Chapter 6
Continuous Probability Distributions

Learning Objectives

1. Understand the difference between how probabilities are computed for discrete and continuous random variables.

2. Know how to compute probability values for a continuous uniform probability distribution and be able to compute the expected value and variance for such a distribution.

3. Be able to compute probabilities using a normal probability distribution. Understand the role of the standard normal distribution in this process.

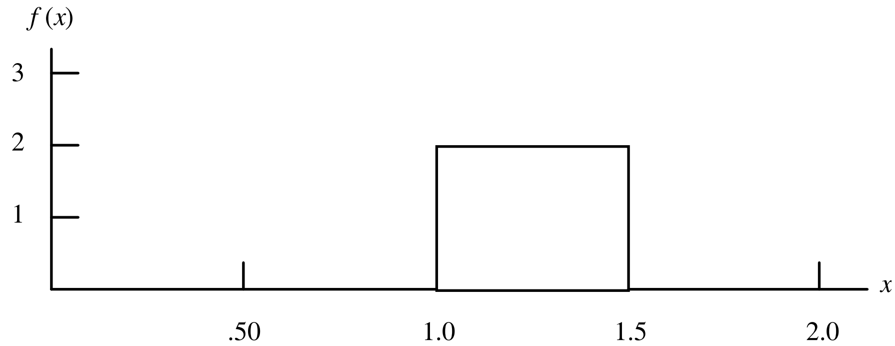
4. Be able to use the normal distribution to approximate binomial probabilities.

5. Be able to compute probabilities using an exponential probability distribution.

6. Understand the relationship between the Poisson and exponential probability distributions.

Chapter 6
Solutions:

1. a.

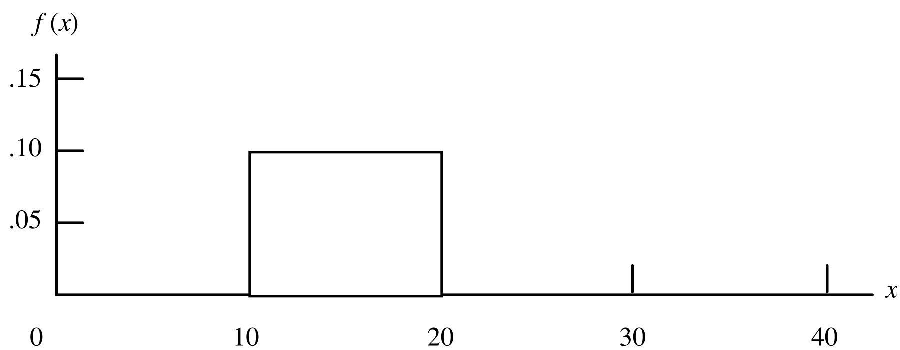


b. $P(x = 1.25) = 0$. The probability of any single point is zero since the area under the curve above any single point is zero.

c. $P(1.0 \le x \le 1.25) = 2(.25) = .50$

d. $P(1.20 < x < 1.5) = 2(.30) = .60$

2. a.



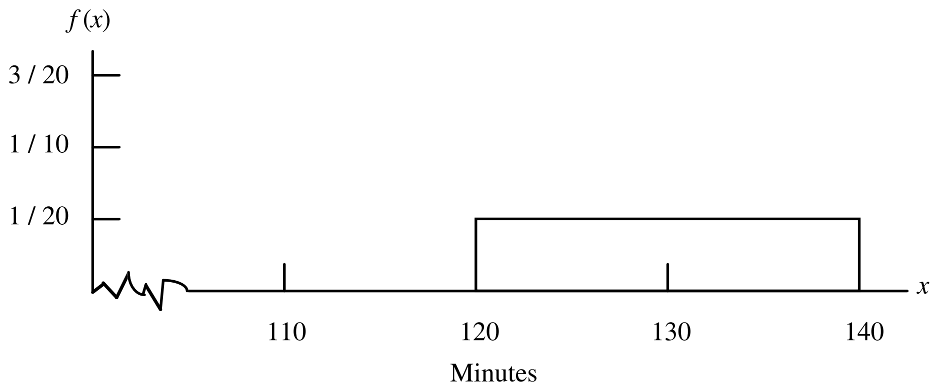
b. $P(x < 15) = .10(5) = .50$

c. $P(12 \le x \le 18) = .10(6) = .60$

d. $E(x) = \dfrac{10+20}{2} = 15$

e. $\text{Var}(x) = \dfrac{(20-10)^2}{12} = 8.33$

3. a.

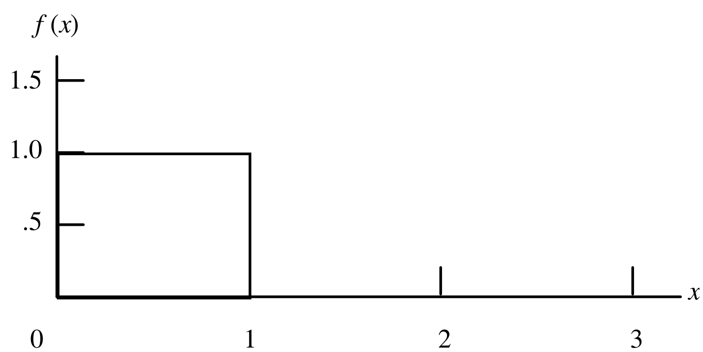


b. $P(x \le 130) = (1/20)(130 - 120) = 0.50$

c. $P(x > 135) = (1/20)(140 - 135) = 0.25$

d. $E(x) = \dfrac{120+140}{2} = 130$ minutes

4. a.



