* Review: Ch4 + 5 :-

- Concepts :

① Experiment.

② Sample space « S »

③ Sample point.

④ Event.

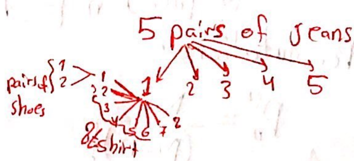
⑤ Prior probability.

⑥ Posterior probability.

- Mutiple-step experiment: the total number of outcomes = $n_1 \cdot n_2 \cdots n_K$

-Ex: How many outfits are possible with 5 pairs of Jeans, 8 t-shirt, and 2 pairs of shoes?

$\longrightarrow 5 \times 8 \times 2 = \underline{80}$



-Ex: If we rolled a dice three times, then the total number of outcomes is.

$\longrightarrow 6 \times 6 \times 6 = \underline{216}$

-Ex: If we want to create a passward of 3 numbers, how many password can we form if:-

1. digits can be repeated.

$\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$
10 10 10

أمام كل منزلة ١٠ اختيارات
«٠، ١، ---، ٩»
لأن التكرار مسموح.

$\rightarrow 10 \times 10 \times 10 = \underline{1000}$

2. digits can't be repeated.

$\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$
10 9 8

$\rightarrow 10 \times 9 \times 8 = \underline{720}$

- Combinations:

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$; order isn't important, repetition isn't allowed.

N $\boxed{nCr}$ n $\boxed{=}$

- Permutations:

$$P_n^N = \frac{N!}{(N-n)!}$$, order is important. repetition isn't allowed.

N $\boxed{shift}$ $\boxed{nCr}$ n $\boxed{=}$

- Ex: In how many ways can a teacher choose $\underline{\underline{2}}$ students from among 5 student?

$$\longrightarrow C^5_2 = 5 \; \boxed{nCr} \; 2 = \underline{\underline{10}}$$

لو كان الطلاب بمراتب يكون الحل على
permutation

- Equally likely experimental outcomes:-
  If n outcomes are possible, $P(E_i) = \frac{1}{n}$; $i = 1, \dots, n$

- Probability = relative frequency $= \frac{f}{n}$.

- $P(S) = 1$, $P(\phi) = 0$; $0 \leq P(A) \leq 1$ for any event

- The complement of A is A^c:-
  $\longrightarrow P(A) + P(A^c) = 1$

- The union of A and B: «or»
  $\longrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  intersection «and»

- Mutually exclusive events: «disjoint events».
  $\longrightarrow A \cap B = \phi$
  $P(A \cap B) = 0$
  $P(A \cup B) = P(A) + P(B)$.

• Conditional probability: "given that ≡ If"

$$P(A|B) = \frac{P(A\cap B)}{P(B}$$

$\longrightarrow P(A|B) \neq P(B|A)$.

• Independent events:

① $P(A|B) = P(A)$.

② $P(B|A) = P(B)$

③ $P(A\cap B) = P(A)P(B)$.

④ $P(A\cup B) = P(A) + P(B) - P(A)P(B)$.

- Ex: If $P(A) = 0.8$, $P(A\cap B) = 0.24$ and $P(A\cup B^c) = 0.94$. If A and B are independent. Find

① $P(B)$

independent

$\longrightarrow P(A\cap B) = P(A)\,P(B)$

$\therefore P(B) = \frac{0.24}{0.8} = 0.3$

② $P(B|A^c) =$

$\longrightarrow P(B|A^c) = \frac{P(B\cap A^c)}{P(A)}$

we have: $P(A \cup B^c) = 0.94$

$\therefore\ P(A \cup B^c)^c = 1 - P(A \cup B^c) = 0.06$

$\rightarrow \boxed{P(A^c \cap B) = 0.06} \rightarrow$ from Demorgan's law.

$\therefore\ P(B \mid A^c) = \dfrac{P(B \cap A^c)}{P(A^c)} = \dfrac{0.06}{0.2} = 0.3$

3. Are B and A^c independent?

Yes, since $P(B \mid A^c) = P(B)$.

- Ex: An insurance company is studing the probability of accidents over many years in a certain city during May. It was noted that the probability it will rain in May is 30%. Moreover, it was noted that on rainy days the probability of car accidents is 60%, while on dry days (days without rains) the probability of car accidents is 30%. If an accident happened, what is the probability it was a dry day?

