Chapter 4
Introduction to Probability

Learning Objectives

1. Obtain an appreciation of the role probability information plays in the decision making process.

2. Understand probability as a numerical measure of the likelihood of occurrence.

3. Know the three methods commonly used for assigning probabilities and understand when they should be used.

4. Know how to use the laws that are available for computing the probabilities of events.

5. Understand how new information can be used to revise initial (prior) probability estimates using Bayes' theorem.

Chapter 4
Solutions:

1. Number of experimental Outcomes = (3)(2)(4) = 24

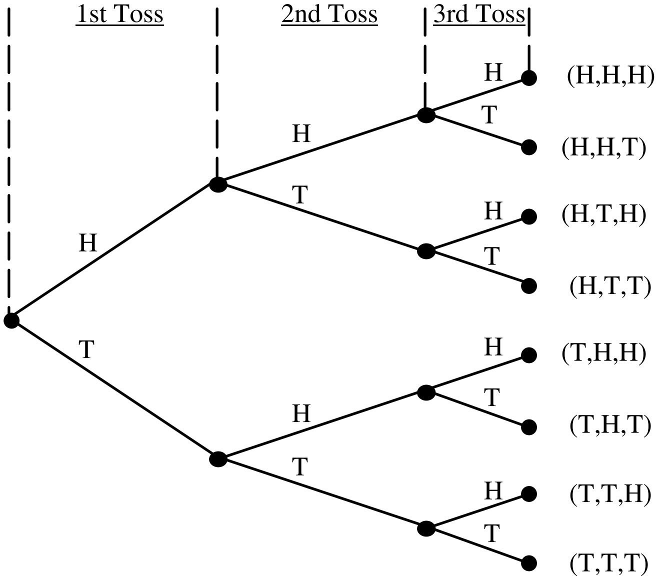
2. $$\binom{6}{3} = \frac{6!}{3!3!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(3\cdot 2\cdot 1)} = 20$$

ABC
ABD
ABE
ABF
ACD
ACE
ACF
ADE
ADF
AEF
BCD
BCE
BCF
BDE
BDF
BEF
CDE
CDF
CEF
DEF

3. $$P_3^6 = \frac{6!}{(6-3)!} = (6)(5)(4) = 120$$

BDF BFD DBF DFB FBD FDB

4. a.



b. Let: H be head and T be tail

(H,H,H) (T,H,H)
(H,H,T) (T,H,T)
(H,T,H) (T,T,H)
(H,T,T) (T,T,T)

c. The outcomes are equally likely, so the probability of each outcomes is 1/8.

5. $P(E_i) = 1/5$ for i = 1, 2, 3, 4, 5

$P(E_i) \geq 0$ for i = 1, 2, 3, 4, 5

$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1/5 + 1/5 + 1/5 + 1/5 + 1/5 = 1$

The classical method was used.

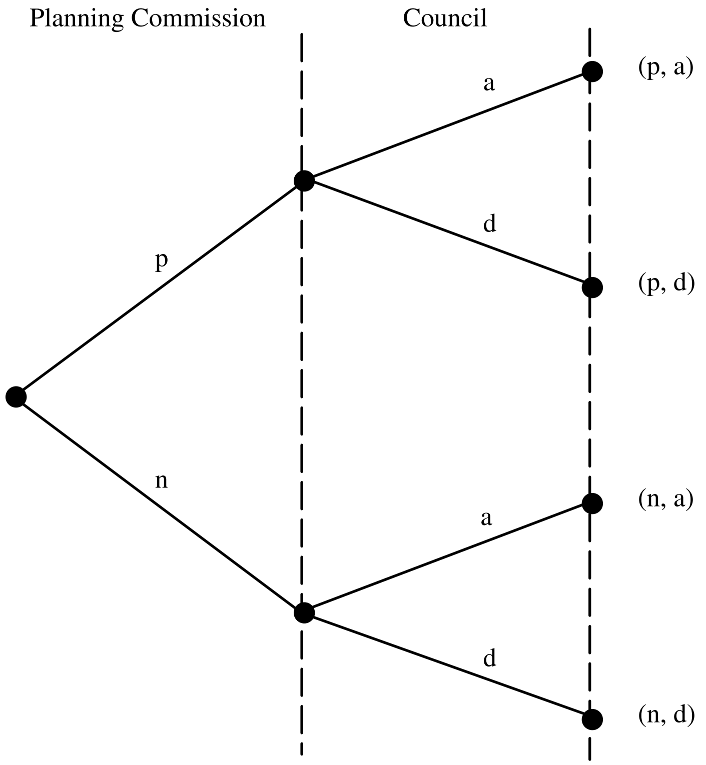
6. $P(E_1) = .40$, $P(E_2) = .26$, $P(E_3) = .34$

   The relative frequency method was used.

7. No. Requirement (4.4) is not satisfied; the probabilities do not sum to 1. $P(E_1) + P(E_2) + P(E_3) + P(E_4) = .10 + .15 + .40 + .20 = .85$

8. a. There are four outcomes possible for this 2-step experiment; planning commission positive - council approves; planning commission positive - council disapproves; planning commission negative - council approves; planning commission negative - council disapproves.

   b. Let p = positive, n = negative, a = approves, and d = disapproves



9. $\binom{50}{4} = \frac{50!}{4!46!} = \frac{50\cdot 49\cdot 48\cdot 47}{4\cdot 3\cdot 2\cdot 1} = 230,300$

10. a. Use the relative frequency approach:

    P(California) = 1,434/2,374 = .60

    b. Number not from 4 states = 2,374 - 1,434 - 390 - 217 - 112 = 221

    P(Not from 4 States) = 221/2,374 = .09

    c. P(Not in Early Stages) = 1 - .22 = .78

    d. Estimate of number of Massachusetts companies in early stage of development = (.22)390 ≈ 86

e. If we assume the size of the awards did not differ by states, we can multiply the probability an award went to Colorado by the total venture funds disbursed to get an estimate.
Estimate of Colorado funds = (112/2374)($32.4) = $1.53 billion

Authors' Note: The actual amount going to Colorado was $1.74 billion.