b. $P(.25 < x < .75) = 1(.50) = .50$

c. $P(x \le .30) = 1(.30) = .30$

d. $P(x > .60) = 1(.40) = .40$

5. a. Length of Interval = 310.6 - 284.7 = 25.9

$$f(x) = \begin{cases} \dfrac{1}{25.9} & \text{for } 284.7 \le x \le 310.6 \\ 0 & \text{elsewhere} \end{cases}$$

b. Note: 1/25.9 = .0386

$P(x < 290) = .0386(290 - 284.7) = .2046$

c. $P(x \ge 300) = .0386(310.6 - 300) = .4092$

d. $P(290 \le x \le 305) = .0386(305 - 290) = .5790$

e. $P(x \geq 290) = .0386(310.6 - 290) = .7952$

Rounding up, we conclude that 80 of the top 100 golfers drive the ball this far.

6. a. $P(x \geq 25) = \frac{1}{8}(26 - 25) = .125$

b. $P(21 \leq x \leq 25) = \frac{1}{8}(25 - 21) = .50$

c. This occurs when programming is 20 minutes or less

$P(x \leq 20) = \frac{1}{8}(20 - 18) = .25$

7. a. $P(10{,}000 \leq x < 12{,}000) = 2000\,(1 / 5000) = .40$

The probability your competitor will bid lower than you, and you get the bid, is .40.

b. $P(10{,}000 \leq x < 14{,}000) = 4000\,(1 / 5000) = .80$

c. A bid of $15,000 gives a probability of 1 of getting the property.

d. Yes, the bid that maximizes expected profit is $13,000.

The probability of getting the property with a bid of $13,000 is

$$P(10{,}000 \leq x < 13{,}000) = 3000\,(1 / 5000) = .60.$$

The probability of not getting the property with a bid of $13,000 is .40.

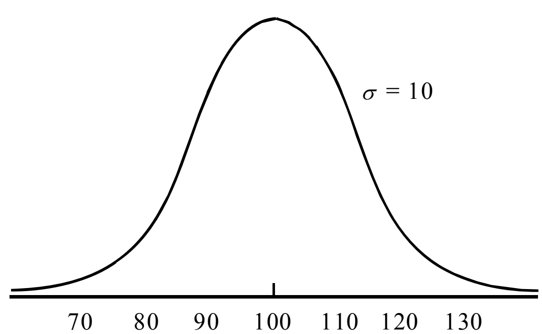
The profit you will make if you get the property with a bid of $13,000 is $3000 = $16,000 - 13,000. So your expected profit with a bid of $13,000 is

EP ($13,000) = .6 ($3000) + .4 (0) = $1800.

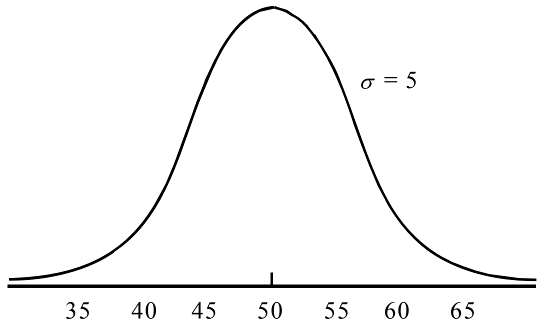
If you bid $15,000 the probability of getting the bid is 1, but the profit if you do get the bid is only $1000 = $16,000 - 15,000. So your expected profit with a bid of $15,000 is

EP ($15,000) = 1 ($1000) + 0 (0) = $1,000.

8.



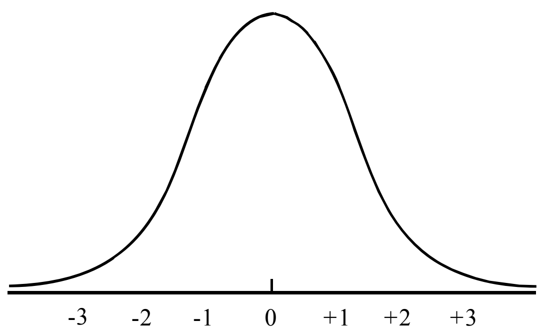
9. a.



b. .683 since 45 and 55 are within plus or minus 1 standard deviation from the mean of 50 (Use the table or see characteristic 7a of the normal distribution).

c. .954 since 40 and 60 are within plus or minus 2 standard deviations from the mean of 50 (Use the table or see characteristic 7b of the normal distribution).

10.



a. $P(z \leq 1.5) = .9332$

b. $P(z \leq 1.0) = .8413$

c. $P(1 \leq z \leq 1.5) = P(z \leq 1.5) - P(z < 1) = .9932 - .8413 = .0919$

d. $P(0 < z < 2.5) = P(z < 2.5) - P(z \leq 0) = .9938 - .5000 = .4938$

11. a. $P(z \leq -1) = .1587$

b. $P(z \geq -1) = 1 - P(z < -1) = 1 - .1587 = .8413$

c. $P(z \geq -1.5) = 1 - P(z < -1.5) = 1 - .0668 = .9332$

d. $P(-2.5 \leq z) = 1 - P(z < -2.5) = 1 - .0062 = .9938$

e. $P(-3 < z \leq 0) = P(z \leq 0) - P(z \leq -3) = .5000 - .0013 = .4987$

12. a. $P(0 \leq z \leq .83) = .7967 - .5000 = .2967$

b. $P(-1.57 \leq z \leq 0) = .5000 - .0582 = .4418$

c. $P(z > .44) = 1 - .6700 = .3300$

d. $P(z \geq -.23) = 1 - .4090 = .5910$

e. $P(z < 1.20) = .8849$

f. $P(z \leq -.71) = .2389$

13. a. $P(-1.98 \leq z \leq .49) = P(z \leq .49) - P(z < -1.98) = .6879 - .0239 = .6640$

b. $P(.52 \leq z \leq 1.22) = P(z \leq 1.22) - P(z < .52) = .8888 - .6985 = .1903$

c. $P(-1.75 \leq z \leq -1.04) = P(z \leq -1.04) - P(z < -1.75) = .1492 - .0401 = .1091$

