2009   19 ??
20 ?? chpt 7 ?

Question # 1   Fill-In the-Blank

Random

I)   A...~~~~ variable..... is a numerical description of the outcome of an experiment.

II)   The name (or shape) of the discrete random variables that express number of cars that arrive at a

Gas station محطة بنزين each hour in a given day is:....paisson........Random

variable

III) A random variable that may assume either a finite number of values or an infinite sequence of

values such as 0, 1, 2,.... Is a ....Simple.... Random variable
      discreat

IV)   The name (or shape) of the continuous random variables that express number of (hours (life time)

of) a lamp can function before failure is:................normal Poshei......Random variable
      Expotential Expontical

Question # 2

A local ambulance service handles 0 to 5 service calls on any given day. The probability distribution for the number of
service calls is shown in the following Table . Use the table to answer questions(I to IV).

X	0	1	2	3	4	.5
X²	0	1	4	9	16	25
p(x)	.15	.10-	.10	.30	.15	.20

(I)   The random variable X is
a)   Discrete
b)   Continuous
c)   Both
d)   neither

$0 + 0.1 + 0.2$
$+ 0.9 + 0.6 +$

(II)   The expected value.
a)   2.05
b)   2.45
c)   2.80
d)   5.05

$E(x^2) - (E(x))^2$

$(x-\mu)^2 f_x$ $2.8^2 \times 0.15$

0

(III) The variance is.
a)   2.2
b)   2.05
c)   1.65
d)   2.76

$0.1 +$
$0.4 +$
2.7
2.4
5
$10.6-$

12

(IV)   The standard deviation of X is
a)   1.05
b)   1.15
c)   1.43
d)   1.66

2.0

Part: Show all your work

16. A life insurance company has determined that each week an average of 10 claims is filed in one of its branch.

5 points   a. What is the probability that during the next week exactly 5 claims will be filed?

5 - //   b. What is the expected number of claims in two weeks?

(a) $\mu = 10$, $x = 5$

$$P(X=5) = \frac{10^5 e^{-10}}{5!} = 0.0378 \checkmark$$

$$\frac{\mu^x e^{-\mu}}{x!} \qquad \text{go=}\mu$$

$$\mu = 10$$
$$x = 5$$

$$\mu = nb$$

(b) $\mu = (2)(10) = 20$ claims

$$\text{SS.} \checkmark \qquad \mu \Rightarrow$$

1 week $\rightarrow$ 5
2 wee k x
$x = 2 * 5 = 10$

17. The average price of personal computers manufactured by MNM Company is $1,200 with a standard deviation of $200. Furthermore, it is known that the computer prices manufactured by MNM are normally distributed. **DO NOT ROUND YOUR NUMBERS.**

5 points   a. What is the probability that a randomly selected computer will have a price of at least $1,500?
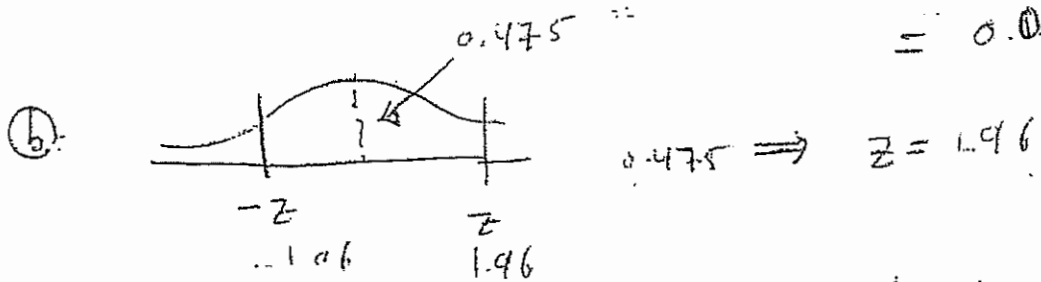
5 //   c. What are the minimum and the maximum values of the middle 95% of computer prices?

5 //   d. If 513 of the MNM computers were priced at or below $647.80, how many computers were produced by MNM?

$\mu = 1200$, $\sigma = 200$

(a) $P(X > 1500) = P\left(Z > \frac{1500 - 1200}{200}\right) = P(Z > 1.5)$

$$= 0.0668$$

(b).

0.475

$-Z$ Z
..1.96   1.96

$0.475 \Rightarrow Z = 1.96$

$+1.96 = \frac{X - 1200}{200} \Rightarrow$ $X = 808$ to $X = 1592$

(c) $P(X < 647.8) = P(Z < -2.76) = \boxed{0.0029} \rightarrow 513$

$$\Rightarrow \# \text{ of chips} = 176\,897$$

$$p = 0.2 \qquad n = 50$$

$$P(x > 12) = P(\bar{p} > 0.24) = P\left(z > \frac{0.24 - 0.2}{0.056}\right)$$

$$= 0.5 - \qquad = 0.2389$$

18. Twenty percent of the applications received for a particular position are rejected. What is the probability that among the next 50 applications,
   a. More than 12 will be rejected?
   b. Determine the expected number of rejected applications and its variance.

$0.5 - P(0 \leq z \leq 0.5)$
$= 0,2981$

$P(z > 0.535) = \quad$ ₥

(a) $\pi = 0.2$, $n = 50 \implies X = B(50, 0.2)$

$n\pi = 10$, $\pi(1-\pi) = 40 > 5.$

Use Normal App. to Binomial

$P(X > 12) \approx P(x > 12.5) = P\left(z > \frac{12.5 - 10}{2.8}\right) = $

(b) $E(X) = n\pi = (50)(0.2) = 10$

$V(x) = n\pi(1-\pi) = (50)(0.2)(0.8) = 8.$

19. Students of a large university spend an average of $8 a day on lunch. The standard deviation of the expenditure is $4. A simple random sample of 64 students is taken. $\implies n > 20 \implies$ Normal

   a. What are the expected value, standard deviation, and shape of the sampling distribution of the sample mean?

   b. What is the probability that the sample mean will be at most $9?