11. a. Total drivers = 858 + 228 = 1086

$$P(\text{Seatbelt}) = \frac{858}{1086} = .79 \text{ or } 79\%$$

b. Yes, the overall probability is up from .75 to .79, or 4%, in one year. Thus .79 does exceed his .78 expectation.

c. Northeast $\frac{148}{200} = .74$

Midwest $\frac{162}{216} = .75$

South $\frac{296}{370} = .80$

West $\frac{252}{300} = .84$

The West with .84 shows the highest probability of use.

d. Probability of selection by region:

Northeast $\frac{200}{1086} = .184$

Midwest $\frac{216}{1086} = .200$

South $\frac{370}{1086} = .340$

West $\frac{300}{1086} = .286$

South has the highest probability (.34) and West was second (.286).

e. Yes, .34 for South + .286 for West = .626 shows that 62.6% of the survey came from the two highest usage regions. The .79 probability may be high.

If equal numbers for each region, the overall probability would have been roughly

$$\frac{.74+.75+.80+.84}{4} = .7825$$

Although perhaps slightly lower, the .78 to .79 usage probability is a nice increase over the prior year.

12. a. Use the counting rule for combinations:

$$\binom{55}{5} = \frac{55!}{5!50!} = \frac{(55)(54)(53)(52)(51)}{(5)(4)(3)(2)(1)} = 3,478,761$$

One chance in 3,489,761

b. Very small: 1/3,478,761 = .000000287

c. Multiply the answer in part (a) by 42 to get the number of choices for the six numbers.

Number of Choices = (3,478,761)(42) = 146,107,962

Probability of Winning = 1/146,107,962 = .00000000684

13. Initially a probability of .20 would be assigned if selection is equally likely. Data does not appear to confirm the belief of equal consumer preference. For example using the relative frequency method we would assign a probability of 5/100 = .05 to the design 1 outcome, .15 to design 2, .30 to design 3, .40 to design 4, and .10 to design 5.

14. a. $P(E_2) = 1/4$

b. P(any 2 outcomes) = 1/4 + 1/4 = 1/2

c. P(any 3 outcomes) = 1/4 + 1/4 + 1/4 = 3/4

15. a. S = {ace of clubs, ace of diamonds, ace of hearts, ace of spades}

b. S = {2 of clubs, 3 of clubs, . . . , 10 of clubs, J of clubs, Q of clubs, K of clubs, A of clubs}

c. There are 12; jack, queen, or king in each of the four suits.

d. For a: 4/52 = 1/13 = .08

For b: 13/52 = 1/4 = .25

For c: 12/52 = .23

16. a. (6)(6) = 36 sample points

b.

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

← Total for Both

c. 6/36 = 1/6

d. 10/36 = 5/18

e. No. *P*(odd) = 18/36 = *P*(even) = 18/36 or 1/2 for both.

f. Classical. A probability of 1/36 is assigned to each experimental outcome.

17. a. (4,6), (4,7), (4,8)

b. .05 + .10 + .15 = .30

c. (2,8), (3,8), (4,8)

d. .05 + .05 + .15 = .25

e. .15

18. a. $P(\text{no meals}) = \dfrac{11}{496} = .022$

b. $P(\text{at least four meals}) = P(4) + P(5) + P(6) + P(7 \text{ or more})$

$$= \frac{36}{496} + \frac{119}{496} + \frac{114}{496} + \frac{139}{496}$$

$$= .073 + .240 + .230 + .280$$

$$= .823$$

c. $P(\text{two or fewer meals}) = P(2) + P(1) + P(0)$

$$= \frac{30}{496} + \frac{11}{496} + \frac{11}{496}$$

$$= .060 + .022 + .022$$

$$= .104$$

19. a/b. Use the relative frequency approach to assign probabilities. For each sport activity, divide the number of male and female participants by the total number of males and females respectively.

Activity	Male	Female
Bicycle Riding	.18	.16
Camping	.21	.19
Exercise Walking	.24	.45
Exercising with Equipment	.17	.19
Swimming	.22	.27

c. $P(\text{Exercise Walking}) = \dfrac{(28.7 + 57.7)}{248.5} = .35$

d. $P(\text{Woman}) = \dfrac{57.7}{(28.7 + 57.7)} = .67$

$P(\text{Man}) = \dfrac{28.7}{(28.7 + 57.7)} = .33$

20. a. $P(\text{N}) = 54/500 = .108$

b. $P(\text{T}) = 48/500 = .096$

c. Total in 5 states = 54 + 52 + 48 + 33 + 30 = 217

$P(\text{B}) = 217/500 = .434$

Almost half the Fortune 500 companies are headquartered in these five states.

21. a. Using the relative frequency method, divide each number by the total population of 281.4 million.