14. a. The z value corresponding to a cumulative probability of .9750 is $z = 1.96$.

b. The z value here also corresponds to a cumulative probability of .9750: $z = 1.96$.

c. The z value corresponding to a cumulative probability of .7291 is $z = .61$.

d. Area to the left of z is 1 - .1314 = .8686. So $z = 1.12$.

e. The z value corresponding to a cumulative probability of .6700 is $z = .44$.

f. The area to the left of z is .6700. So $z = .44$.

15. a. The z value corresponding to a cumulative probability of .2119 is $z = -.80$.

b. Compute .9030/2 = .4515; z corresponds to a cumulative probability of .5000 + .4515 = .9515. So $z = 1.66$.

c. Compute .2052/2 = .1026; z corresponds to a cumulative probability of .5000 + .1026 = .6026. So $z = .26$.

d. The z value corresponding to a cumulative probability of .9948 is z = 2.56.

e. The area to the left of z is 1 - .6915 = .3085. So $z = -.50$.

16. a. The area to the left of z is 1 - .0100 = .9900. The z value in the table with a cumulative probability closest to .9900 is $z = 2.33$.

b. The area to the left of z is .9750. So $z = 1.96$.

c. The area to the left of z is .9500. Since .9500 is exactly halfway between .9495 ($z = 1.64$) and .9505($z = 1.65$), we select $z = 1.645$. However, $z = 1.64$ or $z = 1.65$ are also acceptable answers.

d. The area to the left of z is .9000. So $z = 1.28$ is the closest z value.

17. Let x = debt amount

$\mu = 15{,}015, \quad \sigma = 3540$

a. $$z = \frac{18{,}000 - 15{,}015}{3540} = .84$$

$P(x > 18{,}000) = 1 - P(z \leq .84) = 1 - .7995 = .2005$

b. $z = \dfrac{10,000-15,015}{3540} = -1.42$

$P(x < 10,000) = P(z < -1.42) = .0778$

c. At 18,000, $z = .84$ from part (a)

At 12,000, $z = \dfrac{12,000-15,015}{3540} = -.85$

$P(12,000 < x < 18,000) = P(-.85 < z < .84) = .7995 - .1977 = .6018$

d. $z = \dfrac{14,000-15,015}{3540} = -.29$

$P(x \leq 14,000) = P(z \leq -.29) = .3859$

18. $\mu = 30$ and $\sigma = 8.2$

a. At $x = 40$, $z = \dfrac{40-30}{8.2} = 1.22$

$P(z \leq 1.22) = .8888$

$P(x \geq 40) = 1 - .8888 = .1112$

b. At $x = 20$, $z = \dfrac{20-30}{8.2} = -1.22$

$P(z \leq -1.22) = .1112$

So, $P(x \leq 20) = .1112$

c. A z-value of 1.28 cuts off an area of approximately 10% in the upper tail.

$x = 30 + 8.2(1.28) = 40.50$

A stock price of \$40.50 or higher will put a company in the top 10%

19. We have $\mu = 3.5$ and $\sigma = .8$.

a. $z = \dfrac{5.0-3.5}{.8} \approx 1.88$

$P(x > 5.0) = P(z > 1.88) = 1 - P(z \leq 1.88) = 1 - .9699 = .0301$

The rainfall exceeds 5 inches in 3.01% of the Aprils.

b. $z = \dfrac{3 - 3.5}{.8} \approx -.63$

$P(x < 3.0) = P(z < -.63) = .2643$

The rainfall is less than 3 inches in 26.43% of the Aprils.

c. $z = 1.28$ cuts off approximately .10 in the upper tail of a normal distribution.

$x = 3.5 + 1.28(.8) = 4.524$

If it rains 4.524 inches or more, April will be classified as extremely wet.

20. $\mu = 77$ and $\sigma = 20$

a. At $x = 50$, $z = \dfrac{50 - 77}{20} = -1.35$

$P(z < -1.35) = .0885$

So, $P(x < 50) = .0885$

b. At $x = 100$, $z = \dfrac{100 - 77}{20} = 1.15$

$P(z > 1.15) = 1 - .8749 = .1251$

So, $P(x > 100) = .1251$

12.51% of workers logged on over 100 hours.

c. A z-value of .84 cuts off an area of approximately .20 in the upper tail.

$x = \mu + z\sigma = 77 + 20(.84) = 93.8$

A worker must spend 93.8 or more hours logged on to be classified a heavy user.

21. From the normal probability tables, a z-value of 2.05 cuts off an area of approximately .02 in the upper tail of the distribution.

$x = \mu + z\sigma = 100 + 2.05(15) = 130.75$

A score of 131 or better should qualify a person for membership in Mensa.

