APPENDIX C

Answers to Selected Exercises

CHAPTER 1

1.1 (a) $\{x : x = 0, 1, 2, 3, 4\}$; $\{x : x = 2\}$.
(b) $\{x : 0 < x < 3\}$; $\{x : 1 \leq x < 2\}$.

1.2 (a) $\{x : 0 < x < \frac{5}{8}\}$.

1.7 (a) $\{x : 0 < x < 3\}$.
(b) $\{(x, y) : 0 < x^2 + y^2 < 4\}$.

1.8 (a) $\{x : x = 2\}$.
(b) Null set.
(c) $\{(x, y) : x^2 + y^2 = 0\}$.

1.9 $\frac{80}{81}$; 1.

1.10 $\frac{11}{16}$; 0; 1.

1.11 $\frac{8}{3}$; 0; $\pi/2$.

1.12 $\frac{1}{2}$; 0; $\frac{2}{9}$.

1.13 $\frac{1}{6}$; 0.

1.15 10.

1.18 $\frac{1}{4}$; $\frac{1}{13}$; $\frac{1}{52}$; $\frac{4}{13}$.

1.19 $\frac{31}{32}$; $\frac{3}{64}$; $\frac{1}{32}$; $\frac{63}{64}$.

1.20 0.3.

1.21 e^{-4}; $1 - e^{-4}$; 1.

1.22 $\frac{1}{2}$.

1.26 (a) $\binom{6}{4}\Big/\binom{16}{4}$.

(b) $\binom{10}{4}\Big/\binom{16}{4}$.

1.27 $1 - \binom{990}{5}\Big/\binom{1000}{5}$.

1.29 (b) $1 - \binom{10}{3}\Big/\binom{20}{3}$.

1.34 (a) $\frac{1}{7}$. (b) $\frac{5}{56}$.

(c) $\left[\binom{3}{x}\Big/\binom{8}{x}\right][5/(8 - x)]$.

1.37 $\frac{1}{3}$.

1.38 $\frac{9}{20}$; $\frac{2}{3}$.

1.39 $\frac{5}{14}$.

1.40 $\frac{3}{7}$, $\frac{4}{7}$.

1.42 (a) 0.18. (b) 0.72.
(c) 0.88.

1.45 0.1029 for (a), (b), (c), (d). (e) 0.4116.

1.46 $\frac{1}{4}, \frac{3}{4}$.

1.47 $\frac{9}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}$.

1.48 (a) $\frac{1}{2}$. (b) $\frac{1}{21}$.

1.49 $\frac{1}{5}, \frac{1}{5}, \frac{1}{5}$.

1.51 (a) $\dfrac{\binom{13}{x}\binom{39}{5-x}}{\binom{52}{5}}$, $x=0, 1, \ldots, 5$.

(b) $\dfrac{\binom{39}{5}+\binom{13}{1}\binom{39}{4}}{\binom{52}{5}}$.

1.54 (a) $\frac{1}{10}$, $x=1, 2, \ldots, 9$. (b) $\frac{4}{10}$.

1.56 $\frac{6}{36}$, $x=0$; $\dfrac{12-2x}{36}$, $x=1, 2, 3, 4, 5$.

1.59 $\frac{3}{4}$.

1.61 $\frac{5}{8}; \frac{7}{8}; \frac{3}{8}$.

1.63 $e^{-2}-e^{-3}$.

1.64 $\frac{1}{27}, 1; \frac{2}{9}, \frac{25}{36}$.

1.66 (a) 1. (b) $\frac{2}{3}$. (c) 2.

1.69 (a) 0, $x<0$; $1-(1-x)^3$, $0\le x<1$; 1, $1\le x$; $1-\sqrt[3]{\frac{3}{4}}, 1-\sqrt[3]{\frac{1}{2}}$.

1.71 (a) $\frac{1}{4}$. (b) 0. (c) $\frac{1}{4}$. (d) 0.

1.72 0, $y<0$; y^2, $0\le y<1$; 1, $1\le y$, $2y$, $0<y<1$; 0 elsewhere.

1.74 $\frac{1}{2}; \frac{1}{4}$.

