*Ch5: Discrete probability distributions.

- Sec 5.1: Random variables

A random variable provides a mean for describing experimental outcomes using numerical values.

- Random variable:
  is a numerical description of the outcome of an experiment.

- A random variable can be classified either discrete or continuous.

1 Discrete random variable:-

A random variable that may assume either a finite number or infinite sequence of values such as 0, 1, 2, ...

- Examples:

Experiment	Random variable	Possible values of the r.v
① Contact five customers	Number of customers who place in order	0, 1, 2, 3, 4, 5
② Inspect a shipment of 50 radios	Number of defictive radios	0, 1, 2, 3, ..., 50
③ Operate a restaurant for one day.	Number of customers	0, 1, 2, ...
④ Sell an automobile	Gender of the customer	0 if male, 1 if female

2 Continuous random variable:

A random variable that may assume any numerical value in an interval or collection of intervals. (have decimals).

- Ex: time, weight, distance, temperature, ...

- Ex: let X be the time between consecutive incoming cells in minutes. « $X \geq 0$ »

***Exercises:-**

1 Consider the experiment of tossing a coin twice:

a) Define a random variable that represents the number of heads occuring on the two tosses. $S = \{(HH)(TH)(HT)(TT)\}$.

$X = 0, 1, 2$

b) Is this random variable discrete or continuous?

discrete.

-Sec 5.2 : Discrete probability distribution:

→ the probability distribution: for a random variable describes how probabilities are distributed over the values of the random variables.

→ the probability functions «$f(x)$» : provides the probability for each value of the random variables.

*Required conditions for discrete probability function:

① $f(x) \geq 0$

② $\sum_x f(x) = 1$

-We present probability distributions by graphs, tables, or formula.

-Ex: Tossing 2 coins :

① Let X be a random variable defined by the number of heads.

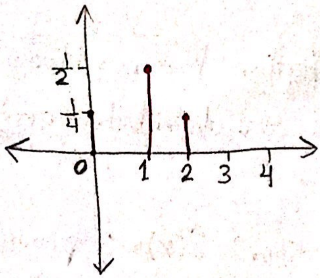
$S = \{(H,H)(H,T)(T,H)(T,T)\}$

→ $X = 0, 1, 2$. «discrete»

② Construct a probability distribution.

X	0	1	2
f(x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

notice that:
$f(x) \geq 0 \quad \forall x$
$\sum f(x) = 1$



* Special discrete probability:

- Discrete uniform probability function:

$$f(x) = \frac{1}{n}$$, for all x ; where n the number of values the random variable may assume.

- Ex: Suppose that for the experiment of rolling a die, we define the random variable X to be the number of dots on the upward face.

→

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

, formula $f(x) = \frac{1}{6} \quad \forall\, x = 1, 2, \dots, 6.$

-Ex: Consider the random variable X with the following discrete probability distribution:

x	$f(x)$
1	1/10
2	2/10
3	3/10
4	4/10

$\longrightarrow f(x) = \frac{x}{10} \quad \forall\, x = 1, 2, 3, 4.$

* The more widely used discrete probability distribution generally are specified formulas. Three important cases are the binomial, Poisson, and hypergeometric distributions.

-Ex: The probability distributions for the random variable X follows:

x	20	25	30	35
$f(x)$	0.2	0.15	0.25	0.4

a) Is this probability distribution valid? Explain?

$0 \leq f(x)$

$\cdot \sum f(x) = 1$

$\therefore$ Yes

b) What is the probability that X is less than or equal to 25?

$P(20)+P(25)=0.2+0.15=0.35$

c) What is the probability that X is greater than 30?

$P(35)=0.4$

d) Find $P(20 \leq X < 35)$.

$P(20)+P(25)+P(30)=0.2+0.15+0.25=0.6$

- Sec 5.3 : Expected value and Variance.

$\boxed{1}$ Expected value:

or mean of a random variable is a measure of the central location for the random variable.

- The expected value is the weighted average of the values the random variable may assume. The weights are the probabilities.

⟶ Expected value of a discrete random variable

$$E(x) = \mu = \sum x f(x)$$

$\boxed{2}$ Variance:

to summarize the variability in the values of a random variable.

- The variance is a weighted average of the squared deviations of a random variable from its mean. The weights are the probabilities.

$\longrightarrow$ $Var(X)$ «σ^2» $= \sum (x-\mu)^2 f(x)$

$$= \sum x^2 f(x) - \mu^2$$

$\boxed{3}$ The standard deviation «σ»:

the positive square root of the variance.

