جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Lab 211

Experiment No. 6

Torsional Torques and the Torsional Modulus

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Abstract:

-Aims: To determine the torsional constant (κ), its dependence of rod geometry, and the shear modulus (G).

-Methods: By measuring the force done on dumbbell at different angles. And by measuring the period of oscillations of the dumbbell, first at a constant rod's length, second at a constant rod's diameter.

-Main Result:

$G = (21.28\ \pm 7.51)10^9\ Pa$

Apparatus:

Metallic rods, scale, and a massive dumbbell System .

Theoretical Background:

The period for small oscillations is given by:

$\boldsymbol{T = 2\pi\sqrt{\frac{I}{k}}}$, where T: the period, I: the moment of inertia of the system, k: is the torsional constant.

For elastic twisting of the rod, the torque τ is related to the twist angle θ by:

$\boldsymbol{\tau = -\kappa\theta}$, where κ is related to the dimension of the rod by the following equation:

$\boldsymbol{\kappa = G\frac{\pi d^n L^m}{32}}$, where $\boldsymbol{G}$ is the shear modulus, $\boldsymbol{d}$ is the rod's diameter and $\boldsymbol{L}$ is the rod's length.

Procedure:

In the first part, using one rod, the system was twisted through different six angles and the force acted on it was measured. In the second part, the length alike rods were set to oscillate at a small angle and the period of the oscillations were measured. In the third part, the same procedure as the second part was done, but this time with rods having the same diameter.

Data Sheet:

Part I:

θ (rad)	F (N)	τ (N.m)
0.174	0.11	0.017
0.349	0.21	0.032
0.524	0.27	0.041
0.698	0.35	0.053
0.873	0.46	0.069

Part II:

Rod	d (mm)	T (sec)	$k\ (kg.\frac{m^2}{s^2})$	$log\ d$	$log\ k$
1	1.960	2.231	0.071	-2.708	-1.146
2	2.960	1.015	0.345	-2.529	-0.462
3	3.960	0.552	1.165	-2.402	0.066

Part III:

Rod	l (cm)	T (sec)	$k\ (kg.\frac{m^2}{s^2})$	$log\ d$	$log\ k$
1	48.0	2.231	0.071	-0.319	-1.146
2	38.0	2.043	0.085	-0.420	-1.070
3	27.2	1.774	0.113	-0.565	-0.947

Calculations:

Part I:

$\boldsymbol{\tau = \kappa\theta}$

We can obtain the value of $'\boldsymbol{\kappa}'$ from the slope of (**Graph I**). And then, we can substitute in the period equation, to calculate the value of $'\boldsymbol{I}'$ (T is measured in part II):

$$\boldsymbol{T = 2\pi\sqrt{\frac{I}{k}} \quad \rightarrow I = \frac{T^2 k}{4\pi^2} \quad \rightarrow I = \frac{(2.231)^2 \times 0.072}{4\pi^2}}$$

$\boxed{\boldsymbol{\rightarrow I = 0.009\ kg.m^2}}$, this value is constant.

Part II:

The length is the same in these three rods and is $l = 48\ cm$, we can find the torsional constant for every one of them using the following equation:

$$\boldsymbol{T = 2\pi\sqrt{\frac{I}{k}} \rightarrow k = \frac{4\pi^2 I}{T^2}}$$

Rod 1: $\boldsymbol{k = \frac{4\pi^2(0.009)}{(2.231)^2} \rightarrow k = 0.071\ kg.m^2/s^2}$

Rod 2: $\boldsymbol{k = \frac{4\pi^2(0.009)}{(1.015)^2} \rightarrow k = 0.345\ kg.m^2/s^2}$

Rod 3: $\boldsymbol{k = \frac{4\pi^2(0.009)}{(0.552)^2} \rightarrow k = 1.165\ kg.m^2/s^2}$

By rearranging the equation: $\boldsymbol{\kappa = G\frac{\pi d^n L^m}{32}}$, we obtain:

$log\, k = n\, log\, d + log\frac{G\pi L^m}{32}$, $\boldsymbol{n}$ is the slope, and $\boldsymbol{log\frac{G\pi L^m}{32}}$ is the y-intercept.