1.76 0, $x<0$; $1-e^{-x/2}; 0\le x$. $\frac{1}{2}e^{-x/2}; 0<x$; 0 elsewhere.

1.79 $1/3\sqrt{y}$, $0<y<1$; $1/6\sqrt{y}$, $1<y<4$; 0 elsewhere.

1.80 2; 86.4; -160.8.

1.81 3; 11; 27.

1.83 (a) $\frac{3}{4}$. (b) $\frac{1}{4}, \frac{1}{2}$.

1.85 \$7.80.

1.88 $\frac{7}{3}$.

1.89 (a) 1.5, 0.75. (b) 0.5, 0.05. (c) 2; does not exist.

1.90 $e^t/(2-e^t)$, $t<\ln 2$; 2; 2.

1.99 10; 0; 2; -30.

1.101 $-\dfrac{2\sqrt{2}}{5}, \dfrac{2\sqrt{2}}{5}$.

1.103 $1/2p; \frac{3}{2}; \frac{5}{2}$; 5; 50.

1.105 $\frac{31}{12}, \frac{167}{144}$.

1.110 $\frac{5}{8}, \frac{37}{192}$.

1.114 0.84.

CHAPTER 2

2.1 $\frac{15}{64}$; 0; $\frac{1}{2}; \frac{1}{2}$.

2.2 $\frac{1}{4}$.

2.6 ze^{-z}, $0<z<\infty$; 0 elsewhere.

2.7 $-\ln z$, $0<z<1$; 0 elsewhere.

2.10 $5x_2^4$, $0<x_2<1$; 0 elsewhere.

2.11 $(3x_1+2)/(6x_1+3)$; $(6x_1^2+6x_1+1)/(2)(6x_1+3)^2$.

2.13 $3x_2/4$; $3x_2^2/80$.

2.18 (b) $1/e$.

2.20 (a) 1. (b) -1. (c) 0.

2.21 (a) $7/\sqrt{804}$.

2.31 $\frac{5}{81}$.

2.32 $\frac{7}{8}$.

2.36 $\frac{1}{2}$.

2.38 (a) $\frac{1}{6}$, 0.

2.39 $1-(1-y)^{12}$, $0\le y<1$; $12(1-y)^{11}$, $0<y<1$.

2.40 $g(y)=[y^3-(y-1)^3]/6^3$, $y=1, 2, 3, 4, 5, 6$.

2.42 $b_2=\sigma_1(\rho_{12}-\rho_{13}\rho_{23})/[\sigma_2(1-\rho_{23}^2)]$; $b_3=\sigma_1(\rho_{13}-\rho_{12}\rho_{23})/[\sigma_3(1-\rho_{23}^2)]$.

CHAPTER 3

3.1 $\frac{40}{81}$.
3.4 $\frac{147}{512}$.
3.6 5.
3.8 $\frac{3}{16}$.
3.10 $\frac{65}{81}$.
3.13 $(\frac{1}{3})(\frac{2}{3})^{x-3}$, $x=3, 4, 5, \ldots$.
3.14 $\frac{5}{72}$.
3.17 $\frac{1}{6}$.
3.18 $\frac{24}{625}$.
3.20 $\frac{11}{6}$; $x_1/2$; $\frac{11}{6}$.
3.21 $\frac{25}{4}$.
3.22 0.09.
3.25 $4^x e^{-4}/x!$, $x=0, 1, 2, \ldots$.
3.26 0.84.
3.31 2.
3.33 (a) $\exp[-2+e^{t/2}(1+e^{t/1})]$.
(b) $\mu_1=1$, $\mu_2=2$, $\sigma_1^2=1$, $\sigma_2^2=2$, $\rho=\sqrt{2}/2$.
(c) $y/2$.
3.34 0.05.
3.35 0.831, 12.8.
3.36 0.90.
3.37 $\chi^2(4)$.
3.39 $3e^{-3y}$, $0<y<\infty$.
3.40 2, 0.95.
3.45 $\frac{11}{16}$.
3.46 $\chi^2(2)$.
3.49 0.067; 0.685.
3.51 71.3, 189.7.
3.52 $\sqrt{\ln 2/\pi}$.
3.57 0.774.
3.58 $\sqrt{2/\pi}$; $(\pi-2)/\pi$.
3.59 0.90.
3.60 0.477.
3.61 0.461.
3.62 $N(0, 1)$.
3.63 0.433.
3.64 0; 3.
3.69 $N(0, 2)$.
3.70 (a) 0.574.
(b) 0.735.
3.71 (a) 0.264. (b) 0.440.
(c) 0.433. (d) 0.642.
3.73 $\rho=\frac{4}{5}$.
3.74 (38.2, 43.4).