Rainy days « R » 0.3 — car accidents « A » 0.6

Dry days « D » 0.7 — car accidents « A » 0.3

$$P(D|A) = \frac{P(D \cap A)}{P(A)}$$

$$\longrightarrow P(A) = P(A \cap R) + P(A \cap D)$$

$$= P(A|R)P(R) + P(A|D)P(D)$$

$$= 0.6(0.3) + 0.3(0.7)$$

$$= 0.39$$

$$\therefore P(D|A) = \frac{0.21}{0.39} = 0.5385$$

– Ex: Consider the following probability distribution

X	-2	0	1	3	4
$f(X)$	0.17	0.22		0.12	0.3

① Find $P(1)$.

$$\sum_{\forall x} P(X) = 1$$

$$\therefore P(1) = 1 - (0.17 + 0.22 + 0.12 + 0.3)$$

$$= 1 - 0.81$$

$$\rightarrow P(1) = 0.19$$

② $E(X)$.

"or by $E(X) = \sum x f(x)$"

$\boxed{\text{Mode}}$ $\boxed{2}$

-2 $\boxed{M+}$ 0 $\boxed{M+}$ 1 $\boxed{M+}$ 3 $\boxed{M+}$ 4 $\boxed{M+}$

$\boxed{\nabla}$ $\boxed{\nabla}$ 0.17 $\boxed{=}$ $\boxed{\nabla}$ $\boxed{\nabla}$ 0.22 $\boxed{=}$ $\boxed{\nabla}$ $\boxed{\nabla}$ 0.19 $\boxed{=}$ $\boxed{\nabla}$ $\boxed{\nabla}$

0.12 $\boxed{=}$ $\boxed{\nabla}$ $\boxed{\nabla}$ 0.3 $\boxed{=}$ $\boxed{\text{ON}}$

$\boxed{\text{shift}}$ $\boxed{2}$ $\boxed{1}$ $\boxed{=}$ $\underline{1.41}$

③ σ.

or by $\sigma^2 = \sum (X - M^2) f(x)$

$\boxed{\text{shift}}$ $\boxed{2}$ $\boxed{2}$ $\boxed{=}$ $\underline{2.18}$

في حال طلب σ^2 فقط نربع $\underline{2.18}$

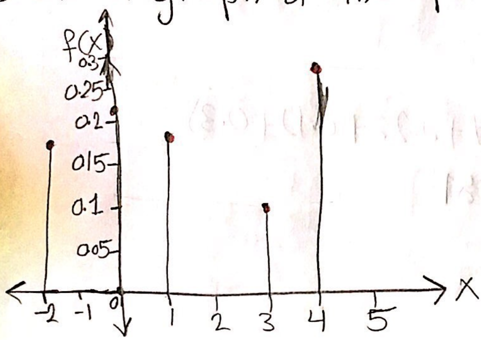
4) Find $P(0 \leq X < 3)$.

$= P(0) + P(1)$

$= 0.22 + 0.19$

$= 0.41$

5) Draw a graph of the probability distribution.



- Ex: In a store, out of all the people who came there 30% bought a shirt. If **20** people came in the store together. عند ما يكون على شكل نسبة مئوية أو نسبة كـ "one_fifth" بيكون binomial

1) Find the probability of 5 of them buying a shirt.

0.17886305

$\rightarrow P(1) = \binom{20}{5}(0.3^5)(0.7^{15}) = 0.1789$

2) Find the probability of no one buying a shirt.

$\rightarrow P(0) = \binom{20}{0}(0.3)^0(0.7)^{20} = 0.0008$

7.979×10^{-4}

$= 0.00079$

- Ex: The number of holes in a pipeline has a poisson distribution with a mean of 8 holes per 2 meter

1) What is the probability of at least 1 holes in 40 cm.

$$8 \rightarrow 2\ m = 200\ cm$$
$$?? \rightarrow 40\ cm$$

$$\rightarrow \mu = \frac{40 \times 8}{200} = 1.6$$

$$\therefore P(1) + P(2) + \cdots = 1 - P(0)$$
$$= 1 - \frac{e^{-1.6}(1.6)^0}{0!}$$
$$= 1 - 0.2019$$
$$= 0.7981$$