

**BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT**

97.5
Excellent

Midterm Exam Stat 236 Summer I 2012

Name (بالعربية).....

Number..... 1110445..... Sec. 40.....

Sample standard deviation:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{\sum x^2 - n(\bar{x})^2}{n-1}}$$

Z - Score: $z = \frac{x - \mu}{\sigma}$

Covariance: $s_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{n-1}$

Correlation coefficient: $r = \frac{s_{xy}}{s_x s_y} = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$

Permutations: $nPr = \frac{n!}{(n-r)!}$

Combinations: $nCr = \frac{n!}{(n-r)!r!}$

Conditional probability: $p(A \setminus B) = \frac{p(A \cap B)}{p(B)}$
 $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

Discrete Random Variable

$E(X) = \mu = \sum xf(x)$

$Var(X) = \sum (x - \mu)^2 f(x)$

Binomial Probability Distribution

$P(X=x) = \binom{n}{x} p^x (q)^{n-x} \quad q = 1 - p$

$E(X) = np, \sigma(X) = \sqrt{np(1-p)}$

• Poisson Probability

$f(x) = \frac{\mu^x e^{-\mu}}{x!}$

Handwritten calculations:
 42
 87544
 87552
 $(12) (7299.5) - (96) (912)$
 $\sqrt{12(870) - (96)^2}$
 10440
 9216
 ✓
 $\sqrt{12(69524) - (69524)^2}$
 834288
 831744
 2544

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{870 - \frac{(96)^2}{12}}{11} = \frac{870 - 768}{11} = \frac{102}{11} = 9.27$$

(1-8) A survey was conducted about a "No smoking" policy at BZU. The students were asked whether they were in favor of (identified by 1) or against (identified by 2) indoor smoking, about monthly expenditure and cumulative average. The following is a sample of the results:

Student #	Gender	No smoking Policy	X = Expenditure (NIS)	Y = Cumulative Average
1.	Male	1	4.5	83
2.	Male	2	5.5	75
3.	Female	2	7.5	70
4.	Female	1	6.5	74
5.	Male	2	7	70
6.	Female	1	7.5	72
7.	Female	1	7.5	82
8.	Male	2	8	78
9.	Male	2	8.5	73
10.	Female	1	9	77
11.	Male	1	9.5	78
12.	Female	1	16.5	80

x - x̄	(x - x̄)²
-6	36
-6	36
-4	16
-3	9
-2	4
-1	1
1	1
1	1
2	4
2	4
4	16
6	36
7	49
7	49
8	64
8	64
16	256
Total	212

[Hint: $\sum y = 912$, $\sum y^2 = 69524$, $\sum x = 96$, $\sum x^2 = 870$, and $\sum xy = 7299.5$]

For the variable "Cumulative Average", find

1. The mean, median, and mode.

$$\bar{x} = \frac{\sum x}{n} = \frac{912}{12} = 76$$

median $\frac{12}{2} \rightarrow 6, 7$

median = $\frac{75 + 77}{2} = 76$

mode = 78, 70

2. The coefficient of variation

$$CV = \frac{s.d}{\text{mean}} \times 100 = \frac{4.006}{76} \times 100 = 5.27\%$$

$$s.d = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{212}{11}} = \sqrt{19.3} = 4.4$$

For the variable "Expenditure", find

3. The five number summary

summary: 4.5, 6.25, 7.5, 8.75, 16.5

$$Q_1 = .25 \times 12 = 3, 4 \rightarrow 6, 6.5 \rightarrow \frac{6 + 6.5}{2} = 6.25$$

$$Q_2 = .50 \times 12 = 6, 7 \rightarrow 7.5, 7.5 \rightarrow 7.5$$

$$Q_3 = .75 \times 12 = 9, 10 \rightarrow 8.5, 9 \rightarrow \frac{8.5 + 9}{2} = 8.75$$