CHAPTER 4

4.2 $\frac{405}{1024}$.
4.3 0.405.
4.6 $\frac{16}{15}$.
4.7 $\frac{1}{8}$.
4.9 $(n+1)/2$; $(n^2-1)/12$.
4.10 $a+b\bar{x}$; $b^2 s_x^2$.
4.11 $\chi^2(2)$.
4.14 $\frac{1}{2}$, $0<y<1$; $1/2y^2$, $1<y<\infty$.
4.15 y^{15}, $0\le y<1$; $15y^{14}$, $0<y<1$.
4.16 $\frac{4}{7}$.
4.17 $\frac{1}{3}$, $y=3, 5, 7$.
4.19 $(\frac{1}{2})^{\sqrt[3]{y}}$, $y=1, 8, 27, \ldots$.
4.20

y_1	$g_1(y_1)$
1	$\frac{1}{36}$
2	$\frac{4}{36}$
3	$\frac{6}{36}$
4	$\frac{4}{36}$
6	$\frac{12}{36}$
9	$\frac{9}{36}$

4.25 $\frac{1}{27}$, $0<y<27$.
4.32 $y_1 e^{-y_1}$, $0<y_1<\infty$.
4.34 $(2y_1)(4y_2^3)$, $0<y_1<1$, $0<y_2<1$.
4.35 $\alpha/(\alpha+\beta)$; $\alpha\beta/[(\alpha+\beta+1)(\alpha+\beta)^2]$.
4.36 (a) 20. (b) 1260. (c) 495.
4.37 $\frac{10}{243}$.
4.40 0.05.
4.43 1/4.74, 3.33.
4.48 $(1/\sqrt{2\pi})^3 y_1^2 e^{-y_1^2/2} \sin y_3$, $0\le y_1<\infty$, $0\le y_2<2\pi$, $0\le y_3\le\pi$.

4.49 $y_2 y_3^2 e^{-y_3}$, $0<y_1<1$, $0<y_2<1$, $0<y_3<\infty$.
4.53 $1/(2\sqrt{y})$, $0<y<1$.
4.54 $e^{-y_1/2}/(2\pi\sqrt{y_1-y_2^2})$, $-\sqrt{y_1}<y_2<\sqrt{y_1}$, $0<y_1<\infty$.
4.56 $1-(1-e^{-3})^4$.
4.57 $\frac{1}{8}$.
4.62 $\frac{5}{16}$.
4.63 $48z_1z_2^3z_3^5$, $0<z_1<1$, $0<z_2<1$, $0<z_3<1$.
4.64 $\frac{7}{12}$.
4.69 $\frac{1}{4}$.
4.70 $6uv(u+v)$, $0<u<v<1$.
4.75

y	$g(y)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

4.76 0.24.
4.79 0.159.
4.82 0.159.
4.88 0.818.
4.91 (b) -1 or 1.
(c) $Z_i=\sigma_i Y_i+\mu_i$.
4.92 $\sum_1^n a_i b_i=0$.
4.94 6.41.
4.95 $n=16$.
4.97 $(n-1)\sigma^2/n$; $2(n-1)\sigma^4/n^2$.
4.98 0.90.
4.100 0.945.
4.102 0.618.
4.103 0.78.
4.104 $\frac{8}{3}$; $\frac{2}{9}$.
4.105 7.
4.107 2.5; 0.25.
4.109 -5; $60-12\sqrt{6}$.
4.110 $\sigma_1/\sqrt{\sigma_1^2+\sigma_2^2}$.
4.113 0.265.
4.115 22.5, $\frac{261}{4}$.
4.116 $r_2>4$.
4.118 $\mu_2\sigma_1/\sqrt{\sigma_1^2\sigma_2^2+\mu_1^2\sigma_2^2+\mu_2^2\sigma_1^2}$.
4.121 $5/\sqrt{39}$.
4.125 $e^{\mu+\sigma^2/2}$; $e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$.