Age	Number	Probability
Under 19	80.5	.2859
20 to 24	19.0	.0674
25 to 34	39.9	.1417
35 to 44	45.2	.1604
45 to 54	37.7	.1339
55 to 64	24.3	.0863
65 and over	35.0	.1243
Total	281.4	1.0000

b. $P(\text{20 to 24}) = .0674$

c. $P(\text{20 to 34}) = .0674 + .1417 = .2091$

d. $P(\text{45 or older}) = .1339 + .0863 + .1243 = .3445$

22. a. $P(\text{A}) = .40$, $P(\text{B}) = .40$, $P(\text{C}) = .60$

b. $P(\text{A} \cup \text{B}) = P(\text{E}_1, \text{E}_2, \text{E}_3, \text{E}_4) = .80$. Yes $P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B})$.

c. $\text{A}^c = \{\text{E}_3, \text{E}_4, \text{E}_5\}$ $\text{C}^c = \{\text{E}_1, \text{E}_4\}$ $P(\text{A}^c) = .60$ $P(\text{C}^c) = .40$

d. $\text{A} \cup \text{B}^c = \{\text{E}_1, \text{E}_2, \text{E}_5\}$ $P(\text{A} \cup \text{B}^c) = .60$

e. $P(\text{B} \cup \text{C}) = P(\text{E}_2, \text{E}_3, \text{E}_4, \text{E}_5) = .80$

23. a. $P(\text{A}) = P(\text{E}_1) + P(\text{E}_4) + P(\text{E}_6) = .05 + .25 + .10 = .40$

$P(\text{B}) = P(\text{E}_2) + P(\text{E}_4) + P(\text{E}_7) = .20 + .25 + .05 = .50$

$P(\text{C}) = P(\text{E}_2) + P(\text{E}_3) + P(\text{E}_5) + P(\text{E}_7) = .20 + .20 + .15 + .05 = .60$

b. $\text{A} \cup \text{B} = \{\text{E}_1, \text{E}_2, \text{E}_4, \text{E}_6, \text{E}_7\}$

$$\begin{aligned} P(\text{A} \cup \text{B}) &= P(\text{E}_1) + P(\text{E}_2) + P(\text{E}_4) + P(\text{E}_6) + P(\text{E}_7) \\ &= .05 + .20 + .25 + .10 + .05 = .65 \end{aligned}$$

c. $\text{A} \cap \text{B} = \{\text{E}_4\}$ $P(\text{A} \cap \text{B}) = P(\text{E}_4) = .25$

d. Yes, they are mutually exclusive.

e. $\text{B}^c = \{\text{E}_1, \text{E}_3, \text{E}_5, \text{E}_6\}$; $P(\text{B}^c) = P(\text{E}_1) + P(\text{E}_3) + P(\text{E}_5) + P(\text{E}_6)$
$= .05 + .20 + .15 + .10 = .50$

24. Let E = experience exceeded expectations
M = experience met expectations

a. Percentage of respondents that said their experience exceeded expectations
= 100 - (4 + 26 + 65) = 5%

$P(\text{E}) = .05$

b. $P(\text{M} \cup \text{E}) = P(\text{M}) + P(\text{E}) = .65 + .05 = .70$

25. Let M = male young adult living in his parents' home
F = female young adult living in her parents' home

a. $P(\text{M} \cup \text{F}) = P(\text{M}) + P(\text{F}) - P(\text{M} \cap \text{F})$
$= .56 + .42 - .24 = .74$

b. $1 - P(\text{M} \cup \text{F}) = 1 - .74 = .26$

26. Let Y = high one-year return
M = high five-year return

a. $P(\text{Y}) = 9/30 = .30$

$P(\text{M}) = 7/30 = .23$

b. $P(\text{Y} \cap \text{M}) = 5/30 = .17$

c. $P(\text{Y} \cup \text{M}) = .30 + .23 - .17 = .36$

$P(\text{Neither}) = 1 - .36 = .64$

27.

		Big Ten		
		Yes	No	
Pac-10	Yes	849	3645	4494
	No	2112	6823	8935
		2,961	10,468	13,429

a. $P(\text{Neither}) = \dfrac{6823}{13{,}429} = .51$

b. $P(\text{Either}) = \dfrac{2961}{13{,}429} + \dfrac{4494}{13{,}429} - \dfrac{849}{13{,}429} = .49$

c. $P(\text{Both}) = \dfrac{849}{13{,}429} = .06$

28. Let: B = rented a car for business reasons
P = rented a car for personal reasons

a. $P(\text{B} \cup \text{P}) = P(\text{B}) + P(\text{P}) - P(\text{B} \cap \text{P})$
$= .54 + .458 - .30 = .698$

b. $P(\text{Neither}) = 1 - .698 = .302$

29. a. $P(\text{E}) = \dfrac{1033}{2851} = .36$

$P(\text{R}) = \dfrac{854}{2851} = .30$

$P(\text{D}) = \dfrac{964}{2851} = .34$

b. Yes; $P(\text{E} \cap \text{D}) = 0$

c. Probability = $\frac{1033}{2375} = .43$

d. 964(.18) = 173.52

Rounding up we get 174 of deferred students admitted from regular admission pool.

