Chapter 5
Discrete Probability Distributions

Learning Objectives

1. Understand the concepts of a random variable and a probability distribution.

2. Be able to distinguish between discrete and continuous random variables.

3. Be able to compute and interpret the expected value, variance, and standard deviation for a discrete random variable.

4. Be able to compute and work with probabilities involving a binomial probability distribution.

5. Be able to compute and work with probabilities involving a Poisson probability distribution.

6. Know when and how to use the hypergeometric probability distribution.

Chapter 5
Solutions:

1. a. Head, Head (H,H)
   Head, Tail (H,T)
   Tail, Head (T,H)
   Tail, Tail (T,T)

   b. x = number of heads on two coin tosses

   c.

Outcome	Values of x
(H,H)	2
(H,T)	1
(T,H)	1
(T,T)	0

   d. Discrete. It may assume 3 values: 0, 1, and 2.

2. a. Let x = time (in minutes) to assemble the product.

   b. It may assume any positive value: $x > 0$.

   c. Continuous

3. Let Y = position is offered
   N = position is not offered

   a. S = {(Y,Y,Y), (Y,Y,N), (Y,N,Y), (Y,N,N), (N,Y,Y), (N,Y,N), (N,N,Y), (N,N,N)}

   b. Let N = number of offers made; N is a discrete random variable.

   c.

Experimental Outcome	(Y,Y,Y)	(Y,Y,N)	(Y,N,Y)	(Y,N,N)	(N,Y,Y)	(N,Y,N)	(N,N,Y)	(N,N,N)
Value of N	3	2	2	1	2	1	1	0

4. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

5. a. S = {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)}

   b.

Experimental Outcome	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
Number of Steps Required	2	3	4	3	4	5

6. a. values: 0,1,2,...,20
   discrete

   b. values: 0,1,2,...
   discrete

   c. values: 0,1,2,...,50
   discrete

   d. values: $0 \leq x \leq 8$
   continuous

e. values: $x > 0$
continuous

7. a. $f(x) \geq 0$ for all values of x.

$\Sigma f(x) = 1$ Therefore, it is a proper probability distribution.

b. Probability $x = 30$ is $f(30) = .25$

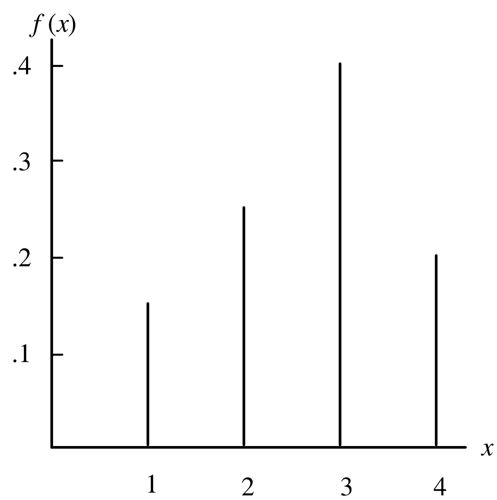
c. Probability $x \leq 25$ is $f(20) + f(25) = .20 + .15 = .35$

d. Probability $x > 30$ is $f(35) = .40$

8. a.

x	$f(x)$
1	3/20 = .15
2	5/20 = .25
3	8/20 = .40
4	4/20 = .20
	Total 1.00

b.



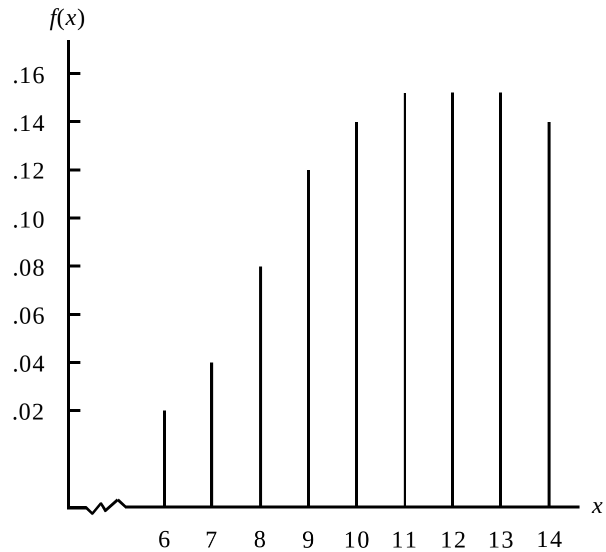
c. $f(x) \geq 0$ for $x = 1,2,3,4$.

$\Sigma f(x) = 1$

9. a.

Age	Number of Children	$f(x)$
6	37,369	0.018
7	87,436	0.043
8	160,840	0.080
9	239,719	0.119
10	286,719	0.142
11	306,533	0.152
12	310,787	0.154
13	302,604	0.150
14	289,168	0.143
	2,021,175	1.001

b.



c. $f(x) \geq 0$ for every x

$\Sigma f(x) = 1$

Note: $\Sigma f(x) = 1.001$ in part (a); difference from 1 is due to rounding values of $f(x)$.

