

Name (بالعربية): Key .....

Student No.: .....

Question (10 points)

Let  $f(x) = \frac{1}{2} \left(\frac{1}{2}\right)^x$ ,  $x = 0, 1, 2, \dots$  zero elsewhere, be the p.d.f. of the random variable  $X$ . Find the m.g.f. and the mean.

$$\begin{aligned} E(e^{tx}) &= \sum_{x \in \mathbb{R}} e^{tx} f(x) = \sum_{x=0}^{\infty} e^{tx} \frac{1}{2} \left(\frac{1}{2}\right)^x \\ &= \frac{1}{2} \sum_{x=0}^{\infty} \left(\frac{e^t}{2}\right)^x = \frac{1}{2} \frac{1}{1 - \frac{e^t}{2}} \quad \text{if } \left|\frac{e^t}{2}\right| < 1 \\ &= \frac{1}{2 - e^t} \quad \text{if } t < \ln 2 \end{aligned}$$

$$M(t) = (2 - e^t)^{-1}, \quad t < \ln 2$$

$$\begin{aligned} M'(t) &= -(2 - e^t)^{-2} (-e^t), \quad t < \ln 2 \\ &= e^t (2 - e^t)^{-2}, \quad t < \ln 2 \end{aligned}$$

$$\begin{aligned} E(X) = \mu &= M'(0) = e^0 (2 - e^0)^{-2} \\ &= 1(2 - 1)^{-2} = 1 \end{aligned}$$