CHAPTER 5

5.1 Degenerate at μ.
5.2 Gamma ($\alpha=1$, $\beta=1$).
5.3 Gamma ($\alpha=1$, $\beta=1$).
5.4 Gamma ($\alpha=2$; $\beta=1$).
5.13 0.682.
5.14 (b) 0.815.
5.17 Degenerate at $\mu_2+(\sigma_2/\sigma_1)(x-\mu_1)$.
5.18 (b) $N(0, 1)$.
5.19 (b) $N(0, 1)$.
5.21 0.954.
5.23 0.840.
5.26 0.08.
5.28 0.267.
5.29 0.682.
5.35 $N(0, 1)$.

CHAPTER 6

6.1 (a) $\bar{X}$.
(b) $-n/\ln(X_1X_2\cdots X_n)$.
(c) $\bar{X}$. (d) The median.
(e) The first order statistic.
6.2 The first order statistic Y_1, $\sum_1^n (X_i-Y_1)/n$.

6.4 $\frac{4}{25}, \frac{11}{25}, \frac{7}{25}$.
6.5 $Y_1 = \min(X_i)$; $n/\ln[(X_1 X_2 \cdots X_n)/Y_1^n]$.
6.7 (b) $\bar{X}/(1-\bar{X})$. (d) $\bar{X}$. (e) $\bar{X}-1$.
6.9 $1-e^{-2/\bar{X}}$.
6.10 Multiply by $n/(n-1)$.
6.12 $(Y_1+Y_n)/2$, $(Y_n-Y_1)/2$; $E[(Y_n-Y_1)/2] = \rho(n-1)/(n+1)$.
6.14 (77.28, 85.12).
6.15 24 or 25.
6.16 (3.7, 5.7).
6.17 160.
6.23 $(5\bar{x}/6, 5\bar{x}/4)$.
6.25 1692.
6.26 3.19 to 3.61.
6.28 3.92 to 31.50.
6.30 $(-3.6, 2.0)$.
6.35 135 or 136.
6.38 $\frac{1}{4}+\frac{3}{4}\ln\frac{3}{4}$; $\frac{7}{16}+\frac{9}{8}\ln\frac{3}{4}$.
6.39 $\frac{11}{64}$; $(31)3^8/4^9$.
6.42 $n = 19$ or 20.
6.43 $K(\frac{1}{2}) = 0.062$; $K(\frac{1}{12}) = 0.920$.
6.44 $n \approx 73$, $c \approx 42$.
6.46 (a) Reject. (b) p-value ≈ 0.005.
6.49 (c) p-value ≈ 0.005.
6.51 23.3.
6.52 2.91.
6.53 $q_3 = \frac{176}{21} > 7.81$, reject H_0.
6.55 $b \le 8$ or $32 \le b$.
6.56 $q_3 = \frac{22}{9} < 11.3$, accept H_0.
6.57 $6.4 < 9.49$, accept H_0.
6.59 $\hat{p} = (X_1 + X_2/2)/(X_1 + X_2 + X_3)$.

CHAPTER 7

7.4 $\frac{1}{3}, \frac{2}{3}$.
7.5 $\delta_1(y)$.
7.6 $b = 0$; does not exist.
7.7 Does not exist.
7.17 $\prod_{i=1}^{n} [X_i(1-X_i)]$.
7.19 $60y_3^2(y_5-y_3)/\theta^5$; $6y_5/5$; $\theta^2/7$; $\theta^2/35$.
7.20 $(1/\theta^2)e^{-y_1/\theta}$, $0 < y_2 < y_1 < \infty$; $y_1/2$; $\theta^2/2$.
7.22 $\Sigma X_i^2/n$; $\Sigma X_i/n$; $(n+1)Y_n/n$
7.24 X; X.
7.25 Y_1/n.
7.27 $Y_1 - 1/n$.
7.29 $Y_1 = \sum_1^n X_i$; $Y_1/4n$; yes.
7.37 $\bar{x}$.
7.40 $\bar{X}^2 - 1/n$.
7.43 $\left(\frac{n-1}{n}\right)^Y \left(1+\frac{Y}{n-1}\right)$.
7.51 $\frac{Y_1+Y_n}{2}$, $\frac{(n+1)(Y_n-Y_1)}{2(n-1)}$.
7.55 Y_1, $\Sigma(Y_i - Y_1)/n$.