10. a.

x	$f(x)$
1	0.05
2	0.09
3	0.03
4	0.42
5	0.41
	1.00

b.

x	$f(x)$
1	0.04
2	0.10
3	0.12
4	0.46
5	0.28
	1.00

c. P(4 or 5) = $f(4) + f(5) = 0.42 + 0.41 = 0.83$

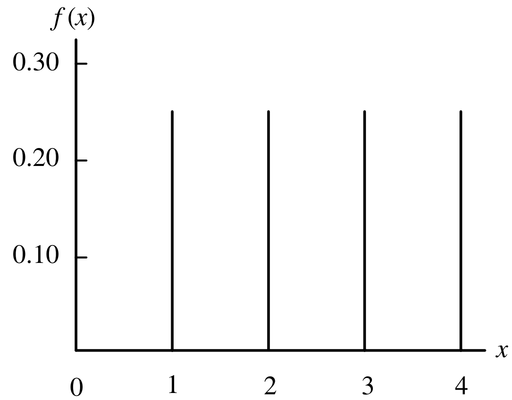
d. Probability of very satisfied: 0.28

e. Senior executives appear to be more satisfied than middle managers. 83% of senior executives have a score of 4 or 5 with 41% reporting a 5. Only 28% of middle managers report being very satisfied.

11. a.

Duration of Call x	$f(x)$
1	0.25
2	0.25
3	0.25
4	0.25
	1.00

b.



c. $f(x) \geq 0$ and $f(1) + f(2) + f(3) + f(4) = 0.25 + 0.25 + 0.25 + 0.25 = 1.00$

d. $f(3) = 0.25$

e. P(overtime) $= f(3) + f(4) = 0.25 + 0.25 = 0.50$

12. a. Yes; $f(x) \geq 0$. $\Sigma f(x) = 1$

b. $f(500{,}000) + f(600{,}000) = .10 + .05 = .15$

c. $f(100{,}000) = .10$

13. a. Yes, since $f(x) \geq 0$ for $x = 1,2,3$ and $\Sigma f(x) = f(1) + f(2) + f(3) = 1/6 + 2/6 + 3/6 = 1$

b. $f(2) = 2/6 = .333$

c. $f(2) + f(3) = 2/6 + 3/6 = .833$

14. a. $f(200) = 1 - f(-100) - f(0) - f(50) - f(100) - f(150)$

$= 1 - .95 = .05$

This is the probability MRA will have a $200,000 profit.

b. P(Profit) $= f(50) + f(100) + f(150) + f(200)$

$= .30 + .25 + .10 + .05 = .70$

c. P(at least 100) $= f(100) + f(150) + f(200)$

$= .25 + .10 + .05 = .40$

15. a.

x	$f(x)$	$x f(x)$
3	.25	.75
6	.50	3.00
9	.25	2.25
	1.00	6.00

$E(x) = \mu = 6$

b.

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
3	-3	9	.25	2.25
6	0	0	.50	0.00
9	3	9	.25	2.25
				4.50

$\text{Var}(x) = \sigma^2 = 4.5$

c. $\sigma = \sqrt{4.50} = 2.12$

16. a.

y	$f(y)$	$y f(y)$
2	.2	.4
4	.3	1.2
7	.4	2.8
8	.1	.8
	1.0	5.2

$E(y) = \mu = 5.2$

b.

y	$y - \mu$	$(y - \mu)^2$	$f(y)$	$(y - \mu)^2 f(y)$
2	-3.20	10.24	.20	2.048
4	-1.20	1.44	.30	.432
7	1.80	3.24	.40	1.296
8	2.80	7.84	.10	.784
				4.560

$\text{Var}(y) = 4.56$

$\sigma = \sqrt{4.56} = 2.14$

17. a/b.

x	$f(x)$	$x f(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
0	.10	.00	-2.45	6.0025	.600250
1	.15	.15	-1.45	2.1025	.315375
2	.30	.60	- .45	.2025	.060750
3	.20	.60	.55	.3025	.060500
4	.15	.60	1.55	2.4025	.360375
5	.10	.50	2.55	6.5025	.650250
		2.45			2.047500

$E(x) = \mu = 2.45$

$\sigma^2 = 2.05$

$\sigma = 1.43$

18. a/b.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
0	0.04	0.00	-1.84	3.39	0.12
1	0.34	0.34	-0.84	0.71	0.24
2	0.41	0.82	0.16	0.02	0.01
3	0.18	0.53	1.16	1.34	0.24
4	0.04	0.15	2.16	4.66	0.17
Total	1.00	1.84			0.79
		↑			↑
		$E(x)$			Var(x)

c/d.

y	$f(y)$	$yf(y)$	$y - \mu$	$(y - \mu)^2$	$(y - \mu)^2 f(y)$
0	0.00	0.00	-2.93	8.58	0.01
1	0.03	0.03	-1.93	3.72	0.12
2	0.23	0.45	-0.93	0.86	0.20
3	0.52	1.55	0.07	0.01	0.00
4	0.22	0.90	1.07	1.15	0.26
Total	1.00	2.93			0.59
		↑			↑
		$E(y)$			Var(y)

e. The number of bedrooms in owner-occupied houses is greater than in renter-occupied houses. The expected number of bedrooms is 1.09 = 2.93 - 1.84 greater. And, the variability in the number of bedrooms is less for the owner-occupied houses.

