



## Physics Lab 211

### Experiment No. 4

## Moment of Inertia of a Flywheel

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## Abstract:

**-Aims:** To calculate the moment of inertia of a flywheel, and to apply the concepts of energy conservation to cases where both rotational and translational motions are effective.

**-Methods:** By counting the number of rotations a flywheel makes when an attached mass free falls before hitting the ground, and after removing the mass.

**-Main Result:**

$$I = (0.038 \pm 0.005) \text{ kg.m}^2$$

## Theoretical Background:

The moment of inertia ( $I$ ) is a quantity expressing a body's tendency to resist angular acceleration, which is the sum of the products of the mass of each particle in the body with the square of its distance from the axis of rotation.

$$I = \int r^2 dm$$

Where  $r$  is the distance of mass element  $dm$  from the axis of rotation.

$$r = \frac{d}{2} \text{ (the radius of the axle).}$$

$R$  is the radius of the wheel itself.

$v$  is the linear velocity of the falling object and  $\omega$  is the angular velocity of the flywheel when the weight reaches the floor.  $n$  is the number of rotations of the wheel before the object reaches ground, and  $N$  is the number of subsequent rotations until the wheel stops.  $W$  is the amount of work done against friction per one rotation, then we have the following equation:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + nW \dots\dots\dots \text{(eq.1)}$$

By using the fact that the flywheel's energy is dissipated against the friction incurred in  $N$  rotations, then:

$$NW = \frac{1}{2}I\omega^2 \dots\dots\dots \text{(eq.2)}$$

$$W = \frac{I\omega^2}{2N}, \text{ where } \omega = \frac{v}{r} \dots\dots\dots \text{(eq.3)}$$

By substituting  $W$  and  $\omega$  in (eq.1):

$$mgh = \frac{v^2}{2}m + \frac{Iv^2}{2r^2} \left(1 + \frac{n}{N}\right) \dots\dots\dots \text{(eq.4)}$$

Also we have:

$$v = 2 \frac{h}{t} = 2 \times (\text{average velocity of fall}) \dots\dots\dots (\text{eq.5})$$

Hence by substituting the above equation in (eq.1):

$$mgh = 2 \frac{h^2}{r^2} m + \frac{2Ih}{r^2} \left(1 + \frac{n}{N}\right) \left(\frac{2h^2}{t^2}\right)$$

Now, we can solve the above equation to  $I$ :

$$I = mr^2 \left(\frac{N}{n+N}\right) \left(\frac{gt^2}{2h} - 1\right) = \frac{1}{4} md^2 \left(\frac{N}{n+N}\right) \left(\frac{gt^2}{2h} - 1\right) \dots\dots\dots (\text{eq.6})$$

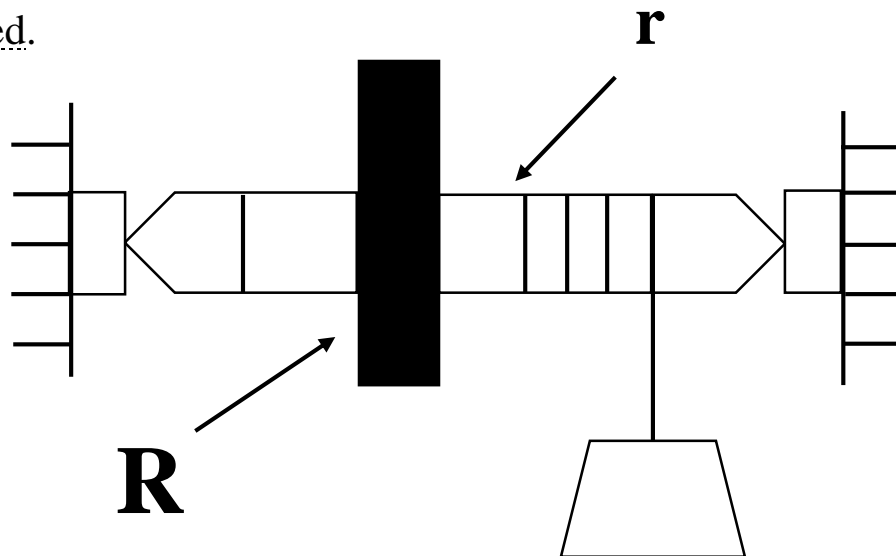
Where  $m$  here is the attached mass.