From (**Graph II**):

$\boxed{\boldsymbol{\rightarrow n = 3.960\ \pm 0.100}}$

Part III:

The diameter is the same in these three rods and is $d = 1.95\ mm$, we can find the torsional constant for every one of them using the following equation:

$$\boldsymbol{T = 2\pi\sqrt{\frac{I}{k}}} \rightarrow \boldsymbol{k = \frac{4\pi^2 I}{T^2}}$$

Rod 1: $\boldsymbol{k = \frac{4\pi^2(0.009)}{(2.231)^2}} \rightarrow \boldsymbol{k = 0.071\ kg.m^2/s^2}$

Rod 2: $\boldsymbol{k = \frac{4\pi^2(0.009)}{(2.043)^2}} \rightarrow \boldsymbol{k = 0.0.85\ kg.m^2/s^2}$

Rod 3: $\boldsymbol{k = \frac{4\pi^2(0.009)}{(1.774)^2}} \rightarrow \boldsymbol{k = 0.113\ kg.m^2/s^2}$

By rearranging again the equation: $\boldsymbol{\kappa = G\frac{\pi d^n L^m}{32}}$, we obtain:

$log\, k = m\, log\, l + log\frac{G\pi d^n}{32}$, $\boldsymbol{m}$ is the slope, and $\boldsymbol{log\frac{G\pi d^n}{32}}$ is the y-intercept.

From (**Graph III**):

$$\boxed{\rightarrow \boldsymbol{m = -0.809\ \pm 0.025}}$$

Now in order to find $'\boldsymbol{G}'$, we must substitute $\boldsymbol{n}$ & $\boldsymbol{m}$ in the y-intercept equation of the two graphs:

From (**Graph II**):

$$y\_intercept = log\frac{G_1 \pi L^m}{32} \rightarrow G_1 = \frac{10^{y\_intercept} \times 32}{\pi L^m} \rightarrow \frac{10^{9.570} \times 32}{\pi(0.48)^{-0.809}}$$

$$\rightarrow G_1 = 21.00\ \times 10^9\ pa$$

$$\frac{\Delta G_1}{G_1} = m\frac{\Delta L}{L} + \ln 10\ (\Delta y\_intercept) \rightarrow \frac{\Delta G_2}{G_2} = -0.809 \times \frac{0.001}{0.48} + \ln 10\ (0.255)$$

$$\frac{\Delta G_1}{G_1} = 0.585$$

$$\rightarrow \Delta G_1 = 1.23\ \times 10^9\ pa$$

$$\boxed{\boldsymbol{G_1 = (21.00\ \pm 12.28) \times 10^9\ Pa}}$$

From (**Graph III**):

$$y\_intercept = log\frac{G_2\pi d^n}{32} \rightarrow G_2 = \frac{10^{y\_intercept} \times 32}{\pi d^n} \rightarrow \frac{10^{-1.406} \times 32}{\pi(0.00195)^{3.960}}$$

$$\rightarrow G_2 = 21.55 \times 10^9\ pa$$

$$\frac{\Delta G_2}{G_2} = n\frac{\Delta d}{d} + \ln 10\ (\Delta y\_intercept) \rightarrow \frac{\Delta G_2}{G_2} = 3.960 \times \frac{0.00005}{0.00195} + \ln 10\ (0.011)$$

$$\frac{\Delta G_2}{G_2} = 0.127$$

$$\rightarrow \Delta G_2 = 2.74 \times 10^9\ pa$$

$$\boxed{\boldsymbol{G_2 = (21.55 \pm 2.74) \times 10^9\ Pa}}$$

$$\bar{G} = \frac{G_1 + G_2}{2} = 21.28 \times 10^9\ pa$$

$$\Delta\bar{G} = \frac{\Delta G_1 + \Delta G_2}{2} = 7.51 \times 10^9\ pa$$

$$\boxed{\boldsymbol{\bar{G} = (21.28 \pm 7.51) \times 10^9\ Pa}}$$

Results & Conclusions:

$$I = 0.009\ kg.m^2$$

$$n = 3.960 \pm 0.100$$

$$m = -0.809 \pm 0.025$$

$$G = (21.28 \pm 7.51)10^9\ Pa$$

The shear modulus for aluminum is 24 $G\ Pa$, so by using the discrepancy test:

$$D = |G_{theo} - G_{exp}| \rightarrow D = |24 - 21.28| \times 10^9 = 2.72 \times 10^9\ Pa$$