19. a. $E(x) = \Sigma x f(x) = 0(.56) + 2(.44) = .88$

b. $E(x) = \Sigma x f(x) = 0(.66) + 3(.34) = 1.02$

c. The expected value of a 3 - point shot is higher. So, if these probabilities hold up, the team will make more points in the long run with the 3 - point shot.

20. a.

x	$f(x)$	$xf(x)$
0	.85	0
500	.04	20
1000	.04	40
3000	.03	90
5000	.02	100
8000	.01	80
10000	.01	100
Total	1.00	430

The expected value of the insurance claim is \$430. If the company charges \$430 for this type of collision coverage, it would break even.

b. From the point of view of the policyholder, the expected gain is as follows:

Expected Gain = Expected claim payout – Cost of insurance coverage
= \$430 - \$520 = -\$90

The policyholder is concerned that an accident will result in a big repair bill if there is no insurance coverage. So even though the policyholder has an expected annual loss of $90, the insurance is protecting against a large loss.

21. a. $E(x) = \Sigma x f(x) = 0.05(1) + 0.09(2) + 0.03(3) + 0.42(4) + 0.41(5) = 4.05$

b. $E(x) = \Sigma x f(x) = 0.04(1) + 0.10(2) + 0.12(3) + 0.46(4) + 0.28(5) = 3.84$

c. Executives: $\sigma^2 = \Sigma (x - \mu)^2 f(x) = 1.25$

Middle Managers: $\sigma^2 = \Sigma (x - \mu)^2 f(x) = 1.13$

d. Executives: $\sigma = 1.12$

Middle Managers: $\sigma = 1.07$

e. The senior executives have a higher average score: 4.05 vs. 3.84 for the middle managers. The executives also have a slightly higher standard deviation.

22. a. $E(x) = \Sigma x f(x) = 300 (.20) + 400 (.30) + 500 (.35) + 600 (.15) = 445$

The monthly order quantity should be 445 units.

b. Cost: 445 @ $50 = $22,250
Revenue: 300 @ $70 = 21,000
$ 1,250 Loss

23. a. Rent Controlled: $E(x) = 1(.61) + 2(.27) + 3(.07) + 4(.04) + 5(.01) = 1.57$

Rent Stabilized: $E(x) = 1(.41) + 2(.30) + 3(.14) + 4(.11) + 5(.03) + 6(.01) = 2.08$

b. Rent Controlled:
$\text{Var}(x) = (-.57)^2.61 + (.43)^2.27 + (1.43)^2.07 + (2.43)^2.04 + (3.43)^2.01 = .75$

Rent Stabilized:
$\text{Var}(x) = (-1.08)^2.41 + (-.08)^2.30 + (.92)^2.14 + (1.92)^2.11 + (2.92)^2.03 + (3.92)^2.01 = 1.41$

c. From the expected values in part (a), it is clear that the expected number of persons living in rent stabilized units is greater than the number of persons living in rent controlled units. For example, comparing a building that contained 10 rent controlled units to a building that contained 10 rent stabilized units, the expected number of persons living in the rent controlled building would be 1.57(10) = 15.7 or approximately 16. For the rent stabilized building, the expected number of persons is approximately 21. There is also more variability in the number of persons living in rent stabilized units.

24. a. Medium $E(x) = \Sigma x f(x)$

$= 50 (.20) + 150 (.50) + 200 (.30) = 145$

Large: $E(x) = \Sigma x f(x)$

$= 0 (.20) + 100 (.50) + 300 (.30) = 140$

Medium preferred.

b. Medium

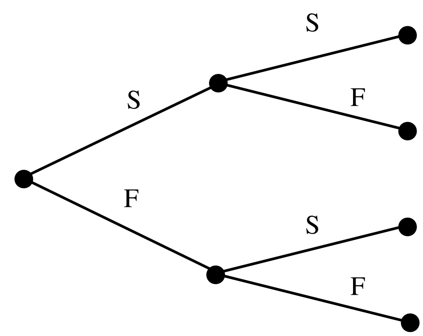
x	$f(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
50	.20	-95	9025	1805.0
150	.50	5	25	12.5
200	.30	55	3025	907.5
				σ^2 = 2725.0

Large

y	$f(y)$	$y - \mu$	$(y - \mu)^2$	$(y - \mu)^2 f(y)$
0	.20	-140	19600	3920
100	.50	-40	1600	800
300	.30	160	25600	7680
				σ^2 = 12,400

Medium preferred due to less variance.