22. Use $\mu = 32.62$ and $\sigma = 2.32$

a. We want to find $P(30 \leq x \leq 35)$

At $x = 35$,

$$z = \frac{35 - 32.62}{2.32} = 1.03$$

At $x = 30$

$$z = \frac{30 - 32.62}{2.32} = -1.13$$

$P(30 \leq x \leq 35) = P(-1.13 \leq z \leq 1.03) = P(z \leq 1.03) - P(z \leq -1.13)$
$= .8485 - .1292$
$= .7193$

The probability a financial manager earns between \$30 and \$35 per hour is .7193.

b. Must find the z-value that cuts off an area of .10 in the upper tail. Using the normal tables, we find $z = 1.28$ cuts off approximately .10 in the upper tail.

So, $x = \mu + z\sigma = 32.62 + 1.28(2.32) = 35.59$

An hourly pay rate \$35.59 or above will put a financial manager in the top 10%.

c. At $x = 28$, $z = \frac{28 - 32.62}{2.32} = -1.99$

$P(x < 28) = P(z < -1.99) = .0233$

The probability a randomly selected financial manager earns less than \$28 per hour is .0233.

23. a. $z = \frac{60 - 80}{10} = -2$ $\quad P(z \leq -2) = .0228$. So $P(x < 60) = .0228$

b. At $x = 60$

$$z = \frac{60 - 80}{10} = -2 \qquad \text{Area to left is .0228}$$

At $x = 75$

$$z = \frac{75 - 80}{10} = -.5 \qquad \text{Area to left is .3085}$$

$P(60 \leq x \leq 75) = .3085 - .0228 = .2857$

c. $z = \frac{90 - 80}{10} = 1$ $\quad P(z \leq 1) = P(x \leq 90) = .1587$

Therefore 15.87% of students will not complete on time.

$$(60)\,(.1587) = 9.522$$

We would expect 9.522 students to be unable to complete the exam in time.

24. a. $\bar{x} = \sum \frac{x_i}{n} = 200$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} = 26.04$$

We will use $\bar{x}$ as an estimate of μ and s as an estimate of σ in parts (b) - (d) below.

b. Remember the data are in thousands of shares.

At $x = 180$

$$z = \frac{180 - 200}{26.04} = -.77$$

$$P(x \leq 180) = P(z \leq -.77) = .2206$$

The probability trading volume will be less than 180 million shares is .2206.

c. At $x = 230$

$$z = \frac{230 - 200}{26.04} = 1.15$$

$P(x > 230) = P(z > 1.15) = 1 - P(z \leq 1.15) = 1 - .8749 = .1251$

The probability trading volume will exceed 230 million shares is .1251.

d. A z-value of 1.645 cuts off an area of .05 in the upper tail

$x = \mu + z\sigma = 200 + 1.645(26.04) = 242.84$

If the early morning trading volume exceeds 242.84 million shares, the day is among the busiest 5%.

25. $\mu = 6.8, \quad \sigma = .6$

a. At $x = 8, \quad z = \frac{8 - 6.8}{.6} = 2.00$

$P(x > 8) = P(z > 2.0) = 1 - .9772 = .0228$

b. At $x = 6, \quad z = \frac{6 - 6.8}{.6} = -1.33$

$P(x \leq 6) = P(z \leq -1.33) = .0918$

c. At $x = 9, \quad z = \frac{9 - 6.8}{.6} = 3.67$

At $x = 7, \quad z = \frac{7 - 6.8}{.6} = .33$

$P(7 < x < 9) = P(.33 < z < 3.67) = 1 - .6293 = .3707$

Only 37.07 percent of the population get the amount of sleep recommended by doctors. Most get less.

26. a. $\mu = np = 100(.20) = 20$

$\sigma^2 = np(1 - p) = 100(.20)(.80) = 16$

$\sigma = \sqrt{16} = 4$

b. Yes because $np = 20$ and $n(1 - p) = 80$

c. $P(23.5 \leq x \leq 24.5)$

$$z = \frac{24.5 - 20}{4} = +1.13 \qquad P(z \leq 1.13) = .8708$$

$$z = \frac{23.5 - 20}{4} = +.88 \qquad P(z \leq .88) = .8106$$

$P(23.5 \leq x \leq 24.5) = .8708 - .8106 = .0602$

d. $P(17.5 \leq x \leq 22.5)$

$$z = \frac{22.5 - 20}{4} = +.63 \qquad P(z \leq .63) = .7357$$

$$z = \frac{17.5 - 20}{4} = -.63 \qquad P(z \leq -.63) = .2643$$

$P(17.5 \leq x \leq 22.5) = .7357 - .2643 = .4714$

e. $P(x \leq 15.5)$

$$z = \frac{15.5 - 20}{4} = -1.13$$

$P(x \leq 15.5) = P(z \leq -1.13) = .1292$

27. a. $\mu = np = 200(.60) = 120$

$\sigma^2 = np(1 - p) = 200(.60)(.40) = 48$

$$\sigma = \sqrt{48} = 6.93$$

b. Yes since $np = 120$ and $n(1 - p) = 80$

c. $P(99.5 \leq x \leq 110.5)$

$$z = \frac{110.5 - 120}{6.93} = -1.37 \qquad P(z \leq -1.37) = .0853$$

$$z = \frac{99.5 - 120}{6.93} = -2.96 \qquad P(z \leq -2.96) = .0015$$

$P(99.5 \leq x \leq 110.5) = .0853 - .0015 = .0838$

d. $P(x \geq 129.5)$

$$z = \frac{129.5 - 120}{6.93} = +1.37 \qquad P(z \geq 1.37) = 1 - .9147 = .0853$$

$P(x \geq 129.5) = .0853$

e. Simplifies computation. By direct computation of binomial probabilities we would have to compute

$$P(x \geq 130) = f(130) + f(131) + f(132) + f(133) + \ldots$$

28. a. In answering this part, we assume it is not known how many Democrats and Republicans are in the group.