$$2 \times \Delta G >?\ D$$

$$2 \times 7.51 \times 10^9 >?\ 2.72 \times 10^9$$

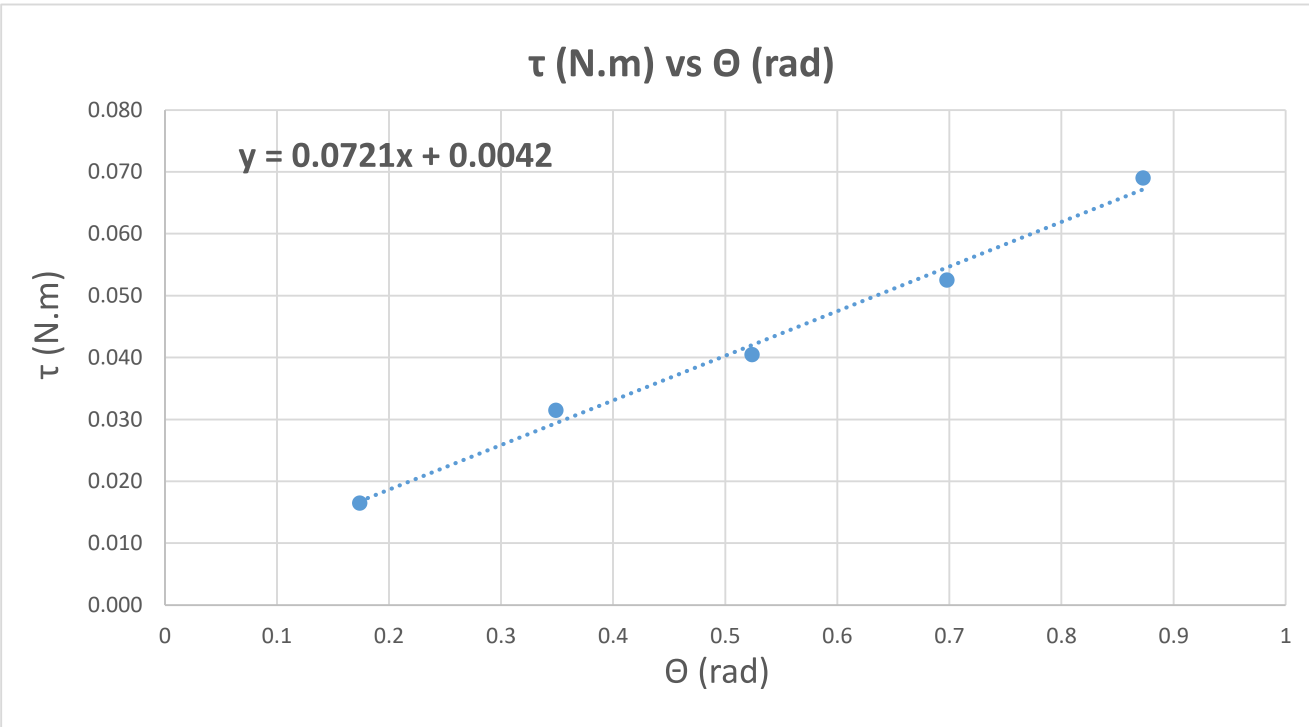
$$15.02 \times 10^9 > 2.72 \times 10^9$$

Therefore, our obtained value is accepted.

The torsional constant accuracy can be indicated from the y-intercept of the first graph. The theoretical y-intercept must equal zero, experimentally it is 0.002 N.m, which is a negligible quantity.