25. a.



b. $f(1) = \binom{2}{1}(.4)^1(.6)^1 = \frac{2!}{1!1!}(.4)(.6) = .48$

c. $f(0) = \binom{2}{0}(.4)^0(.6)^2 = \frac{2!}{0!2!}(1)(.36) = .36$

d. $f(2) = \binom{2}{2}(.4)^2(.6)^0 = \frac{2!}{2!0!}(.16)(1) = .16$

e. $P(x \geq 1) = f(1) + f(2) = .48 + .16 = .64$

f. $E(x) = n\,p = 2\,(.4) = .8$

$\text{Var}(x) = n\,p\;(1 - p) = 2\,(.4)\,(.6) = .48$

$\sigma = \sqrt{.48} = .6928$

26. a. $f(0) = .3487$

b. $f(2) = .1937$

c. $P(x \leq 2) = f(0) + f(1) + f(2) = .3487 + .3874 + .1937 = .9298$

d. $P(x \geq 1) = 1 - f(0) = 1 - .3487 = .6513$

e. $E(x) = np = 10(.1) = 1$

f. $\text{Var}(x) = np(1-p) = 10(.1)(.9) = .9$

$\sigma = \sqrt{.9} = .9487$

27. a. $f(12) = .1144$

b. $f(16) = .1304$

c. $P(x \geq 16) = f(16) + f(17) + f(18) + f(19) + f(20)$

$= .1304 + .0716 + .0278 + .0068 + .0008 = .2374$

d. $P(x \leq 15) = 1 - P(x \geq 16) = 1 - .2374 = .7626$

e. $E(x) = np = 20(.7) = 14$

f. $\text{Var}(x) = np(1-p) = 20(.7)(.3) = 4.2$

$\sigma = \sqrt{4.2} = 2.0494$

28. a. $f(2) = \binom{6}{2}(.23)^2(.77)^4 = .2789$

b. $P(\text{at least } 2) = 1 - f(0) - f(1)$

$$= 1 - \binom{6}{0}(.23)^0(.77)^6 - \binom{6}{1}(.23)^1(.77)^5$$

$= 1 - .2084 - .3735 = .4181$

c. $f(0) = \binom{10}{0}(.23)^0(.77)^{10} = .0733$

29. a. $f(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$

$$f(3) = \frac{10!}{3!(10-3)!}(.30)^3(1-.30)^{10-3}$$

$$f(3) = \frac{10(9)(8)}{3(2)(1)}(.30)^3(1-.30)^7 = .2668$$

b. $P(x \geq 3) = 1 - f(0) - f(1) - f(2)$

$$f(0) = \frac{10!}{0!(10)!}(.30)^0(1-.30)^{10} = .0282$$

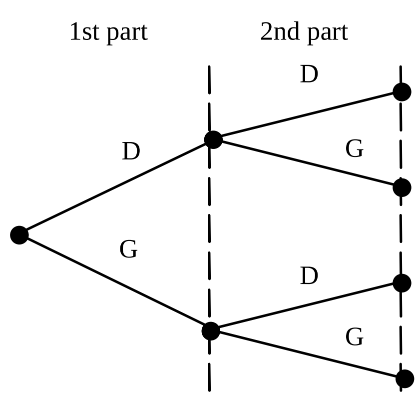
$$f(1) = \frac{10!}{1!(9)!}(.30)^1(1-.30)^9 = .1211$$

$$f(2) = \frac{10!}{2!(8)!}(.30)^2(1-.30)^8 = .2335$$

$P(x \geq 3) = 1 - .0282 - .1211 - .2335 = .6172$

30. a. Probability of a defective part being produced must be .03 for each part selected; parts must be selected independently.

b. Let: D = defective
G = not defective



Experimental Outcome	Number Defective
(D, D)	2
(D, G)	1
(G, D)	1
(G, G)	0

c. 2 outcomes result in exactly one defect.

d. P(no defects) = (.97) (.97) = .9409

P (1 defect) = 2 (.03) (.97) = .0582

P (2 defects) = (.03) (.03) = .0009

31. Binomial $n = 10$ and $p = .09$

$$f(x) = \frac{10!}{x!(10-x)!}(.09)^x(.91)^{10-x}$$

a. Yes. Since they are selected randomly, p is the same from trial to trial and the trials are independent.

b. $f(2) = .1714$

c. $f(0) = .3894$

d. $1 - f(0) - f(1) - f(2) = 1 - (.3894 + .3851 + .1714) = .0541$

32. a. .90

b. $P(\text{at least 1}) = f(1) + f(2)$

$$f(1) = \frac{2!}{1!\ 1!}(.9)^1(.1)^1$$

$$= 2(.9)(.1) = .18$$

$$f(2) = \frac{2!}{2!\ 0!}(.9)^1(.1)^0$$

$$= 1(.81)(1) = .81$$

$\therefore P(\text{at least 1}) = .18 + .81 = .99$

Alternatively

$P(\text{at least 1}) = 1 - f(0)$

$$f(0) = \frac{2!}{0!\ 2!}(.9)^0(.1)^2 = .01$$

Therefore, $P(\text{at least 1}) = 1 - .01 = .99$

c. $P(\text{at least 1}) = 1 - f(0)$

$$f(0) = \frac{3!}{0!\ 3!}(.9)^0(.1)^3 = .001$$

Therefore, $P(\text{at least 1}) = 1 - .001 = .999$

d. Yes; P(at least 1) becomes very close to 1 with multiple systems and the inability to detect an attack would be catastrophic.