Total admitted = 1033 + 174 = 1207

P(Admitted) = 1207/2851 = .42

30. a. $P(\text{A} \mid \text{B}) = \frac{P(\text{A} \cap \text{B})}{P(\text{B})} = \frac{.40}{.60} = .6667$

b. $P(\text{B} \mid \text{A}) = \frac{P(\text{A} \cap \text{B})}{P(\text{A})} = \frac{.40}{.50} = .80$

c. No because $P(\text{A} \mid \text{B}) \neq P(\text{A})$

31. a. $P(\text{A} \cap \text{B}) = 0$

b. $P(\text{A} \mid \text{B}) = \frac{P(\text{A} \cap \text{B})}{P(\text{B})} = \frac{0}{.4} = 0$

c. No. $P(\text{A} \mid \text{B}) \neq P(\text{A})$; $\therefore$ the events, although mutually exclusive, are not independent.

d. Mutually exclusive events are dependent.

32. a. Total sample size = 2000

Dividing each entry by 2000 provides the following joint probability table.

	Health Insurance		
Age	Yes	No	Total
18 to 34	.375	.085	.46
35 and over	.475	.065	.54
	.850	.150	1.00

Let A = 18 to 34 age group
B = 35 and over age group
Y = Insurance coverage
N = No insurance coverage

b. $P(\text{A}) = .46$
$P(\text{B}) = .54$

Of population age 18 and over

46% are ages 18 to 34
54% are ages 35 and over

c. $P(\text{N}) = .15$

d. $P(\text{N}|\text{A}) = \dfrac{P(\text{N} \cap \text{A})}{P(\text{A})} = \dfrac{.085}{.46} = .1848$

e. $P(\text{N}|\text{B}) = \dfrac{P(\text{N} \cap \text{B})}{P(\text{B})} = \dfrac{.065}{.54} = .1204$

f. $P(\text{A}|\text{N}) = \dfrac{P(\text{A} \cap \text{N})}{P(\text{N})} = \dfrac{.085}{.150} = .5677$

g. Probability of no health insurance coverage is .15. A higher probability exists for the younger population. Ages 18 to 34: .1848 or approximately 18.5% of the age group. Ages 35 and over: .1204 or approximately 12% of the age group. Of the no insurance group, more are in the 18 to 34 age group: .5677, or approximately 57% are ages 18 to 34.

33. a.

	Reason for Applying			
	Quality	Cost/Convenience	Other	Total
Full Time	.218	.204	.039	.461
Part Time	.208	.307	.024	.539
	.426	.511	.063	1.00

b. It is most likely a student will cite cost or convenience as the first reason - probability = .511. School quality is the first reason cited by the second largest number of students - probability = .426.

c. $P(\text{Quality} \mid \text{full time}) = .218/.461 = .473$

d. $P(\text{Quality} \mid \text{part time}) = .208/.539 = .386$

e. For independence, we must have $P(\text{A})P(\text{B}) = P(\text{A} \cap \text{B})$.

From the table, $P(\text{A} \cap \text{B}) = .218$, $P(\text{A}) = .461$, $P(\text{B}) = .426$

$P(\text{A})P(\text{B}) = (.461)(.426) = .196$

Since $P(\text{A})P(\text{B}) \neq P(\text{A} \cap \text{B})$, the events are not independent.

34. a. Let O = flight arrives on time
O^c = flight arrives late
S = Southwest flight
U = US Airways flight
J = JetBlue flight

Given: $P(\text{O} \mid \text{S}) = .834$ $\quad P(\text{O} \mid \text{U}) = .751$ $\quad P(\text{O} \mid \text{J}) = .701$

$P(\text{S}) = .40$ $\quad P(\text{U}) = .35$ $\quad P(\text{J}) = .25$

$$P(\text{O} \mid \text{S}) = \frac{P(\text{O} \cap \text{S})}{P(\text{S})}$$

$$\therefore\ P(\text{O} \cap \text{S}) = P(\text{O} \mid \text{S})P(\text{S}) = (.834)(.4) = .3336$$

Similarly

$$P(\mathrm{O} \cap \mathrm{U}) = P(\mathrm{O} \mid \mathrm{U})P(\mathrm{U}) = (.751)(.35) = .2629$$

$$P(\mathrm{O} \cap \mathrm{J}) = P(\mathrm{O} \mid \mathrm{J})P(\mathrm{J}) = (.701)(.25) = .1753$$

Joint probability table

	On time	Late	Total
Southwest	.3336	.0664	.40
US Airways	.2629	.0871	.35
JetBlue	.1753	.0747	.25
Total:	.7718	.2282	1.00

b. Southwest Airlines; $P(\mathrm{S}) = .40$

c. $P(\mathrm{O}) = P(\mathrm{S} \cap \mathrm{O}) + P(\mathrm{U} \cap \mathrm{O}) + P(\mathrm{J} \cap \mathrm{O}) = .3336 + .2629 + .1753 = .7718$

d. $$P(\mathrm{S} \mid \mathrm{O}^c) = \frac{P(\mathrm{S} \cap \mathrm{O}^c)}{P(\mathrm{O}^c)} = \frac{.0664}{.2282} = .2910$$

Similarly, $$P(\mathrm{U} \mid \mathrm{O}^c) = \frac{.0871}{.2282} = .3817$$

$$P(\mathrm{J} \mid \mathrm{O}^c) = \frac{.0747}{.2282} = .3273$$

Most likely airline is US Airways; least likely is Southwest

35. a. The joint probability table is given.