$\mu = np = 250(.47) = 117.5$

$\sigma^2 = np(1\text{-}p) = 250(.47)(.53) = 62.275$

$\sigma = \sqrt{62.275} = 7.89$

Half the group is 125 people. We want to find $P(x \geq 124.5)$.

At $x = 124.5$, $z = \dfrac{124.5 - 117.5}{7.89} = .89$

$P(z \geq .89) = 1 - .8133 = .1867$

So, $P(x \geq 124.5) = .1867$

We estimate a probability of .1867 that at least half the group is in favor of the proposal.

b. For Republicans: $np = 150(.64) = 96$

For Democrats: $np = 100(.29) = 29$

Expected number in favor = 96 + 29 = 125

c. It's a toss up. From part (b) we see that we can expect just as many in favor of the proposal as opposed.

29. a/b. $\mu = np = (100)(.058) = 5.8$

$\sigma^2 = np(1\text{-}p) = 100(.058)(.942) = 5.4636$

$\sigma = \sqrt{5.4636} = 2.34$

c. Must compute $P(5.5 \leq x \leq 6.5)$

At $x = 6.5$, $z = \dfrac{6.5 - 5.8}{2.34} = .30$

$P(z \leq .30) = .6179$

At $x = 5.5$, $z = \dfrac{5.5 - 5.8}{2.34} = -.13$

$P(z \leq -.13) = .4483$

$P(5.5 \leq x \leq 6.5) = .6179 - .4483 = .1696$

Our estimate of the probability is .1696.

d. Must compute $P(x \geq 3.5)$

At $x = 3.5$, $z = \dfrac{3.5-5.8}{2.34} = -.98$

$P(z \geq -.98) = 1 - .1635 = .8365$

So, $P(x \geq 3.5) = .8365$

Our estimate of the probability that at least 4 are unemployed is .8365.

30. a. $\mu = np = 500(.44) = 220$

b. $\sigma = \sqrt{np(1-p)} = \sqrt{(500)(.44)(.56)} = 11.10$

Find $P(x \leq 200.5)$

At $x = 200.5$,

$z = \dfrac{200.5-220}{11.10} = -1.76$ $\quad P(z \leq -1.76) = .0392$

$P(x \leq 200.5) = .0392$

The probability that 200 or fewer individuals will say they read every word is .0392.

c. $\mu = np = 500(.04) = 20$

$\sigma = \sqrt{np(1-p)} = \sqrt{(500)(.04)(.96)} = 4.38$

Find $P(x \geq 14.5)$

At $x = 14.5$,

$z = \dfrac{14.5-20}{4.38} = -1.26$ $\quad P(z \geq -1.26) = 1 - .1038 = .8962$

$P(x \geq 14.5) = .8962$

The probability that at least 15 individuals say they do not read credit card contracts is .8962.

31. a. $\mu = np = 120(.75) = 90$

$\sigma = \sqrt{np(1-p)} = \sqrt{(120)(.75)(.25)} = 4.74$

The probability at least half the rooms are occupied is the normal probability: $P(x \geq 59.5)$.
At $x = 59.5$

$z = \dfrac{59.5-90}{4.74} = -6.43$

Therefore, probability is approximately 1

b. Find the normal probability: $P(x \geq 99.5)$

At $x = 99.5$,

$$z = \frac{99.5 - 90}{4.74} = 2.00 \qquad P(z \geq 2.00) = 1 - .9772 = .0228$$

$P(x \geq 99.5) = P(z \geq 2.00) = .0228$

c. Find the normal probability: $P(x \leq 80.5)$

At $x = 80.5$,

$$z = \frac{80.5 - 90}{4.74} = -2.00 \qquad P(z \leq -2.00) = .0228$$

$P(x \leq 80.5) = P(z \leq -2.00) = .0228$

32. a. $P(x \leq 6) = 1 - e^{-6/8} = 1 - .4724 = .5276$

b. $P(x \leq 4) = 1 - e^{-4/8} = 1 - .6065 = .3935$

c. $P(x \geq 6) = 1 - P(x \leq 6) = 1 - .5276 = .4724$

d. $P(4 \leq x \leq 6) = P(x \leq 6) - P(x \leq 4) = .5276 - .3935 = .1341$

33. a. $P(x \leq x_0) = 1 - e^{-x_0/3}$

b. $P(x \leq 2) = 1 - e^{-2/3} = 1 - .5134 = .4866$

c. $P(x \geq 3) = 1 - P(x \leq 3) = 1 - (1 - e^{-3/3}) = e^{-1} = .3679$

d. $P(x \leq 5) = 1 - e^{-5/3} = 1 - .1889 = .8111$

e. $P(2 \leq x \leq 5) = P(x \leq 5) - P(x \leq 2) = .8111 - .4866 = .3245$

34. a. $P(x < 10) = 1 - e^{-10/12.1} = .5624$

b. $P(x > 20) = 1 - P(x \leq 20) = 1 - (1 - e^{-20/12.1}) = e^{-20/12.1} = .1915$