- Ex: The following table provides a probability distribution for the random variable X.

x	3	6	9
$f(x)$	0.25	0.5	0.25

a) Compute the expected value of X.

$$E(x) = \sum x f(x)$$
$$= 3(0.2) + 6(0.5) + 9(0.25)$$
$$= 4.5$$

b) Compute the variance.

$$\sigma^2 = \sum (x-\mu)^2 f(x)$$
$$= (3-4.5)^2(0.25) + (6-4.5)^2(0.5) + (9-4.5)^2(0.25)$$
$$= 6.75$$

c) Compute the standard deviation.

$\longrightarrow \sigma = \sqrt{6.75} = 2.598$

* Using calculator:

Expected value and Variance for discrete random variables using calculator:- $f_x - 82$

X	X_1	X_2	....	X_n
$f(x)$	$f(X_1)$	$f(X_2)$	....	$f(X_n)$

Mode 2

X_1 M+ X_2 M+ ---- X_n M+

▽ ▽ $f(X_1)$ = ▽ ▽ $f(X_2)$ = ---- ▽ ▽ $f(X_n)$ =

ON

1. to find the expected value.

shift 2 1 =

2. to find the standar

shift 2 2 =

3. to find the variance
We just square σ

- Ex: The following table provides a probability distribution for the random variable X.

X	3	1	2	6
f(x)	0.2	0.3	0.35	

a) Find P(6).

→ $P(6) = 1 - (0.2 + 0.3 + 0.35)$
$= 0.15$

b) Find E(X).

mode 2

3 M+ 1 M+ 2 M+ 6 M+

▽ ▽ 0.2 = ▽ ▽ 0.3 = ▽ ▽ 0.35 = ▽ ▽ 0.15 =

ON shift 2 1 =

$E(X) = 2.5$

c) Find σ.

shif 2 2 =

$\sigma = 1.628$

d) find σ^2.

$\sigma^2 = (1.628)^2 = 2.65$

*Sec 5.4 : Binomial probability distribution:-

The binomial probability distribution is a discrete probability distribution that provides many applications.

→ It is associated with a multiple-step experiment.

- Properties of Binomial experiment.

1) The experiment consists of a sequence of $\underbrace{n \text{ identical trials.}}_{n=\text{sample size}}$

2) 2 outcomes are possible on each trial.

We refer to one outcome as success and the other as a failure.

S: The outcome we study

F: the complement

3) The probability of a success, denoted by p, does not change (fixed) from trial to trial, the probability of a failure, denoted by 1-P doesn't change.

4) The trials are independent.

- Ex: Tossing a coin 3 times:-

let X be the number of heads.

→ $X = 0, 1, 2, 3$

we have 3 trials, $P(S) = \frac{1}{2}$, $P(F) = \frac{1}{2}$

the trials are independent.

∴ the experiment is called a binomial experiment.

* Binomial probability function:-

let X be a random variable, and X has a binomial distribution, then:

X: # of successes.

① $P(x) = \binom{n}{x} p^x (1-p)^{n-x}$ where n: the number of trials

$\binom{n}{x} = \frac{n!}{x!(n-x)!} = n\ \boxed{nCr}\ x$

p: the probability of a success.

$1-p$: the probability of a failure.

② $P(0) + P(1) + \cdots + P(n) = \sum_{x=1}^{n} P(x) = 1$

③ The expected value "mean":-

$E(x) = \mu = np$.

④ The variance :

$var(x) = \sigma^2 = np(1-p)$.

⑥ The standard deviation:

$\sigma = \sqrt{np(1-p)}$.

-Ex: 30% of workers take public transportation daily, in a sample of 10 workers:-

a) What is the probability that exactly 3 workers take public transportation?-

X = # of workers take public transportation.

$n = 10$

$\therefore X = 0, \ldots, 10$.

$P = 30\% = 0.3$

$$\longrightarrow P(3) = \binom{10}{3}(0.3)^3(0.7)^7 = 0.2668$$

b) What is the probability that all of them take public transportation?

$$\longrightarrow P(10) = \binom{10}{10}(0.3)^{10}(0.7)^0 = 5.9049 \times 10^{-6}$$
$$= 0.000006$$

c) What is the probability that at least 3 workers take public transportation daily?

$$\longrightarrow P(3) + P(4) + \cdots + P(10) = 1 - P(0) - P(1) - P(2)$$

$$P(0) = \binom{10}{0}(0.3)^0(0.7)^{10} = 0.0282$$

$$P(1) = \binom{10}{1}(0.3)^1(0.7)^9 = 0.1211$$

$$P(2) = \binom{10}{2}(0.3)^2(0.7)^8 = 0.2335$$

$$\therefore \longrightarrow 1 - 0.0282 - 0.1211 - 0.2335 = 0.6172$$

d) What is the probability that at most 3 workers take public transportation?