Occupation	Male	Female	Total
Managerial/Professional	0.17	0.17	0.34
Tech./Sales/Admin.	0.10	0.17	0.27
Service	0.04	0.07	0.12
Precision Production	0.11	0.01	0.12
Oper./Fabricator/Labor	0.10	0.03	0.13
Farming/Forestry/Fishing	0.02	0.00	0.02
Total	0.54	0.46	1.00

b. Let MP = Managerial/Professional
F = Female

$$P(\mathrm{MP} \mid \mathrm{F}) = \frac{P(\mathrm{MP} \cap \mathrm{F})}{P(\mathrm{F})} = \frac{.17}{.46} = .37$$

c. Let PP = Precision Production
M = Male

$$P(\mathrm{PP} \mid \mathrm{M}) = \frac{P(\mathrm{PP} \cap \mathrm{M})}{P(\mathrm{M})} = \frac{.11}{.54} = .20$$

d. No. From part (c), $P(\mathrm{PP} \mid \mathrm{M}) = .20$. But, $P(\mathrm{PP}) = .12$, so $P(\mathrm{PP} \mid \mathrm{M}) \neq P(\mathrm{PP})$

36. a. Let A = makes 1st free throw
B = makes 2nd free throw

Assuming independence, $P(\text{A} \cap \text{B}) = P(\text{A})P(\text{B}) = (.89)(.89) = .7921$

b. $P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B}) = (.89)(.89) - .7921 = .9879$

c. $P(\text{Miss Both}) = 1 - P(\text{at least one}) = 1 - .9878 = .0121$

d. For this player use $P(\text{A}) = .58$

$P(\text{A} \cap \text{B}) = (.58)(.58) = .3364$

$P(\text{A} \cup \text{B}) = .58 + .58 - .3364 = .8236$

$P(\text{Miss Both}) = 1 - .8236 = .1764$

The probability Reggie Miller makes both free throws is .7921, while the center's probability is .3364. The probability Reggie Miller misses both free throws is only .0121, while the center's probability is a much higher, .1764. The opponent's strategy should be to foul the center and not Reggie Miller.

37. Let C = event consumer uses a plastic card
B = event consumer is 18 to 24 years old
B^c = event consumer is over 24 years old

Given information:

$$P(\text{C}) = .37$$

$$P(\text{B} \mid \text{C}) = .19$$

$$P(\text{B}^c \mid \text{C}) = .81$$

$$P(\text{B}) = .14$$

a. $$P(\text{C} \mid \text{B}) = \frac{P(\text{C} \cap \text{B})}{P(\text{B})}$$

but $P(\text{C} \cap \text{B})$ is unknown. So first compute

$$\begin{aligned} P(\text{C} \cap \text{B}) &= P(\text{C})P(\text{B} \mid \text{C}) \\ &= .37(.19) = .0703 \end{aligned}$$

Then

$$P(\text{C} \mid \text{B}) = \frac{P(\text{C} \cap \text{B})}{P(\text{B})} = \frac{.0703}{.14} = .5021$$

b. $P(\mathrm{C}|\mathrm{B}^c) = \dfrac{P(\mathrm{C} \cap \mathrm{B}^c)}{P(\mathrm{B}^c)}$

but $P(\mathrm{C} \cap \mathrm{B}^c)$ and $P(\mathrm{B}^c)$ are unknown. However, they can be computed as follows.

$$P(\mathrm{C} \cap \mathrm{B}^c) = P(\mathrm{C})P(\mathrm{B}^c|\mathrm{C})$$
$$= .37(.81) = .2997$$

$$P(\mathrm{B}^c) = 1 \text{-} P(\mathrm{B}) = 1 - .14 = .86$$

Then

$$P(\mathrm{C}|\mathrm{B}^c) = \frac{P(\mathrm{C} \cap \mathrm{B}^c)}{P(\mathrm{B}^c)} = \frac{.2997}{.86} = .3485$$

c. There is a higher probability that the younger consumer, age 18 to 24, will use plastic when making a purchase. The probability that the 18 to 24 year old consumer uses plastic is .5021 and the probability that the older than 24 year old consumer uses plastic is .3485. Note that there is greater than .50 probability that the 18 to 24 years old consumer will use plastic.

d. Companies such as Visa, Mastercard and Discovery want their cards in the hands of consumers who will have a high probability of using the card. So yes, these companies should get their cards in the hands of young consumers even before these consumers have established a credit history. The companies should place a low limit of the amount of credit charges until the young consumer has demonstrated the responsibility to handle higher credit limits.

38. Let M = event consumer is a man
Let W = event consumer is a woman
Let B = event preferred plain bottled water
Let S = event preferred sports drink

a. $P(\mathrm{B}) = 280/400 = .70$

b. Sports drink: 80 + 40 = 120

$P(\mathrm{S}) = 120/400 = .30$

c. $P(\mathrm{M}|\mathrm{S}) = 80/120 = .67$

$P(\mathrm{W}|\mathrm{S}) = 40/120 = .33$

d. $P(\mathrm{S})P(\mathrm{M}|\mathrm{S}) = .30(.67) = .20$

$P(\mathrm{S})P(\mathrm{W}|\mathrm{S}) = .30(.33) = .10$

e. $P(\text{M}) = .50$

$$P(\text{S}|\text{M}) = \frac{P(\text{M} \cap \text{S})}{P(\text{M})} = \frac{.20}{.50} = .40$$

f. $P(\text{W}) = .50$

$$P(\text{S}|\text{W}) = \frac{P(\text{W} \cap \text{S})}{P(\text{W})} = \frac{.10}{.50} = .20$$

g. No;

$$P(\text{S}|\text{M}) \neq P(\text{S})$$

$$P(\text{S}|\text{W}) \neq P(\text{S})$$

39. a. Yes, since $P(\text{A}_1 \cap \text{A}_2) = 0$

b. $P(\text{A}_1 \cap \text{B}) = P(\text{A}_1)P(\text{B} \mid \text{A}_1) = .40(.20) = .08$