c. $P(10 \leq x \leq 20) = P(x \leq 20) - P(x \leq 10)$

$$= (1 - e^{-20/12.1}) - (1 - e^{-10/12.1}) = e^{-10/12.1} - e^{-20/12.1} =$$

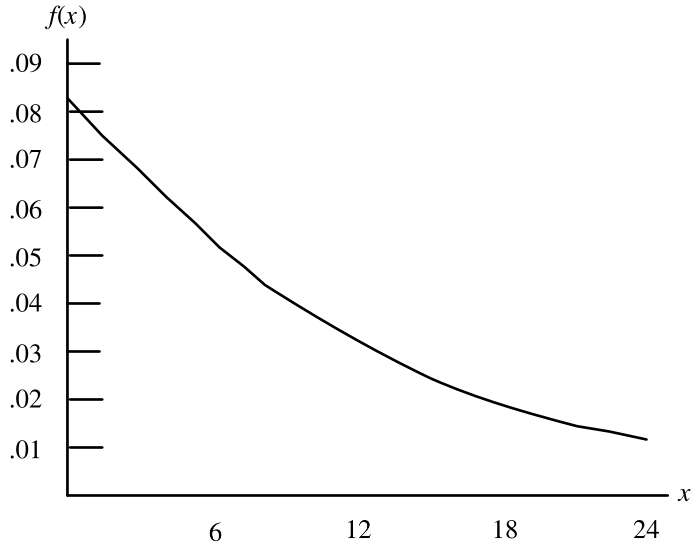
$$= .4376 - .1915 = .2461$$

d. To make your flight you must get through security within 18 minutes.

$P(x < 18) = 1 - e^{-18/12.1} = .7741$

Thus, the probability you will not make your flight is 1 - .7741 = .2259.

35. a.



b. $P(x \le 12) = 1 - e^{-12/12} = 1 - .3679 = .6321$

c. $P(x \le 6) = 1 - e^{-6/12} = 1 - .6065 = .3935$

d. $P(x \ge 30) = 1 - P(x < 30)$

$= 1 - (1 - e^{-30/12})$

$= .0821$

36. a. 50 hours

b. $P(x \le 25) = 1 - e^{-25/50} = 1 - .6065 = .3935$

c. $P(x \ge 100) = 1 - (1 - e^{-100/50})$

$= .1353$

37. a. $P(x < 2) = 1 - e^{-2/2.78} = .5130$

b. $P(x > 5) = 1 - P(x \le 5) = 1 - (1 - e^{-5/2.78}) = e^{-5/2.78} = .1655$

c. $P(x > 2.78) = 1 - P(x \le 2.78) = 1 - (1 - e^{-2.78/2.78}) = e^{-1} = .3679$

This may seem surprising since the mean is 2.78 minutes. But, for the exponential distribution, the probability of a value greater than the mean is significantly less than the probability of a value less than the mean. This is because the distribution is skewed to the right.

38. a. If the mean number of interruptions per hour follows the Poisson distribution, the time between interruptions follows the exponential distribution. So,

$\mu = \dfrac{1}{5.5}$ of an hour and $\dfrac{1}{\mu} = \dfrac{1}{1/5.5} = 5.5$

Thus, $f(x) = 5.5e^{-5.5x}$.

Here x is the time between interruptions in hours.

b. Fifteen minutes is 1/4 of an hour so,

$$P\left(x > \frac{1}{4}\right) = 1 - P\left(x \leq \frac{1}{4}\right) = 1 - (1 - e^{-5.5/4}) = e^{-5.5/4} = .2528$$

The probability of no interruptions during a15-minute period is .2528.

c. Since 10 minutes is 1/6 of an hour, we compute,

$$P\left(x \leq \frac{1}{6}\right) = 1 - e^{-5.5/6} = 1 - .3998 = .6002$$

Thus, the probability of being interrupted within 10 minutes is .6002.

39. a. Let x = sales price ($1000s)

$$f(x) = \begin{cases} \dfrac{1}{25} & \text{for } 200 \leq x \leq 225 \\ 0 & \text{elsewhere} \end{cases}$$

b. $P(x \geq 215) = (1 / 25)(225 - 215) = .40$

c. $P(x < 210) = (1 / 25)(210 - 200) = .40$

d. $E(x) = (200 + 225)/2 = 212{,}500$

If the executive leaves the house on the market for another month, the expected sales price will be $2,500 higher than if the house is sold back to the company for $210,000. However, if the house is left on the market for another month, there is a .40 probability that the executive will get less than the company offer of $210,000. It is a close call. But the expected value of $212,500 suggests the executive should leave the house on the market another month.

40. a. Find the z value that cuts off an area of .10 in the lower tail.

From the standard normal table $z \approx -1.28$. Solve for x,

$$-1.28 = \frac{x - 5700}{1500}$$

$$x = 5700 - 1.28(1500) = 3780$$

10% of families spend $3780 or less.

b. $$z = \frac{7000 - 5700}{1500} = .87$$

$P(x > 7000) = 1 - P(z \leq .87) = 1 - .8078 = .1922$

19.22% of families spend more than $7000 annually for food and drink.

c. Find the z value that cuts off an area of .05 in the upper tail: z = 1.645. Solve for x,

$$1.645 = \frac{x - 5700}{1500}$$

$$x = 5700 + 1.645(1500) = 8167.5$$

5% of families spend more than $8,167.50 annually on food and drink.

