

10

Name (بالعربية): *Key*

Student No.:

Question (10 points)

Let $0 < p < 1$. A $(100p)$ th percentile, denoted by ξ_p , of the distribution of a continuous random variable X with p.d.f. $f(x)$ is defined as

$$\Pr(X \leq \xi_p) = \Pr(X < \xi_p) = \int_{-\infty}^{\xi_p} f(x) dx = p$$

Using the above information find the 20th percentile, denoted by $\xi_{0.20}$, of the distribution which has the following p.d.f.

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_{-\infty}^{\xi_{0.20}} f(x) dx = 0.20$$

$$\Rightarrow \int_0^{\xi_{0.20}} 3x^2 dx = 0.20, \quad 0 < \xi_{0.20} < 1$$

$$\Rightarrow x^3 \Big|_0^{\xi_{0.20}} = 0.20$$

$$\Rightarrow \xi_{0.20}^3 = 0.20$$

$$\Rightarrow \xi_{0.20}^3 - 0.20 = 0$$

$$\Rightarrow \left(\xi_{0.20} - (0.20)^{1/3} \right) \left(\xi_{0.20}^2 + (0.20)^{1/3} \xi_{0.20} + (0.20)^{2/3} \right) = 0$$

has no solution in \mathbb{R}

$$\Rightarrow \xi_{0.20} = (0.20)^{1/3} = 0.58$$