$P(\text{A}_2 \cap \text{B}) = P(\text{A}_2)P(\text{B} \mid \text{A}_2) = .60(.05) = .03$

c. $P(\text{B}) = P(\text{A}_1 \cap \text{B}) + P(\text{A}_2 \cap \text{B}) = .08 + .03 = .11$

d. $$P(\text{A}_1|\text{B}) = \frac{.08}{.11} = .7273$$

$$P(\text{A}_2|\text{B}) = \frac{.03}{.11} = .2727$$

40. a. $P(\text{B} \cap \text{A}_1) = P(\text{A}_1)P(\text{B} \mid \text{A}_1) = (.20)(.50) = .10$

$P(\text{B} \cap \text{A}_2) = P(\text{A}_2)P(\text{B} \mid \text{A}_2) = (.50)(.40) = .20$

$P(\text{B} \cap \text{A}_3) = P(\text{A}_3)P(\text{B} \mid \text{A}_3) = (.30)(.30) = .09$

b. $$P(\text{A}_2|\text{B}) = \frac{.20}{.10 + .20 + .09} = .51$$

c.

Events	$P(\text{A}_i)$	$P(\text{B} \mid \text{A}_i)$	$P(\text{A}_i \cap \text{B})$	$P(\text{A}_i \mid \text{B})$
A_1	.20	.50	.10	.26
A_2	.50	.40	.20	.51
A_3	.30	.30	.09	.23
	1.00		.39	1.00

41. S_1 = successful, S_2 = not successful and B = request received for additional information.

a. $P(\text{S}_1) = .50$

b. $P(\text{B} \mid \text{S}_1) = .75$

c. $P(S_1|B) = \dfrac{(.50)(.75)}{(.50)(.75)+(.50)(.40)} = \dfrac{.375}{.575} = .65$

42. M = missed payment
D$_1$ = customer defaults
D$_2$ = customer does not default

$P(D_1) = .05 \quad P(D_2) = .95 \quad P(M \mid D_2) = .2 \quad P(M \mid D_1) = 1$

a. $P(D_1|M) = \dfrac{P(D_1)P(M|D_1)}{P(D_1)P(M|D_1)+P(D_2)P(M|D_2)} = \dfrac{(.05)(1)}{(.05)(1)+(.95)(.2)} = \dfrac{.05}{.24} = .21$

b. Yes, the probability of default is greater than .20.

43. Let: S = small car
S^c = other type of vehicle
F = accident leads to fatality for vehicle occupant

We have $P(S) = .18$, so $P(S^c) = .82$. Also $P(F \mid S) = .128$ and $P(F \mid S^c) = .05$. Using the tabular form of Bayes Theorem provides:

Events	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
S	.18	.128	.023	.36
S^c	.82	.050	.041	.64
	1.00		.064	1.00

From the posterior probability column, we have $P(S \mid F) = .36$. So, if an accident leads to a fatality, the probability a small car was involved is .36.

44. a. $P(A_1) = .47 \qquad P(W \mid A_1) = .50$

$P(A_2) = .53 \qquad P(W \mid A_2) = .45$

b. Using tabular approach

	Prior Probabilities	Conditional Probability	Joint Probability	Posterior Probability
Events	$P(A_i)$	$P(W \mid A_i)$	$P(A_i \cap W)$	$P(A_i \mid W)$
Graduate	.47	.50	.2350	.4963
Not a Graduate	.53	.45	.2385	.5037
		$P(W) =$	.4735	1.0000

$P(A_i \mid W) = .4963$

c.

	Prior Probabilities	Conditional Probability	Joint Probability	Posterior Probability
Events	$P(A_i)$	$P(M \mid A_i)$	$P(A_i \cap W)$	$P(A_i \mid M)$
Graduate	.47	.50	.2350	.4463
Not a Graduate	.53	.45	.2915	.5537
		$P(M) =$	.5265	1.0000

$P(A_i \mid M) = .4463$

About a .05 higher probability a woman student will graduate compared to a man.

d. $P(\text{W}) = .4735$

$P(\text{M}) = .5265$

Approximately 47% women and 53% men.

45. a. Let S = person is age 65 or older

$$P(\text{S}) = \frac{34,991,753}{281,421,906} = .12$$

b. Let D = takes prescription drugs regularly

$$\begin{aligned} P(\text{D}) &= P(\text{D} \cap \text{S}) + P(\text{D} \cap \text{S}^c) = P(\text{D} \mid \text{S})P(\text{S}) + P(\text{D} \mid \text{S}^c)P(\text{S}^c) \\ &= .82(.12) + .49(.88) = .53 \end{aligned}$$

c. Let D_5 = takes 5 or more prescriptions

$$P(\text{D}_5 \cap \text{S}) = P(\text{D}_5 \mid \text{S})P(\text{S}) = .40(.12) = .048$$

d. $P(\text{S} \mid \text{D}_5) = \dfrac{P(\text{S} \cap \text{D}_5)}{P(\text{D}_5)}$