41. a. $P(defect) = 1 - P(9.85 \leq x \leq 10.15)$

$$= 1 - P(-1 \leq z \leq 1) = 1 - .6826 = .3174$$

Expected number of defects = 1000(.3174) = 317.4

b. $P(defect) = 1 - P(9.85 \leq x \leq 10.15)$

$$= 1 - P(-3 \leq z \leq 3) = 1 - .9974 = .0026$$

Expected number of defects = 1000(.0026) = 2.6

c. Reducing the process standard deviation causes a substantial reduction in the number of defects.

42. $\mu = 6{,}312$

a. $z = -1.645$ cuts off .05 in the lower tail

So,

$$-1.645 = \frac{1000 - 6312}{\sigma}$$

$$\sigma = \frac{1000 - 6312}{-1.645} = 3229$$

b. At 6000, $z = \frac{6000 - 6312}{3229} = -.10$

At 4000, $z = \frac{4000 - 6312}{3229} = -.72$

$P(4000 < x < 6000) = P(-.72 < z < -.10) = .4602 - .2358 = .2244$

c. z = 1.88 cuts off approximately .03 in the upper tail

$x = 6312 + 1.88(3229) = 12{,}382.52$

The households with the highest 3% of expenditures spent more than $12,382.

43. a. $\mu = 670 \quad \sigma = 30$

All rooms will be rented if demand is at least 700.

At $x = 700$

$$z = \frac{x-\mu}{\sigma} = \frac{700-670}{30} = 1$$

$P(x \geq 700) = P(z \geq 1) = 1 - P(z < 1) = 1 - .8413 = .1587$

b. 50 or more rooms will be unrented is demand falls to 650 or less.

At $x = 650$

$$z = \frac{650-670}{30} = -.67$$

$P(x \leq 650) = P(z \leq -.67) = .2514$

c. A promotion might be a good idea if it isn't too expensive. Things to consider:

- The probability of renting all the rooms without a promotion is approximately .16.
- The probability is about .25 that 50 or more rooms will go unrented. This is significant lost revenue.
- To be successful, a promotion should increase the expected value of demand above 670.

44. a. At $x = 200$

$$z = \frac{200-150}{25} = 2$$

$P(x > 200) = P(z > 2) = 1 - P(z \leq 2) = 1 - .9772 = .0228$

b. Expected Profit = Expected Revenue - Expected Cost

= 200 - 150 = \$50

45. $\mu = 1550$ $\sigma = 300$

a. At $x = 1000$,

$$z = \frac{x-\mu}{\sigma} = \frac{1000-1550}{300} = -1.83$$

$P(x < 1000) = P(z < -1.83) = .0336$

b. At $x = 2000$,

$$z = \frac{2000-1550}{300} = 1.50$$

$P(x < 2000) = P(z < 1.50) = .9332$

$P(x < 1000) = .0336$ (from part (a))

$P(1000 < x < 2000) = .9332 - .0336 = .8996$

c. Find the z value that cuts off an area of .05 in the upper tail: $z = 1.645$

Solve for x,

$$1.645 = \frac{x - 1550}{300}$$

$x = 1550 + 1.645(300) = 2043.5$

Rounding up, we would say that 2044 or more crashes would put a year in the top 5% for fatal crashes. It would be a bad year.

46. a. At 400,

$$z = \frac{400 - 450}{100} = -.500$$

Area to left is .3085

At 500,

$$z = \frac{500 - 450}{100} = +.500$$

Area to left is .6915

$P(400 \leq x \leq 500) = .6915 - .3085 = .3830$

38.3% will score between 400 and 500.

b. At 630,

$$z = \frac{630 - 450}{100} = 1.80$$

96.41% do worse and 3.59% do better .

c. At 480,

$$z = \frac{480 - 450}{100} = .30$$

Area to left is .6179

38.21% are acceptable.

47. a. At 75,000

$$z = \frac{75{,}000 - 67{,}000}{7{,}000} \approx 1.14$$

$P(x > 75{,}000) = P(z > 1.14) = 1 - P(z \leq 1.14) = 1 - .8729 = .1271$

The probability of a woman receiving a salary in excess of $75,000 is .1271

b. At 75,000

$$z = \frac{75{,}000 - 65{,}500}{7{,}000} \approx 1.36$$

$P(x > 75{,}000) = P(z > 1.36) = 1 - P(z \leq 1.36) = 1 - .9131 = .0869$

The probability of a man receiving a salary in excess of $75,000 is .0869

c. At $x = 50{,}000$

$$z = \frac{50{,}000 - 67{,}000}{7{,}000} \approx -2.43$$

$$P(x < 50{,}000) = P(z < -2.43) = .0075$$

The probability of a woman receiving a salary below $50,000 is very small: .0075

d. The answer to this is the male copywriter salary that cuts off an area of .01 in the upper tail of the distribution for male copywriters.

Use $z = 2.33$

$x = 65{,}500 + 2.33(7{,}000) = 81{,}810$

A woman who makes $81,810 or more will earn more than 99% of her male counterparts.