33. a $f(12) = \frac{20!}{12!8!}(.5)^{12}(.5)^8$

Using the binomial tables, $f(12) = .1201$

b. $f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$

.0000 + .0000 + .0002 + .0011 + .0046 + .0148 = .0207

c. $E(x) = np = 20(.5) = 10$

d. $\text{Var}(x) = \sigma^2 = np(1 - p) = 20(.5)(.5) = 5$

$\sigma = \sqrt{5} = 2.24$

34. a. $f(x) = \binom{n}{x} (p)^x (1-p)^{n-x}$

$$f(4) = \frac{15!}{4!(15-4)!} (.28)^4 (1-.28)^{15-4}$$

$$f(4) = \frac{15(14)(13)(12)}{4(3)(2)(1)} (.28)^4 (1-.28)^{11} = .2262$$

b. $P(x \geq 3) = 1 - f(0) - f(1) - f(2)$

$$f(0) = \frac{15!}{0!(15)!} (.28)^0 (1-.28)^{15} = .0072$$

$$f(1) = \frac{15!}{1!(14)!} (.28)^1 (1-.28)^{14} = .0423$$

$$f(2) = \frac{15!}{2!(13)!} (.28)^2 (1-.28)^{13} = .1150$$

$P(x \geq 3) = 1 - .0072 - .0423 - .1150 = .8355$

35. a. $f(0) + f(1) + f(2) = .0115 + .0576 + .1369 = .2060$

b. $f(4) = .2182$

c. $1 - [f(0) + f(1) + f(2) + f(3)] = 1 - .2060 - .2054 = .5886$

d. $\mu = np = 20(.20) = 4$

36. a. $p = ¼ = .25$

$$f(x) = \binom{n}{x} (p)^x (1-p)^{n-x}$$

$$f(4) = \binom{20}{4} (.25)^4 (1-.25)^{20-4}$$

$$f(4) = \frac{20!}{4!(20-4)!} (.25)^4 (.75)^{16} = f(4) = \frac{20(19)(18)(17)}{4(3)(2)(1)} (.25)^4 (.75)^{16} = .1897$$

b. $P(x \geq 2) = 1 - f(0) - f(1)$

Using the binomial tables $f(0) = .0032$ and $f(1) = .0211$

$P(x \geq 2) = 1 - .0032 - .0211 = .9757$

c. Using the binomial tables $f(12) = .0008$

And, with $f(13) = .0002$, $f(14) = .0000$, and so on, the probability of finding that 12 or more investors have exchange-traded funds in their portfolio is so small that it is highly unlikely that $p = .25$. In such a case, we would doubt the accuracy of the results and conclude that p must be greater than .25.

d. $\mu = np = 20(.25) = 5$

37. $E(x) = np = 35(.23) = 8.05$ (8 automobiles)

$\text{Var}(x) = np(1 - p) = 35(.23)(1-.23) = 6.2$

$\sigma = \sqrt{6.2} = 2.49$

38. a. $f(x) = \dfrac{3^x e^{-3}}{x!}$

b. $f(2) = \dfrac{3^2 e^{-3}}{2!} = \dfrac{9(.0498)}{2} = .2241$

c. $f(1) = \dfrac{3^1 e^{-3}}{1!} = 3(.0498) = .1494$

d. $P(x \geq 2) = 1 - f(0) - f(1) = 1 - .0498 - .1494 = .8008$

39. a. $f(x) = \dfrac{2^x e^{-2}}{x!}$

b. $\mu = 6$ for 3 time periods

c. $f(x) = \dfrac{6^x e^{-6}}{x!}$

d. $f(2) = \dfrac{2^2 e^{-2}}{2!} = \dfrac{4(.1353)}{2} = .2706$

e. $f(6) = \dfrac{6^6 e^{-6}}{6!} = .1606$

f. $f(5) = \dfrac{4^5 e^{-4}}{5!} = .1563$

40. a. $\mu = 48(5/60) = 4$

$f(3) = \dfrac{4^3 e^{-4}}{3!} = \dfrac{(64)(.0183)}{6} = .1952$

b. $\mu = 48\ (15 / 60) = 12$

$$f(10) = \frac{12^{10} e^{-12}}{10!} = .1048$$

c. $\mu = 48\ (5 / 60) = 4$ I expect 4 callers to be waiting after 5 minutes.

$$f(0) = \frac{4^{0} e^{-4}}{0!} = .0183$$

The probability none will be waiting after 5 minutes is .0183.

d. $\mu = 48\ (3 / 60) = 2.4$

$$f(0) = \frac{2.4^{0} e^{-2.4}}{0!} = .0907$$

The probability of no interruptions in 3 minutes is .0907.

