* Review: (Ch6, 8, 9)

Q1 Assume a random sample size of 40 has a mean of 110. let the population standard deviations b 20.

Construct a 90% confidence interval for the population mean. $\alpha = 0.1$, $n = 40$, $\overline{X} = 110$, $\sigma = 20$
«Known»

A 90 % C.I for $\mu$ is $(\overline{X} - E, \overline{X} + E)$

$$E = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = Z_{0.05} \left(\frac{20}{\sqrt{40}}\right) =$$

$Z_{0.05} = t_{0.05}$, $df = \infty$
$= 1.645$

$$\therefore E = 1.645 \left(\frac{20}{\sqrt{40}}\right) = 5.2$$

$$A \ 90\% \ CI \ for \ \mu = (110 - 5.2, 110 + 5.2)$$
$$= (104.8, 115.2)$$

**Q2** Assume a random sample of size 15 has a mean of 58.6 and a standard deviation of 7.54. Construct a 90% confidence interval for the population mean. $\alpha = 0.1$

$\longrightarrow n = 15, \ \bar{x} = 58.6, \ s = 7.54.$

$\sigma$ unknown

$$E = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} ; \quad t_{\frac{\alpha}{2}} = t_{0.05} \quad df = n-1 = 14$$

$$= 1.761$$

$$= 1.761\left(\frac{7.54}{\sqrt{15}}\right)$$

$$= 3.4283\ldots = 3.43$$

A 90% C.I is $(58.6 - 3.43, 58.6 + 3.43)$

$$= (55.17, 62.03).$$

Q3 A healt care professional wishes to estimate the birth weights of infants. What is the minimum sample size must be obtained if he desired to be 90% confident that the true mean is within 0.5 Kg of the sample mean? Assume the range is 5.

$\longrightarrow n = \left( Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{E} \right)^2$ ; $Z_{\frac{\alpha}{2}} = Z_{0.05} = t_{0.05}$ $df = \infty$

$= \left( \frac{1.645(1.25)}{0.5} \right)^2$ $\sigma = \frac{Range}{4} = \frac{5}{4} = 1.25$

$= 16.9 \xrightarrow{\text{round up}} 17$ or more.

Q4 The prices of a particular model of digital camera for some online retailers are shown below: 225 // 240 // 206 // 193 // 250 // 202

a) Find the point estimate for the population mean.

$\overline{X} = 219.33$

Mode 2

225 M+

⋮

202 M+

shift 2 1 =

b) Find the point estimate for the population standard deviation.

Shift | 2 | 3 | =

$S = 22.68$

c) Construct a 99% confidence interval for the true mean price.

$(\overline{X} - E, \overline{X} + E)$ ; $E = t_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$ ; $df = n-1 = 5$

⋮

$\boxed{Q5}$ If the confidence interval for the true pop mean is $(11.5, 20.3)$, then find :-

① the sample mean. $(\overline{X} - E, \overline{X} + E) = (11.5, 20.3)$

$$\overline{X} = \frac{\overline{X} - E + \overline{X} + E}{2} = \frac{11.5 + 20.3}{2} = \boxed{15.9}$$

② the margin of error.

$$E = \frac{(\overline{X} + E) - (\overline{X} - E)}{2} = \frac{20.3 - 11.5}{2} = 4.4$$

Q6 Find the critical value(s) in each case below:-

(to test the hypothesized mean).

1) $\alpha = 0.01$, $\sigma$ Known, lower tailed test.

$$-Z_{\alpha} = -t_{\alpha} \qquad df = \infty$$

$$= -t_{0.01} \quad , \quad df = \infty$$

$$= -2.326$$

المساواة دائماً مع الرفض

reject $H_0$ if $Z_{test} \leq -Z_{\alpha}$

b) $\alpha = 0.1$, $\sigma$ Known, upper tailed.

$$Z_{\alpha} = t_{\alpha} \qquad df = \infty$$

$$= t_{0.1} \qquad df = \infty$$

$$= 1.282$$

c) $\alpha = 0.05$, $\sigma$ Known, 2-tailed test

$$Z_{\frac{\alpha}{2}} \; , \; -Z_{\frac{\alpha}{2}}$$

$$\longrightarrow Z_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} \quad ; \quad df = \infty$$

$$= t_{0.025} \quad , \quad df = \infty$$

$$= 1.96$$

$\therefore$ the critical values are $-1.96$, $1.96$

d) $\alpha = 0.1$ , $\sigma$ unknown , $n = 18$ . 2-tailed test

$\longrightarrow -t_{\frac{\alpha}{2}} \, , \, t_{\frac{\alpha}{2}}$ $\quad df = n - 1 = 17$

$t_{\frac{\alpha}{2}} = t_{0.05}$ ; $df = 17$

$= 1.74$

$\boxed{Q4}$ Find the p-value in each case:-

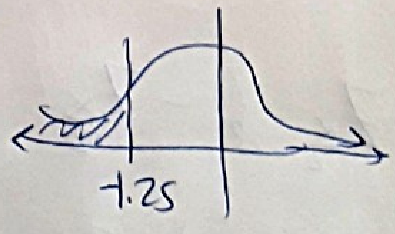
a) $\mu_0 = 80$ , $\bar{X} = 78.5$ , $\sigma = 12$ , $n = 100$ ,

lower tailed test.

$$\longrightarrow Z_{test} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{78.5 - 80}{12 / \sqrt{100}} = -1.25$$

$\text{p-value} = P(Z < Z_{test})$ lower لأنه

$= P(Z < -1.25)$




$= 1 - P(Z > 1.25)$

$= 1 - 0.8944$

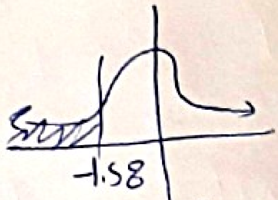
$= 0.1056$

| | 0.05 |
|---|---|
| 1.2 | 0.8944 |

b) $\mu_0 = 125500$, $\bar{x} = 118000$, $\sigma = 30000$,
$n = 40$, 2-tailed test.

$$\rightarrow Z_{test} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{118000 - 125500}{30000/\sqrt{40}} = -1.58$$

p-value $= 2P(Z < Z_{test})$




$= 2(0.0571)$

$= 0.1142$

$1 - P(Z < 1.58)$

$= 1 - 0.9429$

$=$