4. The limits of the box plot. Is there any outliers?

$$IQR = Q_3 - Q_1 = 8.75 - 6.25 = 2.5$$

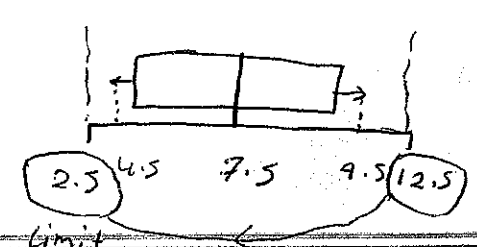
$$1.5 \times IQR = 3.75$$

$$Q_1 - 3.75 = 2.5$$

$$Q_3 + 3.75 = 12.5$$

Extrem = 16.5

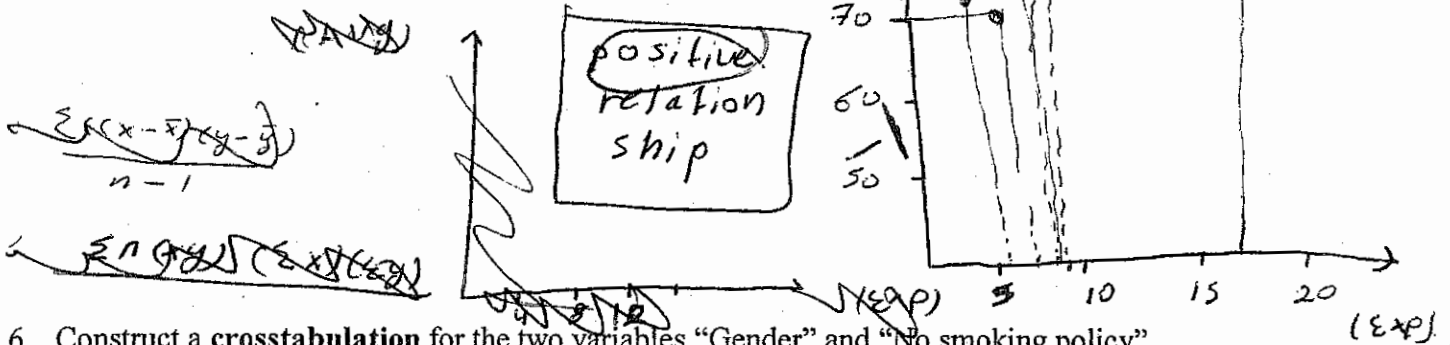
Limit



(Avg)

-Scatter

5. Construct a scatter diagram for the two variables "Expenditure" and "Average". Does the scatter diagram give an indication about the relationship between the two variables?



6. Construct a crosstabulation for the two variables "Gender" and "No smoking policy"

Gender \ do smok	1	2	Total
Male	2	4	6
Female	5	1	6
Total	7	5	12

7. Find the probability that a randomly selected student is a male or he is against indoor smoking.

$$P(M \cup 2) = P(M) + P(2) - P(M \cap 2)$$

$$= \frac{1}{2} \frac{6}{12} + \frac{5}{12} - \frac{4}{12} = 0.5 + 0.42 - 0.33 = 0.59$$

8. Are the two variables "Gender" and "No smoking policy" independent. Explain using probability male against smok

$$P(M \cap 2) = P(M) \times P(2)$$

$$0.33 \neq 0.5 \times 0.42 \quad \text{they are dependent}$$

9. A six-sided die is tossed 3 times. Find the probability of observing three ones in a row

$$P(1, 1, 1) = \frac{1}{216}$$

$$n = 6^3 = 216$$

10. If $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cup B) = 0.88$, find $P(B/A)$? Are A, B independent?

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.02}{0.5} = 0.04$$

$$P(A \cup B) = 0.88 = P(A) + P(B) - P(A \cap B)$$

$$0.88 = 0.5 + 0.4 - P(A \cap B)$$

$$0.88 = 0.9 - P(A \cap B)$$

$$P(A \cap B) = 0.02$$

11. A sample of the monthly amounts spent for food by BZU students has a symmetric distribution. The sample mean is \$100 and the standard deviation is \$10. Find the percent of the monthly food expenditures less than \$120 is

$\frac{120 - 100}{10} = \frac{20}{10} = 2$

$P(A \cap B) = P(A) \times P(B)$

$$0.02 \neq 0.5 \times 0.4$$

$$0.02 \neq 0.2$$

they are dependent

(12 - 13) A sample of 100 BZU students was asked how many cups of coffee they drink in the morning. You are given the following sample information.