41. a. 30 per hour

b. $\mu = 1\ (5/2) = 5/2$

$$f(3) = \frac{(5/2)^3 e^{-(5/2)}}{3!} = .2138$$

c. $$f(0) = \frac{(5/2)^0 e^{-(5/2)}}{0!} = e^{-(5/2)} = .0821$$

42. a. $$f(0) = \frac{7^0 e^{-7}}{0!} = e^{-7} = .0009$$

b. probability = 1 - [f(0) + f(1)]

$$f(1) = \frac{7^1 e^{-7}}{1!} = 7e^{-7} = .0064$$

probability = 1 - [.0009 + .0064] = .9927

c. $\mu = 3.5$

$$f(0) = \frac{3.5^0 e^{-3.5}}{0!} = e^{-3.5} = .0302$$

probability = 1 - f(0) = 1 - .0302 = .9698

d. probability = 1 - [f(0) + f(1) + f(2) + f(3) + f(4)]

= 1 - [.0009 + .0064 + .0223 + .0521 + .0912] = .8271

Note: The Poisson tables were used to compute the Poisson probabilities f(0), f(1), f(2), f(3) and f(4) in part (d).

43. a. $f(0) = \frac{10^0 e^{-10}}{0!} = e^{-10} = .000045$

b. $f(0) + f(1) + f(2) + f(3)$

$f(0) = .000045$ (part a)

$f(1) = \frac{10^1 e^{-10}}{1!} = .00045$

Similarly, $f(2) = .00225$, $f(3) = .0075$

and $f(0) + f(1) + f(2) + f(3) = .010245$

c. 2.5 arrivals / 15 sec. period Use $\mu = 2.5$

$f(0) = \frac{2.5^0 e^{-2.5}}{0!} = .0821$

d. $1 - f(0) = 1 - .0821 = .9179$

44. Poisson distribution applies

a. $\mu = 1.25$ per month

b. $f(0) = \frac{1.25^0 e^{-1.25}}{0!} = 0.2865$

c. $f(1) = \frac{1.25^1 e^{-1.25}}{1!} = 0.3581$

d. $P(\text{More than } 1) = 1 - f(0) - f(1) = 1 - 0.2865 - 0.3581 = 0.3554$

45. a. $f(x) = \frac{\mu^x e^{-\mu}}{x!}$

$f(0) = \frac{3^0 e^{-3}}{0!} = e^{-3} = .0498$

b. $P(x \geq 2) = 1 - f(0) - f(1)$

$f(1) = \frac{3^1 e^{-3}}{1!} = .1494$

$P(x \geq 2) = 1 - .0498 - .1494 = .8008$

c. $\mu = 3$ per year

$\mu = 3/2 = 1.5$ per 6 months

d. $f(0) = \frac{1.5^0 e^{-1.5}}{0!} = .2231$

46. a. $$f(1)=\frac{\binom{3}{1}\binom{10-3}{4-1}}{\binom{10}{4}}=\frac{\left(\frac{3!}{1!2!}\right)\left(\frac{7!}{3!4!}\right)}{\frac{10!}{4!6!}}=\frac{(3)(35)}{210}=.50$$

b. $$f(2)=\frac{\binom{3}{2}\binom{10-3}{2-2}}{\binom{10}{2}}=\frac{(3)(1)}{45}=.067$$

c. $$f(0)=\frac{\binom{3}{0}\binom{10-3}{2-0}}{\binom{10}{2}}=\frac{(1)(21)}{45}=.4667$$

d. $$f(2)=\frac{\binom{3}{2}\binom{10-3}{4-2}}{\binom{10}{4}}=\frac{(3)(21)}{210}=.30$$

47. $$f(3)=\frac{\binom{4}{3}\binom{15-4}{10-3}}{\binom{15}{10}}=\frac{(4)(330)}{3003}=.4396$$

48. Hypergeometric with $N = 10$ and $r = 7$

a. $$f(2)=\frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}}=\frac{(21)(3)}{120}=.5250$$

b. Must be 0 or 1 prefer basketball.

$$f(1)=\frac{\binom{7}{1}\binom{3}{2}}{\binom{10}{3}}=\frac{(7)(3)}{120}=.1750$$

$$f(0)=\frac{\binom{7}{0}\binom{3}{3}}{\binom{10}{3}}=\frac{(1)(1)}{120}=.0083$$

P(Majority football) $= f(1)+f(0) = .1833$

49. Parts a, b & c involve the hypergeometric distribution with $N = 52$ and $n = 2$

a. $r = 20, x = 2$

$$f(2) = \frac{\binom{20}{2}\binom{32}{0}}{\binom{52}{2}} = \frac{(190)(1)}{1326} = .1433$$

b. $r = 4, x = 2$

$$f(2) = \frac{\binom{4}{2}\binom{48}{0}}{\binom{52}{2}} = \frac{(6)(1)}{1326} = .0045$$

c. $r = 16, x = 2$

$$f(2) = \frac{\binom{16}{2}\binom{36}{0}}{\binom{52}{2}} = \frac{(120)(1)}{1326} = .0905$$

d. Part (a) provides the probability of blackjack plus the probability of 2 aces plus the probability of two 10s. To find the probability of blackjack we subtract the probabilities in (b) and (c) from the probability in (a).