The mass of the wheel used in this lab is  $(7.12 \pm 0.01)$  kg

**Apparatus:** Flywheel mounted on a horizontal axis, weights, meter scale, caliper, and a stop watch.

## Procedure:

Several turns of the string were winded around the axle  
raising the weight up a few centimeters from the ground. Then, the  
weight was allowed to fall from rest, while the time of fall, number  
of turns the wheel made until it hit the ground, and the number of  
turns the wheel made after it hit the ground until stopped, were all  
measured.



### Flywheel and axle

## Data Sheet:

$$R = (10.0 \pm 0.1) \text{ cm}$$

$$d = (2.55 \pm 0.05) \text{ cm}$$

$$r = (1.27 \pm 0.05) \text{ cm}$$

$$h = (80.0 \pm 0.5) \text{ cm}$$

$$g = 9.78 \text{ m/s}^2$$

$$m = (0.98 \pm 0.01) \text{ kg}$$

$$M = (7.12 \pm 0.01) \text{ kg}$$

| <i>Attempt</i> | <i>t(s)</i> | <i>n(turns)</i> | <i>N(turns)</i> | <i>I(kg.m<sup>2</sup>)</i> |
|----------------|-------------|-----------------|-----------------|----------------------------|
| 1              | 7.5         | 9               | 19              | 0.037                      |
| 2              | 7.5         | 9               | 21              | 0.038                      |
| 3              | 7.5         | 9               | 21              | 0.038                      |
| 4              | 7.5         | 9               | 21              | 0.038                      |

## Calculations:

$$\bar{I} = \frac{1}{4} m d^2 \left( \frac{N(\text{avg})}{n(\text{avg}) + N(\text{avg})} \right) \left( \frac{g t^2(\text{avg})}{2h} - 1 \right)$$

$$N_{(\text{avg})} = 20.5 \text{ turns}$$

$$n_{(\text{avg})} = 9 \text{ turns}$$

$$t_{(\text{avg})} = 7.5 \text{ seconds}$$

By substituting the above equation we have:

$$\bar{I} = 0.038 \text{ kg.m}^2$$

In order to find the error:

$$\frac{\Delta \bar{I}}{\bar{I}} = \frac{\Delta m}{m} + \frac{2\Delta d}{d} + \frac{2\Delta t(\text{avg})}{t(\text{avg})} + \frac{\Delta N(\text{avg})}{N(\text{avg})} + \frac{(\Delta n + \Delta N)(\text{avg})}{(n + N)(\text{avg})} + \frac{\Delta h}{h} + \frac{2\Delta t(\text{avg})}{t(\text{avg})}$$

Where:

$$\Delta m = 0.01 \text{ kg}, \quad \Delta N = 0.5 \text{ turn}, \quad \Delta n = 0 \text{ turn}, \quad \Delta h = 0.005 \text{ m}, \\ \Delta t = 0 \text{ sec}$$

By Substitution we find  $\Delta \bar{I} = 0.005 \text{ kg.m}^2$

## Results & Conclusions:

$$\bar{I} = (0.038 \pm 0.005) \text{ kg} \cdot \text{m}^2$$

The accepted value of  $I$ :

$$I_{(theoretical)} = \frac{1}{2} MR^2$$

By substituting:  $M = 7.12 \text{ kg}$  ,  $R = 0.1 \text{ m}$

We obtain:

$$I_{(theoretical)} \approx 0.036 \text{ kg} \cdot \text{m}^2$$

$$\text{Discrepancy} = |I_{(theoretical)} - \bar{I}_{(experimental)}|$$

$$\text{Discrepancy} = |0.036 - 0.038| = 0.002 \text{ kg} \cdot \text{m}^2$$

$2 \times \text{Error} \geq ? \text{ Discrepancy}$

$$0.01 \geq 0.002$$

It is clear now that our experimental value is accepted within the errors estimated.

Our experiment might have had some random errors. These errors are due to many reasons, such as the delay in reading the time, also the distance which the mass was released from was not the same in all the attempts since it was manually established. As well as, there exist a frictional force from the wheel that was not taken in consideration.