(a) $\boxed{E(\bar{x}) = \mu = 8}$ ✓ , $\boxed{\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4/8 = 1/2}$ ✓

$n = 64 > 30 \overset{C.L.T}{\implies}$ & shape is approximately normal

(b) $\cancel{P(\bar{x} > 9) = P(z > \frac{9-8}{1/2}) = P(z > 2) = 0.0228}$

$\sqrt{\quad} P(\bar{x} \leq 9) = P(z \leq \frac{9-8}{1/2}) = P(z \leq 2) = 0.5 + P(0 < z < 2)$

$= 0.9772.$ ✓

20. Ten percent of the items produced by a machine are defective. A random sample of 100 items is selected and checked for defects.

   a. Determine the standard error of the sample proportion.
   b. What is the probability that the sample will contain more than 2.5% defective units?

(a) $\sigma_{\bar{p}} = \sqrt{\frac{(0.1)(0.9)}{100}} = 0.03$ ✓

(b) $P(\bar{p} > 0.025) = P\left(z > \frac{0.025 - 0.1}{0.03}\right)$

$= P(z \geq 2.5)$

$\cancel{= 0.0062}$

$= 0.5 + P(0 < z \leq 2.5).$

$= 0.5 + 0.4938 = 0.9938.$

5 points
5 //

5 points
5 //

Question # 6-

A simple random sample of size $\underline{144}$ is selected from a population with $\pi = 0.40$ (n labeled above 144)
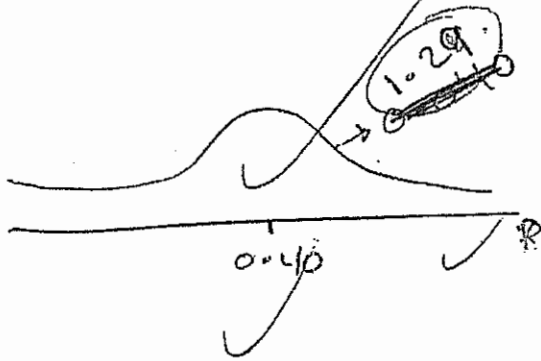
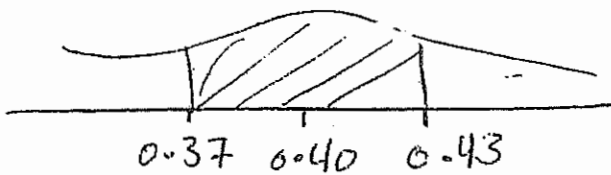a. What is the expected value of the sample proportion p

$$E(p) = \pi = 0.40$$

b. What is the standard error of p

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.40(0.6)}{144}} = \sqrt{1.64} = 0.28$$

c. Sketch the sampling distribution of p

0.40

d. What is the probability that the sample proportion will be within ± 0.03 of the population proportion

0.37   0.40   0.43

$P(0.37 \subseteq p \subseteq 0.43)$

$P\left(\frac{0.37-0.40}{1.28} \subseteq z \subseteq \frac{0.43-0.40}{0.40}{1.28}\right)$

$P(-0.023 \subseteq z \subseteq 0.023) = 2 \ast 0.0910$

$P(0.017 \subseteq 0.023) = 0.182$

Question # 5

The time needed to finish the second exam in Stat236 is normally distributed with mean of 60 minutes and a standard deviation of 10 minutes. Answer the following questions.

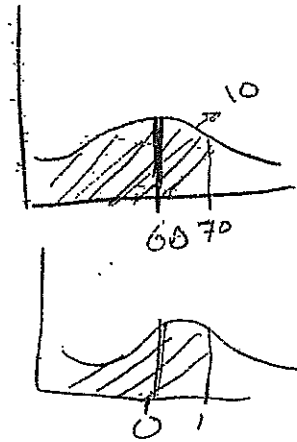a. What is the probability of completing the exam in 70 minutes or less?

$X \leq 70$ $\dfrac{70-60}{10}$

$P(X \leq 70)$

$P\left(Z \leq \dfrac{70-60}{10}\right)$

$P(Z \leq 1) \Rightarrow P(0 \leq Z \leq 1)$

$0.5 + 0.3413$

$= 0.8413$

b. What is the probability that a random sample of 100 student will have a sample mean time of completing the exam of more than 59 minutes but in less than 60.5 minutes?

$P(59 \leq \overline{X} \leq 60.5)$

$P\left(\dfrac{59-60}{10} \leq Z \leq \dfrac{60.5-60}{10}\right)$

$P(-0.1 \leq Z \leq 0.05)$

$P(0 \leq Z \leq 0.1) + 0 \leq Z \leq 0$

$= 0.0398 + 0.0199$

$= 0.0597$

c. Assume that the class has 180 students and that the examination period مدة الامتحان is 80 minutes. in How many students do you expect will be unable to finish the exam in the allowable time (you're your work)
غير قادرين على إنهاء الامتحان في الوقت المحدد؟

$P(X \geq 80)$

$P\left(Z \geq \dfrac{80-60}{10}\right)$

$P(Z \geq 2) \Rightarrow P(0 \leq Z \leq 2)$

$0.5 - 0.4772$