P(blackjack) = .1433 - .0045 - .0905 = .0483

50. $N = 60 \quad n = 10$

a. $r = 20 \quad x = 0$

$$f(0) = \frac{\binom{20}{0}\binom{40}{10}}{\binom{60}{10}} = \frac{(1)\left(\frac{40!}{10!30!}\right)}{\frac{60!}{10!50!}} = \left(\frac{40!}{10!30!}\right)\left(\frac{10!50!}{60!}\right)$$

$$= \frac{40\cdot 39\cdot 38\cdot 37\cdot 36\cdot 35\cdot 34\cdot 33\cdot 32\cdot 31}{60\cdot 59\cdot 58\cdot 57\cdot 56\cdot 55\cdot 54\cdot 53\cdot 52\cdot 51} \approx .01$$

b. $r = 20 \quad x = 1$

$$f(1) = \frac{\binom{20}{1}\binom{40}{9}}{\binom{60}{10}} = 20\left(\frac{40!}{9!31!}\right)\left(\frac{10!50!}{60!}\right)$$

$$\approx .07$$

c. $1 - f(0) - f(1) = 1 - .08 = .92$

d. Same as the probability one will be from Hawaii. In part b that was found to equal approximately .07.

51. a. $$f(0) = \frac{\binom{5}{0}\binom{10}{3}}{\binom{15}{3}} = \frac{(1)(120)}{455} = .2637$$

b. $$f(1) = \frac{\binom{5}{1}\binom{10}{2}}{\binom{15}{3}} = \frac{(5)(45)}{455} = .4945$$

c. $$f(2) = \frac{\binom{5}{2}\binom{10}{1}}{\binom{15}{3}} = \frac{(10)(10)}{455} = .2198$$

d. $$f(3) = \frac{\binom{5}{3}\binom{10}{0}}{\binom{15}{3}} = \frac{(10)(1)}{455} = .0220$$

52. Hypergeometric with $N = 10$ and $r = 2$.

Focus on the probability of 0 defectives, then the probability of rejecting the shipment is $1 - f(0)$.

a. $n = 3, x = 0$

$$f(0) = \frac{\binom{2}{0}\binom{8}{3}}{\binom{10}{3}} = \frac{56}{120} = .4667$$

$P(\text{Reject}) = 1 - .4667 = .5333$

b. $n = 4, x = 0$

$$f(0) = \frac{\binom{2}{0}\binom{8}{4}}{\binom{10}{4}} = \frac{70}{210} = .3333$$

$P(\text{Reject}) = 1 - .3333 = .6667$

c. $n = 5, x = 0$

$$f(0) = \frac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}} = \frac{56}{252} = .2222$$

$P(\text{Reject}) = 1 - .2222 = .7778$

d. Continue the process. $n = 7$ would be required with the probability of rejecting $= .9333$

53. a. $$f(1) = \frac{\binom{7}{1}\binom{3}{1}}{\binom{10}{2}} = \frac{(7)(3)}{45} = .4667$$

b. $$f(2) = \frac{\binom{7}{2}\binom{3}{0}}{\binom{10}{2}} = \frac{(21)(1)}{45} = .4667$$

c. $$f(0) = \frac{\binom{7}{0}\binom{3}{2}}{\binom{10}{2}} = \frac{(1)(3)}{45} = .0667$$

54. a. $E(x) = np = 100(.041) = 4.1$

b. $\text{Var}(x) = np(1 - p) = 100(.041)(.959) = 3.93$

$$\sigma = \sqrt{3.93} = 1.98$$

55. Hypergeometric $N = 52, n = 5$ and $r = 4$.

a. $$\frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}} = \frac{6(17296)}{2{,}598{,}960} = .0399$$

b. $$\frac{\binom{4}{1}\binom{48}{4}}{\binom{52}{5}} = \frac{4(194580)}{2{,}598{,}960} = .2995$$

c. $\dfrac{\binom{4}{0}\binom{48}{5}}{\binom{52}{5}} = \dfrac{1{,}712{,}304}{2{,}598{,}960} = .6588$

d. $1 - f(0) = 1 - .6588 = .3412$

56. Since the shipment is large we can assume that the probabilities do not change from trial to trial and use the binomial probability distribution.

a. $n = 5$

$$f(0) = \binom{5}{0}(0.01)^0(0.99)^5 = 0.9510$$

b. $f(1) = \binom{5}{1}(0.01)^1(0.99)^4 = 0.0480$

c. $1 - f(0) = 1 - .9510 = .0490$

d. No, the probability of finding one or more items in the sample defective when only 1% of the items in the population are defective is small (only .0490). I would consider it likely that more than 1% of the items are defective.