$$P(3) + P(2) + P(1) + P(0)$$

$$= \binom{10}{3}(0.3)^3(0.7)^7 + \binom{10}{2}(0.3)^2(0.7)^8 + \binom{10}{1}(0.3)^1(0.7)^9 + \binom{10}{0}(0.3)^0(0.7)^{10}$$

$$= 0.2668 + 0.2335 + 0.1211 + 0.0282$$

$$= 0.6496$$

e) What is the probability that 6 workers don't take public transportation?

$$P(6) = \binom{10}{6}(0.7)^6(0.3)^4 = 0.2001$$

f) What is the expected number of workers that take a public transportation?

$$E(X) = np = 10(0.3) = 3$$

g) What is the variance of workers who take a public transportation?

$$\sigma^2 = np(1-p) = 10(0.3)(0.7) = 2.1$$

ملاحظة: standard deviation ← فقط نأخذ الجذر التربيعي "square root" لـ variance

-Sec5.5: Poisson Probability Distribution:-

In this section, we consider a discrete random variable that is useful in estimating the number of occurances over a specified interval of time or space.

*Propability of a Poisson distributions experiment:-

(1) The probability of an occurrence is the same for any two intervals of equal length.

(2) The occurrence or nonoccurence in any interval is independent of the occurence or nonoccurence in any other interval.

* Poisson probability function:

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \quad ;$$

$f(x)$ = the probability of x occurence in an interval.

μ = the expected value or mean number of occurences in an interval.

$e = 2.718....$ ((Euler number)).

- Ex:

1. the number of arrivals at a car wash in one hour.
2. the number of leaks in 100 miles of pipeline.
3. the number of calls received by Jawwal center per hour.

→ Poisson probability distribution:-

let X be arandom variable, and X has a poisson distribution, then:-

1. $P(x) = \frac{\mu^x e^{-\mu}}{x!}$; $x = 0, 1, 2, \ldots$
2. $P(0) + P(1) + P(2) + \cdots = \sum_{x=0}^{\infty} P(x) = 1$
3. A property of the Poisson distribution is that mean and variance are equal, that is:-

   $\mu = E(x) = \sigma^2$

4. the standard deviation of X « σ » $= \sqrt{\mu}$.

⑤ μ changes according to the interval.

-Ex:

During the period of time that a local university takes phone-in registrations, calls come in at the rate of one every 2 minutes.

one call every 2 minute.

$X = \#$ of calls $= 0, 1, 2, \ldots$

a) What is the expected number of call in one hour?

$$1 \longrightarrow 2 \text{ minutes}$$

$$\boxed{\mu} \longrightarrow 1 \text{ hour} = 60 \text{ minutes}$$

$E(X) = \mu = 30$ calls in one hour.

b) What is the probability of 3 calls in 5 minutes?

$$P(X) = \frac{\mu^x e^{-\mu}}{x!} \quad ; \quad X = 3$$

$$1 \longrightarrow 2 \text{ min}$$

$$?? \longrightarrow 5 \text{ min}$$

$\therefore \mu = \frac{5}{2} = 2.5$ calls in 5 minutes.

$$\rightarrow P(3) = \frac{2.5^3 e^{-2.5}}{3!} =$$

« from calculator :

2.5 $\boxed{\wedge}$ 3 $\boxed{\text{shift}}$ $\boxed{\ln}$ −2.5 $\boxed{\div}$ 3 $\boxed{\text{shift}}$ $\boxed{x^{-1}}$ $\boxed{=}$

$= 0.2138$

c) What is the probability of no calls in a 5 minute period?

$X = 0$
$\mu = 2.5$

$\rightarrow P(0) = \frac{2.5^0 e^{-2.5}}{0!} = 0.0821$

d) What is the variance of calls in a day?

$1 \rightarrow 2 \text{ min.}$
$?? \rightarrow 1 \text{ day} = 24 \times 60 = 1440$

$\therefore \mu = \frac{1440}{2} = 720$ call

$\rightarrow \sigma^2 = 720$.

e) What is the standard deviation of calls in a day?

$\sigma = \sqrt{\mu} = \sqrt{720} = 26.83$

f) What is the probability that at least 2 calls in 5 minutes? $\mu = 2.5$

$P(2) + P(3) + \cdots \}$ complement.

$\rightarrow 1 - P(0) - P(1) =$

$$= 1 - \frac{2.5^0 \cdot e^{-2.5}}{0!} - \frac{2.5^1 e^{-2.5}}{1!}$$

$$= 0.9179$$