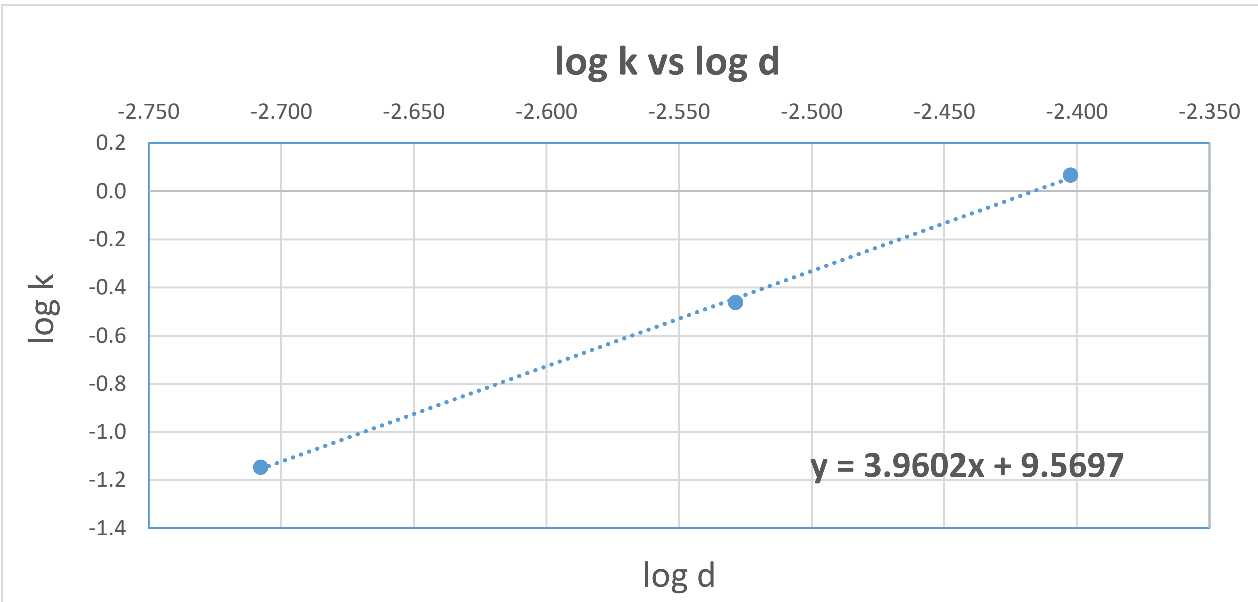
This experiment is very sensitive and lots of random errors may have occurred. Most likely, the rods are not completely straight which affects the whole experiment by giving a wrong value for the torsional force which leads to an error in the torsional constant and eventually in the moment of inertia. Also, the difference in time between completing the period and stopping the stop watch could have affected the experiment as well. Another source of error is that the rods considered to be with the same length or same diameter were not actually like that, which increases the possibility of errors in the values of the constants n & m.

Graph I



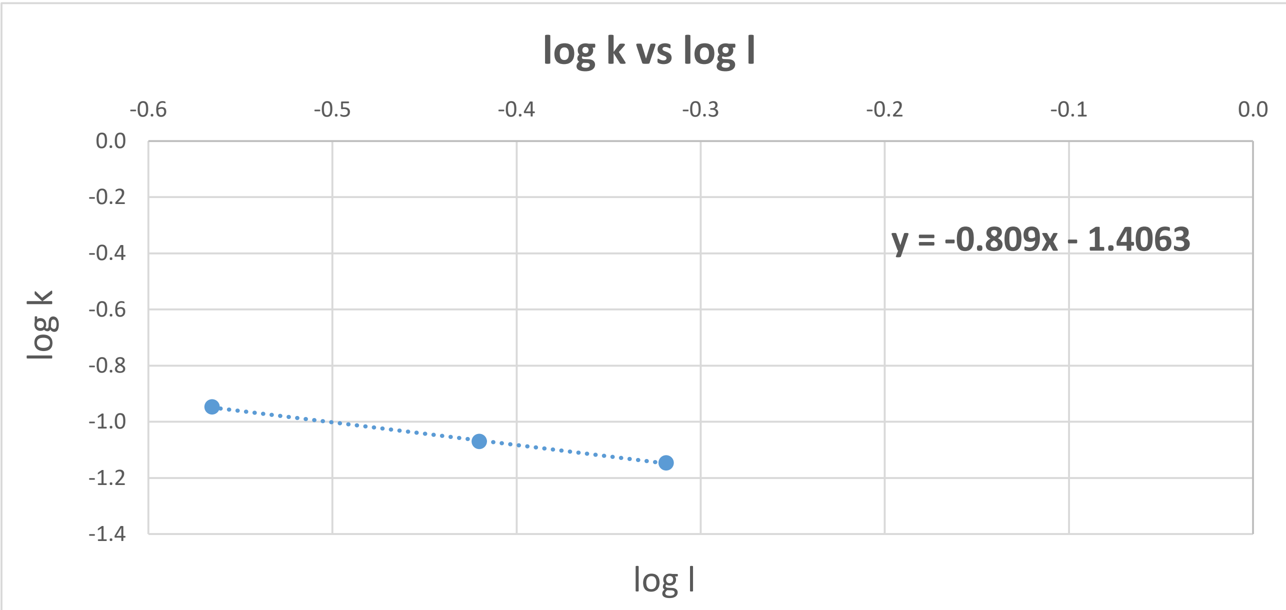
	slope	y-intercept
	0.072	0.004
error	0.004	0.002

Graph II



	slope	y-intercept
	3.960	9.570
error	0.100	0.255

Graph III



	slope	y-intercept
	-0.809	-1.406
error	0.025	0.011