57. a. $f(3) = \dfrac{3^3 e^{-3}}{3!} = 0.2240$

b. $f(3) + f(4) + \cdots = 1 - [f(0) + f(1) + f(2)]$

$$f(0) = \frac{3^0 e^{-3}}{0!} = e^{-3} = .0498$$

Similarly, $f(1) = .1494$, $f(2) = .2240$

$\therefore 1 - [.0498 + .1494 + .2241] = .5767$

58. a. We must have $E(x) = np \geq 10$

With $p = .4$, this leads to:
$n(.4) \geq 10$
$n \geq 25$

b. With $p = .12$, this leads to:
$n(.12) \geq 10$
$n \geq 83.33$

So, we must contact 84 people in this age group to have an expected number of internet users of at least 10.

c. $\sigma = \sqrt{25(.4)(.6)} = 2.45$

d. $\sigma = \sqrt{84(.12)(.88)} = 2.97$

59. $\mu = 10 \quad f(4) = .0189$

60. a. $n = 20$ and $x = 3$

$$f(3) = \binom{20}{3}(.05)^3(.95)^{17} = .0596$$

b. $n = 20$ and $x = 0$

$$f(0) = \binom{20}{0}(.05)^0(.95)^{20} = .3585$$

c. $E(x) = n\,p = 2000(.05) = 100$

The expected number of employees is 100.

d. $\sigma^2 = np\,(1 - p) = 2000(.05)(.95) = 95$

$\sigma = \sqrt{95} = 9.75$

61. $\mu = 1.5$

prob of 3 or more breakdowns is $1 - [\,f(0) + f(1) + f(2)\,]$.

$1 - [\,f(0) + f(1) + f(2)\,]$

$= 1 - [\,.2231 + .3347 + .2510]$

$= 1 - .8088 = .1912$

62. a.

x	$f(x)$
9	.30
10	.20
11	.25
12	.05
13	.20

b. $E(x) = \Sigma x\,f(x)$

$= 9(.30) + 10(.20) + 11(.25) + 12(.05) + 13(.20) = 10.65$

Expected value of expenses: $10.65 million

c. $\text{Var}(x) = \Sigma(x - \mu)^2 f(x)$

$= (9 - 10.65)^2\,(.30) + (10 - 10.65)^2\,(.20) + (11 - 10.65)^2\,(.25)$

$+ (12 - 10.65)^2\,(.05) + (13 - 10.65)^2\,(.20) = 2.13$

d. Looks Good: E(Profit) = 12 - 10.65 = 1.35 million

However, there is a .20 probability that expenses will equal $13 million and the college will run a deficit.

63. $\mu = 15$

prob of 20 or more arrivals $= f(20) + f(21) + \cdots$

$= .0418 + .0299 + .0204 + .0133 + .0083 + .0050 + .0029 + .0016 + .0009 + .0004 + .0002 + .0001 + .0001 = .1249$

64. a/b.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
1	0.24	0.24	-2.00	4.00	0.97
2	0.21	0.41	-1.00	1.00	0.21
3	0.10	0.31	0.00	0.00	0.00
4	0.21	0.83	1.00	1.00	0.21
5	0.24	1.21	2.00	4.00	0.97
Total	1.00	3.00			2.34
		↑			↑
		$E(x)$			Var(x)

c. For the bond fund categories: $E(x) = 1.36$ Var(x) = .23

For the stock fund categories: $E(x) = 4$ Var(x) = 1.00

The total risk of the stock funds is much higher than for the bond funds. It makes sense to analyze these separately. When you do the variances for both groups (stocks and bonds), they are reduced.

65. a. $E(x) = 800(.30) = 240$

b. $\sigma = \sqrt{np(1-p)} = \sqrt{800(.30)(.70)} = 12.9615$

c. For this one $p = .70$ and $(1-p) = .30$, but the answer is the same as in part (b). For a binomial probability distribution, the variance for the number of successes is the same as the variance for the number of failures. Of course, this also holds true for the standard deviation.

66. a/b/c.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
1	0.07	0.07	-2.12	4.49	0.31
2	0.21	0.42	-1.12	1.25	0.26
3	0.29	0.87	-0.12	0.01	0.00
4	0.39	1.56	0.88	0.77	0.30
5	0.04	0.20	1.88	3.53	0.14
Total	1.00	3.12			1.03
		↑			↑
		$E(x)$			Var(x)

$\sigma = \sqrt{1.03} = 1.01$

d. The expected level of optimism is 3.12. This is a bit above neutral and indicates that investment managers are somewhat optimistic. Their attitudes are centered between neutral and bullish with the consensus being closer to neutral.