \frac{210}{400} = 0.525$



b) A student was selected at random, what is the probability the student is an engineering student?

$\longrightarrow P(Eng) = \frac{80}{400} = 0.2$



c) A student was selected at random, what is the probability the student is a male science student?

$\longrightarrow P(M \cap S) = \frac{40}{400} = 0.1$

d) A student was selected at random, what is the probability the student is female or a science student?

$$\longrightarrow P(F \cup S) = P(F) + P(S) - P(F \cap S)$$
$$= \frac{210}{400} + \frac{100}{400} - \frac{60}{400}$$
$$= \frac{250}{400} = 0.625$$

e) An information technology student was selected, what is the probability that the student is female?

$$\longrightarrow P(F \mid IT) = \frac{P(F \cap IT)}{P(IT)} = \frac{50/400}{120/400} = 0.4167$$

f) A male student was selected, what is the probability that the student is an education student?

$$\longrightarrow P(Edu \mid M) = \frac{P(Edu \cap M)}{P(M)} = \frac{30/400}{190/400} = 0.1579$$

g) Using the above answer, show if gender and faculty are independent ?

$$P(M|S) \overset{??}{=} P(M)$$

$$\frac{40}{100} \overset{??}{=} \frac{190}{400}$$

$0.4 \neq 0.475$ $\therefore$ Gender and faculty are not independent.

h) Two students were selected at random, what is the probability that both are education students ?

$$\rightarrow P(EE) = P(E)\,P(E)$$

$$= \frac{100}{400} \cdot \frac{100}{400} = 0.0625$$

اذا اخترنا طالبين فإن احتمال ان يكون الأول تربية لا يؤثر على احتمال كون الثاني تربية لذا فهما independent .