Cups of Coffee	Frequency	$f(x)$	$x \cdot f(x)$
0	10	$\frac{10}{100}$	0
1	<u>40</u>	$\frac{40}{100}$	0.4
2	<u>20</u>	$\frac{20}{100}$	0.4
3	<u>30</u>	$\frac{30}{100}$	0.9
			<u>1.7</u>

12. Find the probability that a selected random person drunk at least 1 cup of coffee in a morning.

~~PROBABLY~~ $P(X \geq 1) = P(1) + P(2) + P(3)$

~~$1 - P(X < 1) = 1 - P(0) = 1 - 0.1 = 0.9$~~

13. Find the expected number of drunk cups of coffee.

~~$E(X) = \sum x f(x)$
 $= 1.7$~~

(14 - 16) A production process produces 90% non-defective parts.

0.10 defective

14. If a sample of 10 parts from the production process is selected. What is the probability that the sample will contain at least 4 defective parts?

~~$P(X \geq 4) = 1 - P(X < 4)$
 $= 1 - (P(3) + P(2) + P(1) + P(0))$
 $= 1 - \left[\binom{10}{3} (0.1)^3 (0.9)^7 + \binom{10}{2} (0.1)^2 (0.9)^8 + \binom{10}{1} (0.1) (0.9)^9 + \binom{10}{0} (0.1)^0 (0.9)^{10} \right]$~~

15. If a sample of 15 parts from the production process is selected. What is the probability that the sample will contain 7 non-defective parts?

~~$P(X=7) = P(7)$
 $= \binom{15}{7} (0.4)^7 (0.1)^8$~~

16. If a sample of 2500 parts from the production process is selected. What is the expected number and the standard deviation of the defective parts.

~~$E(P) = np$
 $= 2500 \times 0.1 = 250$~~

~~$S.d = \sqrt{var}$~~

~~$\sqrt{np(1-p)} = \sqrt{2500 \times 0.9}$
 $= \sqrt{2250} = 15$~~

$$\mu = 8 \quad 8 \rightarrow \text{hour}$$

(17 - 20) Ramallah Hospital has noted that they admit an average of 8 patients per hour.

17. What is the probability that during the next hour **less than 3** patients will be admitted?

$$\begin{aligned}
 P(X < 3) &= P(2) + P(1) + P(0) \\
 &= \frac{8^2 e^{-8}}{2!} + \frac{8^1 e^{-8}}{1!} + \frac{8^0 e^{-8}}{0!} = \frac{1}{4}
 \end{aligned}$$

18. What is the probability that during the next half hour **exactly 5** patients will be admitted?

$$\begin{aligned}
 f(5) &= \frac{4^5 e^{-4}}{5!} = 0.16 \quad \checkmark \\
 &\quad \begin{array}{l} 8 \rightarrow \text{hour} \\ ?? \rightarrow \frac{1}{2} \\ \mu = 4 \end{array}
 \end{aligned}$$

19. What is the expected number of admitted patients in any given day?

$$\begin{array}{l}
 \mu = 8 \leftarrow 1 \text{ hour} \\
 ?? \leftarrow 24 \text{ hour}
 \end{array}$$

$$\# \text{ of patients} = 192 \text{ patients}$$

20. Which of the following is a discrete and which is a continuous variable?

- The volume of water released from a dam continuous
- The distance you drove yesterday. continuous
- The number of employees of an insurance company discrete
- The amount of milk produced by a cow in one 24-hour period continuous
- The number of gallons of milk sold at the local grocery store yesterday discrete