$$\begin{aligned} P(\text{D}_5) &= P(\text{S} \cap \text{D}_5) + P(\text{S}^c \cap \text{D}_5) = P(\text{D}_5 \mid \text{S})P(\text{S}) + P(\text{D}_5 \mid \text{S}^c)P(\text{S}^c) \\ &= .40(.12) + (.28)(.88) = .048 + .246 = .294 \end{aligned}$$

$$P(\text{S} \mid \text{D}_5) = \frac{.048}{.294} = .16$$

46. a. $P(24 \text{ to } 26 \mid \text{Yes}) = .1482/.4005 = .3700$

b. $P(\text{Yes} \mid 36 \text{ and over}) = .0253/.1090 = .2321$

c. $.1026 + .1482 + .1878 + .0917 + .0327 + .0253 = .5883$

d. $P(31 \text{ or more} \mid \text{No}) = (.0956 + .0837)/.5995 = .2991$

e. No, because the conditional probabilities do not all equal the marginal probabilities. For instance,

$$P(24 \text{ to } 26 \mid \text{Yes}) = .3700 \neq P(24 \text{ to } 26) = .3360$$

47. a. Let F = female. Using past history as a guide, $P(\text{F}) = .40$.

b. Let D = Dillard's

$$P(\text{F}|\text{D}) = \frac{.40\left(\frac{3}{4}\right)}{.40\left(\frac{3}{4}\right) + .60\left(\frac{1}{4}\right)} = \frac{.30}{.30 + .15} = .67$$

The revised (posterior) probability that the visitor is female is .67.

We should display the offer that appeals to female visitors.

48. a.

	Yes	No	Total
23 and Under	.1026	.0996	.2022
24 - 26	.1482	.1878	.3360
27 - 30	.0917	.1328	.2245
31 - 35	.0327	.0956	.1283
36 and Over	.0253	.0837	.1090
Total	.4005	.5995	1.0000

b. .2022

c. .2245 + .1283 + .1090 = .4618

d. .4005

49. a. $P(\text{Oil}) = .50 + .20 = .70$

b. Let S = Soil test results

Events	$P(A_i)$	$P(S \mid A_i)$	$P(A_i \cap S)$	$P(A_i \mid S)$
High Quality (A_1)	.50	.20	.10	.31
Medium Quality (A_2)	.20	.80	.16	.50
No Oil (A_3)	.30	.20	.06	.19
	1.00		$P(S) = .32$	1.00

$P(\text{Oil}) = .81$ which is good; however, probabilities now favor medium quality rather than high quality oil.

50. a. The first probability table is given.

	Household Income ($1000s)					
Education Level	Under 24.9	25.0-49.9	50.0-74.9	75.0-99.9	100 or more	Total
Not H.S. Graduate	0.093	0.041	0.016	0.005	0.004	0.159
H.S. Graduate	0.101	0.098	0.060	0.027	0.020	0.308
Some College	0.060	0.082	0.058	0.032	0.031	0.264
Bachelor's Degree	0.021	0.040	0.040	0.027	0.047	0.175
Beyond Bachelor's Degree	0.008	0.015	0.018	0.016	0.038	0.095
Total	0.284	0.276	0.192	0.108	0.140	1.000

b. This is a marginal probability.

P(Not H.S. graduate) = .159

c. This is the sum of 2 marginal probabilities.

P(Bachelor's Degree $\cup$ Beyond Bachelor's Degree) = .175 + .095 = .27

d. This is a conditional probability.

$$P(\text{100 or more} \mid \text{Bachelor's Degree}) = \frac{.047}{.175} = .269$$

e. This is a marginal probability.

P(Under 25) = P(Under 24.9) = .284

f. This is a conditional probability.

$$P(\text{Under 25} \mid \text{Bachelor's Degree}) = \frac{.021}{.175} = .12$$

g. No. Note that the probabilities in parts (e) and (f) are not equal.

51. Let: B = blogger
B^c = non blogger
Y = young adult (18-29)
Y^c = older adult

Given: $P(B) = .08 \quad P(Y \mid B) = .54 \quad P(Y \mid B^c) = .24$

$$P(Y \mid B) = \frac{P(Y \cap B)}{P(B)}$$

$$\therefore\ P(Y \cap B) = P(Y \mid B)P(B) = (.54)(.08) = .0432$$

$$P(Y \mid B^c) = \frac{P(Y \cap B^c)}{P(B^c)}$$

$$\therefore\ P(Y \cap B^c) = P(Y \mid B^c)P(B^c) = (.24)(.92) = .2208$$

	Young Adult	Older Adult	Total
Blogger	.0432	.0368	.08
Non Blogger	.2208	.6992	.92
Total:	.2640	.7360	1.00

b. $P(Y) = P(B \cap Y) + P(B^c \cap Y) = .0432 + .2208 = .2640$

c. $P(Y \cap C) = .0432$

d. $$P(B \mid Y) = \frac{P(B \cap Y)}{P(Y)} = \frac{.0432}{.2640} = .1636$$

52. a. Probability of the event $= P(\text{average}) + P(\text{above average}) + P(\text{excellent})$

$$= \frac{11}{50} + \frac{14}{50} + \frac{13}{50} = .22 + .28 + .26 = .76$$

b. Probability of the event $= P(\text{poor}) + P(\text{below average})$

$$= \frac{4}{50} + \frac{8}{50} = .24$$

53. Let A = lost time accident in current year
B = lost time accident previous year

Given: $P(B) = .06$, $P(A) = .05$, $P(A \mid B) = .15$

a. $P(A \cap B) = P(A \mid B)P(B) = .15(.06) = .009$

b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .06 + .05 - .009 = .101$ or 10.1%