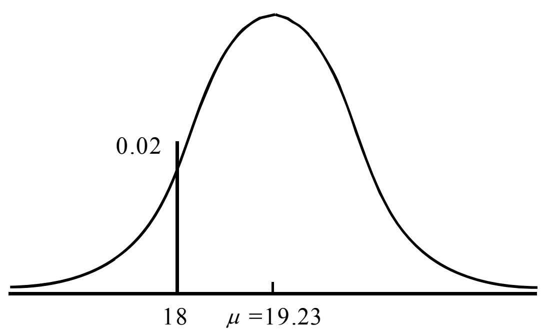
48. $\sigma = .6$

At 2%

$$z \approx -2.05 \quad x = 18$$

$$z = \frac{x - \mu}{\sigma} \qquad \therefore -2.05 = \frac{18 - \mu}{.6}$$

$$\mu = 18 + 2.05\ (.6) = 19.23 \text{ oz.}$$



The mean filling weight must be 19.23 oz.

49. Use normal approximation to binomial.

a. $\mu = np = 50\ (.75) = 37.5$

$$\sigma = \sqrt{np(1-p)} = \sqrt{50(.75)(.25)} = 3.06$$

At $x = 42.5$

$$z = \frac{x - \mu}{\sigma} = \frac{42.5 - 37.5}{3.06} = 1.63$$

$$P(z \leq 1.63) = .9484$$

Probability of an A grade = 1 - .9484 = .0516 or 5.16% will obtain an A grade.

b. At $x = 34.5$

$$z = \frac{34.5 - 37.5}{3.06} = -.98$$

At $x = 39.5$

$$z = \frac{39.5 - 37.5}{3.06} = .65$$

$P(-.98 \leq z \leq .65) = .7422 - .1635 = .5787$

or 57.87% will obtain a C grade.

c. At $x = 29.5$

$$z = \frac{29.5 - 37.5}{3.06} = -2.61$$

$P(z \geq -2.61) = 1 - .0045 = .9955$

or 99.55%of the students who have done their homework and attended lectures will pass the examination.

d. $\mu = np = 50\,(.25) = 12.5$ (We use $p = .25$ for a guess.)

$$\sigma = \sqrt{np(1-p)} = \sqrt{50(.25)(.75)} = 3.06$$

At $x = 29.5$

$$z = \frac{29.5 - 12.5}{3.06} = 5.55$$

$P(z \geq 5.55) \approx 0$

Thus, essentially no one who simply guesses will pass the examination.

50. a. $\mu = np = (240)(0.49) = 117.6$

Expected number of wins is 117.6

Expected number of losses = 240(0.51) = 122.4

Expected payoff = 117.6(50) - 122.4(50) = (-4.8)(50) = -240.

The player should expect to lose $240.

b. To lose $1000, the player must lose 20 more hands than he wins. With 240 hands in 4 hours, the player must win 110 or less in order to lose $1000. Use normal approximation to binomial.

$\mu = np = (240)(0.49) = 117.6$

$$\sigma = \sqrt{240(.49)(.51)} = 7.7444$$

Find $P(x \leq 110.5)$

At x = 110.5

$$z = \frac{110.5 - 117.6}{7.7444} = -.92$$

$P(x \leq 110.5) = 0.1788$

The probability he will lose $1000 or more is 0.1788.

c. In order to win, the player must win 121 or more hands.

Find $P(x \geq 120.5)$

At x = 120.5

$$z = \frac{120.5 - 117.6}{7.7444} = .37$$

$P(x \geq 120.5) = 1 - .6443 = 0.3557$

The probability that the player will win is 0.3557. The odds are clearly in the house's favor.

d. To lose $1500, the player must lose 30 hands more than he wins. This means he wins 105 or fewer hands.

Find $P(x \leq 105.5)$

At x = 105.5

$$z = \frac{105.5 - 117.6}{7.7444} = -1.56$$

$P(x \leq 105.5) = 0.0594$

The probability the player will go broke is 0.0594.

51. a. $P(x \leq 15) = 1 - e^{-15/36} = 1 - .6592 = .3408$

b. $P(x \leq 45) = 1 - e^{-45/36} = 1 - .2865 = .7135$

Therefore $P(15 \leq x \leq 45) = .7135 - .3408 = .3727$

c. $P(x \geq 60) = 1 - P(x < 60)$

$= 1 - (1 - e^{-60/36}) = .1889$

52. a. Mean time between arrivals = 1/7 minutes

b. $f(x) = 7e^{-7x}$

c. $P(x > 1) = 1 - P(x < 1) = 1 - [1 - e^{-7(1)}] = e^{-7} = .0009$

d. 12 seconds is .2 minutes

$P(x > .2) = 1 - P(x < .2) = 1- [1- e^{-7(.2)}] = e^{-1.4} = .2466$

53. a. $\frac{1}{36.5} e^{-x/36.5} \approx .0274 e^{-.0274x}$

b. $P(x < 40) = 1 - e^{-.0274(40)} = 1 - .3342 = .6658$

$P(x < 20) = 1 - e^{-.0274(20)} = 1 - .5781 = .4219$

$P(20 < x < 40) = .6658 - .4219 = .2439$

c. From part (b), $P(x < 40) = .6658$

$P(x > 40) = P(x \geq 40) = 1 - P(x < 40) = 1 - .6658 = .3342$

54. a. $\frac{1}{\mu} = 0.5$ therefore μ = 2 minutes = mean time between telephone calls

b. Note: 30 seconds = .5 minutes

$P(x \leq .5) = 1 - e^{-.5/2} = 1 - .7788 = .2212$

c. $P(x \leq 1) = 1 - e^{-1/2} = 1 - .6065 = .3935$

d. $P(x \geq 5) = 1 - P(x < 5) = 1 - (1 - e^{-5/2}) = .0821$