CHAPTER 8

8.2 $[y\tau^2 + \mu\sigma^2/n]/(\tau^2 + \sigma^2/n)$.
8.3 $\beta(y+\alpha)/(n\beta+1)$.
8.9 $\sqrt[6]{2}$ if $y_4 < 1$, $\sqrt[6]{2}\, y_4$ if $1 \le y_4$.
8.13 θ^2/n; $\theta^2/n(n+2)$.
8.15 (a) $4/\theta^2$.
8.17 (d) $\text{var}(\hat{\theta}) = \frac{1}{nI(\theta)} = \frac{\theta^2}{5n}$.
8.22 2.17; 2.44.
8.25 2.20.

CHAPTER 9

9.4 $\sum_1^{10} x_i^2 \ge 18.3$; yes; yes.

9.6 $3\sum_{1}^{10} x_i^2 + 2\sum_{1}^{10} x_i \geq c.$

9.7 95 or 96; 76.7.
9.9 38 or 39; 15.
9.10 0.08; 0.875.
9.11 $(1-\theta)^9(1+9\theta)$.
9.12 1, $0<\theta\leq\frac{1}{2}$; $1/(16\theta^4)$, $\frac{1}{2}<\theta<1$; $1-15/(16\theta^4)$, $1\leq\theta$.
9.14 53 or 54, 5.6.
9.17 Reject H_0 if $\bar{x}\geq 77.564$.
9.18 26 or 27; reject H_0 if $\bar{x}\leq 24$.
9.19 220 or 221; reject H_0 if $y\geq 17$.
9.23 $t=3>2.262$, reject H_0.
9.24 $|t|=2.27>2.145$, reject H_0.
9.37 $c_0(n)=(14.4)\times(n\ln 1.5-\ln 9.5)$; $c_1(n)=(14.4)\times(n\ln 1.5+\ln 18)$.
9.38 $c_0(n)=(0.05n-\ln 8)/\ln 3.5$; $c_1(n)=(0.05n-\ln 4.5)/\ln 3.5$.
9.41 (b) $c=0.18$; 0.64, (c) $c=0.5$; 0.16; 0.84. (d) $c=0.23$; 0.06; 0.68.
9.44 $(9y-20x)/30\leq c$.

CHAPTER 10

10.9 6.39.
10.12 $r+\theta$, $2r+4\theta$.
10.13 $r_2(\theta+r_1)/[r_1(r_2-2)]$.
10.23 7.00, 9.98.
10.25 4.79, 22.82, 30.73.
10.26 (a) $4.483x+6.483$.
10.28 $\hat{\beta}=\sum(X_i/nc_i)$, $\sum[(X_i-\hat{\beta}c_i)^2/nc_i^2]$.
10.32 Reject H_0.
10.44 $a_i=0$, $i=1, 2, 3, 4$.
10.45 $\sum_{j=1}^{n} a_{ij}=0$, $i=1, 2, \ldots, n$.

CHAPTER 11

11.2 (a) $\frac{15}{16}$. (b) 675/1024; (c) $(0.8)^4$.
11.4 8.
11.6 0.954; 0.92; 0.788.
11.9 8.
11.12 (a) Beta $(n-j+1, j)$. (b) Beta $(n-j+i-1, j-i+2)$.
11.15 0.067.
11.18 Reject H_0.
11.25 0; $4(4^n-1)/3$; no.
11.37 $\frac{2}{99}$.
11.44 98; $\frac{686}{3}$.

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