54. Let I = treatment-caused injury
D = death from injury
N = injury caused by negligence
M = malpractice claim filed
\$ = payment made in claim

We are given $P(I) = 0.04$, $P(N \mid I) = 0.25$, $P(D \mid I) = 1/7$, $P(M \mid N) = 1/7.5 = 0.1333$, and $P(\$ \mid M) = 0.50$

a. $P(N) = P(N \mid I)\, P(I) + P(N \mid I^c)\, P(I^c)$
$= (0.25)(0.04) + (0)(0.96) = 0.01$

b. $P(D) = P(D \mid I)\, P(I) + P(D \mid I^c)\, P(I^c)$
$= (1/7)(0.04) + (0)(0.96) = 0.006$

c. $P(M) = P(M \mid N)\, P(N) + P(M \mid N^c)\, P(N^c)$
$= (0.1333)(0.01) + (0)(0.99) = 0.001333$

$P(\$) = P(\$ \mid M)\, P(M) + P(\$ \mid M^c)\, P(M^c)$
$= (0.5)(0.001333) + (0)(0.9987) = 0.00067$

55. a. $P(A) = 200/800 = .25$

b. $P(B) = 100/800 = .125$

c. $P(A \cap B) = 10/800 = .0125$

d. $P(A \mid B) = P(A \cap B)/P(B) = .0125/.125 = .10$

e. No, $P(A \mid B) \neq P(A) = .25$

56. a. Number favoring elimination $= .47(671) \approx 315$

b. Let F = in favor of proposal
D = Democrat

$P(F \mid D) = .29$

c. $P(\text{F}) = .47$ and $P(\text{F} \mid \text{D}) = .29$

Since $P(\text{F}) \neq P(\text{F} \mid \text{D})$ they are not independent.

d. I would expect Republicans to benefit most because they are the ones who had the most people in favor of the proposal.

57. a. $P(\text{B}|\text{S}) = \dfrac{P(\text{B} \cap S)}{P(\text{S})} = \dfrac{.12}{.40} = .30$

We have $P(\text{B} \mid \text{S}) > P(\text{B})$.

Yes, continue the ad since it increases the probability of a purchase.

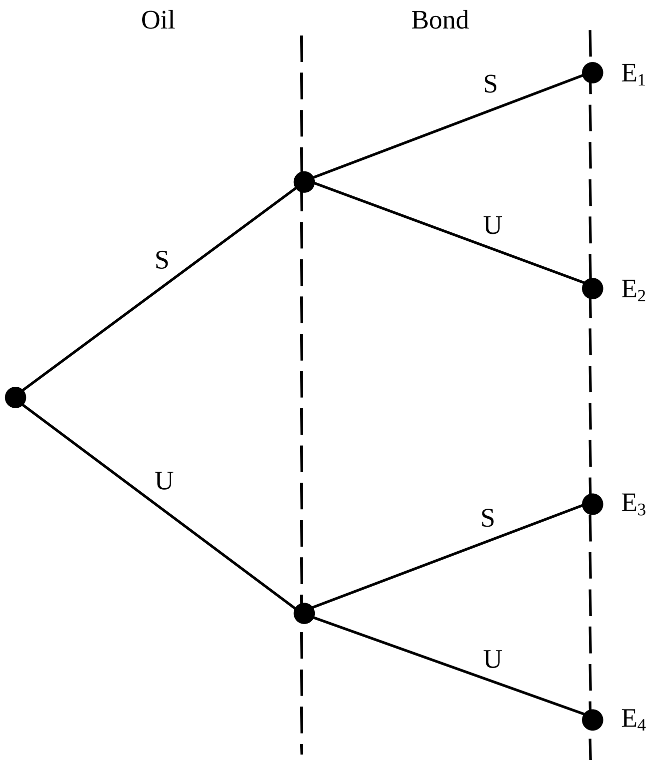
b. Estimate the company's market share at 20%. Continuing the advertisement should increase the market share since $P(\text{B} \mid \text{S}) = .30$.

c. $P(\text{B}|\text{S}) = \dfrac{P(\text{B} \cap S)}{P(\text{S})} = \dfrac{.10}{.30} = .333$

The second ad has a bigger effect.

58. a. (2)(2) = 4

b. Let S = successful
U = unsuccessful



c. O = {E_1, E_2}

M = {E_1, E_3}

d. $O \cup M = \{E_1, E_2, E_3\}$

e. $O \cap M = \{E_1\}$

f. No; since $O \cap M$ has a sample point.

59. Let I = important or very important
M = male
F = female

a. $P(I) = .49$ (a marginal probability)

b. $P(I \mid M) = .22/.50 = .44$ (a conditional probability)

c. $P(I \mid F) = .27/.50 = .54$ (a conditional probability)

d. It is not independent

$P(I) = .49 \neq P(I \mid M) = .44$

and

$P(I) = .49 \neq P(I \mid F) = .54$

e. Since level of importance is dependent on gender, we conclude that male and female respondents have different attitudes toward risk.

60. a. $P(\text{Excellent}) = .18$

$P(\text{Pretty Good}) = .50$

$P(\text{Pretty Good} \cup \text{Excellent}) = .18 + .50 = .68$

Note: Events are mutually exclusive since a person may only choose one rating.

b. 1035(.05) = 51.75

We estimate 52 respondents rated US companies poor.

c. 1035(.01) = 10.35

We estimate 10 respondents did not know or did not answer.