BIRZEIT UNIVERSITY

Faculty of Science

Physics Department

Physics 212

Measurement of the specific charge of the electron e/m

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Date: 2 Mar 2019

– Abstract:

The aim of this experiment is to measure the charge to mass ratio of the electron. Using a potential difference to produce an electric field to work as an accelerator for the electrons, the electrons got some kinetic energy. Then, they were exposed to a magnetic field using the Helmlioltz pair consists of two circular loops that are separated by a distance equal to their radius. The electrons took a circular motion. By plotting the curve V vs I^2 at fixed radius, the ratio e/m can be obtained. The ratio e/m in this experiment was $(1.78 \pm 0.12) \times 10^{11} C/kg$ and this result is acceptable.

– Theory:

In 1897, J.J.Thomson measured the ratio between the charge of the electron to its mass. Thomson's experiment involves the using of an electric field to accelerate electrons up to high velocity, and a magnetic field to then steer the electrons in a circular trajectory.

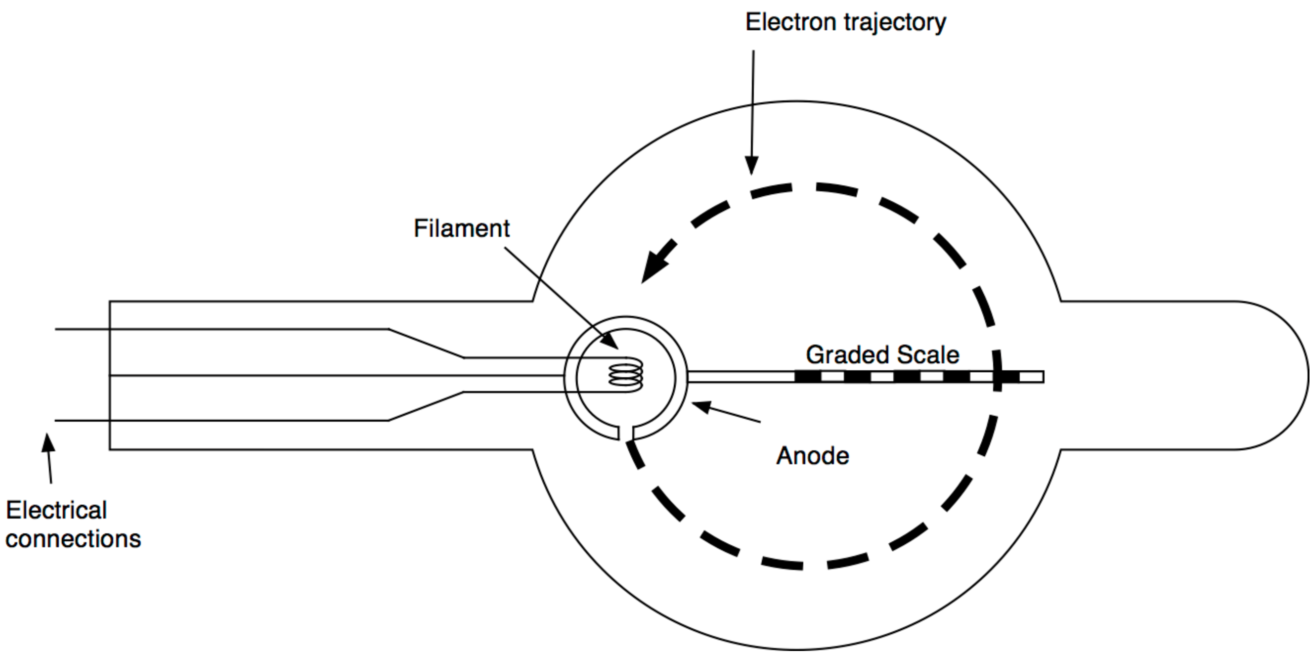


Figure 1: Schematic of the e/m apparatus.

To calculate e/m from the electron's trajectory, we must consider the motion of a charged particle in electric and magnetic fields. An electron emitted from the cathode experiences a total force:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{1}$$

Where,

$\vec{E}$: Electric field.

$\vec{B}$: Magnetic field.

q: Electron's charge.

As the electrons accelerated from cathode to anode, they gain a kinetic energy in the opposite of their potential energy following the law of conservation of energy.

$$U_e = \int_{Cathode}^{Anode} \vec{F} \cdot d\vec{r} = e \int_{Cathode}^{Anode} \vec{E} \cdot d\vec{r} = eV$$

$$\Delta K_e = \frac{1}{2} m_e v_{ef}^2 - \frac{1}{2} m_e v_{ei}^2$$

Where,

U_e: Electron’s potential energy.

e: Electron’s charge.

V: Potential difference.

ΔK_e: The increasing in the electron’s kinetic energy.

m_e: Electron’s mass.

v_{ef}: Electron’s final velocity.

v_{ei}: Electron’s initial velocity.

Assume the initial kinetic energy of the electrons is too small $(v_{ei} \ll v_{ef})$, then

$$eV = \frac{1}{2} m_e v_e^2 \tag{2}$$

Therefore, the velocity of the electron is

$$v_e = \sqrt{\frac{2eV}{m_e}} \tag{3}$$

Then, the electrons enter the region of magnetic field. The magnetic force on the electrons will equal the centripetal force. The direction of propagation would be perpendicular to the magnetic field for circularity motion.

$$ev_eB = m_e\frac{v_e^2}{R} \tag{4}$$

From equation (2) we get:

$$v_e^2 = 2V\left(\frac{e}{m_e}\right) \tag{5}$$

From equation (4) we get:

$$v_e = BR\left(\frac{e}{m_e}\right) \tag{6}$$

From equations (5) & (6) we get:

$$\frac{e}{m_e} = \frac{2V}{B^2R^2} \tag{7}$$

The magnetic field must be calculated from the current applied to the coils and the coil geometry. Applying the Biot-Savart law to each coil, we have:

$$\vec{B} = \frac{\mu_0}{4\pi}\oint_C \frac{Id\overrightarrow{\ell_1} \times \overrightarrow{R_1}}{R_1^3} + \frac{\mu_0}{4\pi}\oint_C \frac{Id\overrightarrow{\ell_2} \times \overrightarrow{R_2}}{R_2^3}$$

Analyzing the previous term geometrically at the center of coils, knowing that the coil separation is equal to the coil radius, we get:

$$B = \left(\frac{5}{4}\right)^{\frac{3}{2}}\frac{\mu_0 I}{a} \tag{8}$$

Where,

μ_0: Vacuum permeability = $4\pi \times 10^{-7} N/A^2$.

I: Current.

a: Coil radius.

If each coil consists of N turns of wire, then:

$$B = \left(\frac{5}{4}\right)^{\frac{3}{2}} \frac{\mu_0 N I}{a} \tag{9}$$

Substitute the results in equation (7) by equation (9), we get:

$$V = \left(\frac{e}{m_e}\right) \frac{125 \mu_0^2 N^2 R^2}{128 a^2} I^2 \tag{10}$$

Let,

$$R_{em} = \left(\frac{e}{m_e}\right) \tag{11}$$

$$\delta = \frac{125 \mu_0^2 N^2 R^2}{128 a^2} \tag{12}$$

Then,

$$V = R_{em} \delta I^2 \tag{13}$$

In Graph V vs I^2:

$$R_{em} = \frac{Slope}{\delta} \tag{14}$$

$$\Delta R_{em} = \frac{\Delta Slope}{\delta} \tag{15}$$

– Procedure:

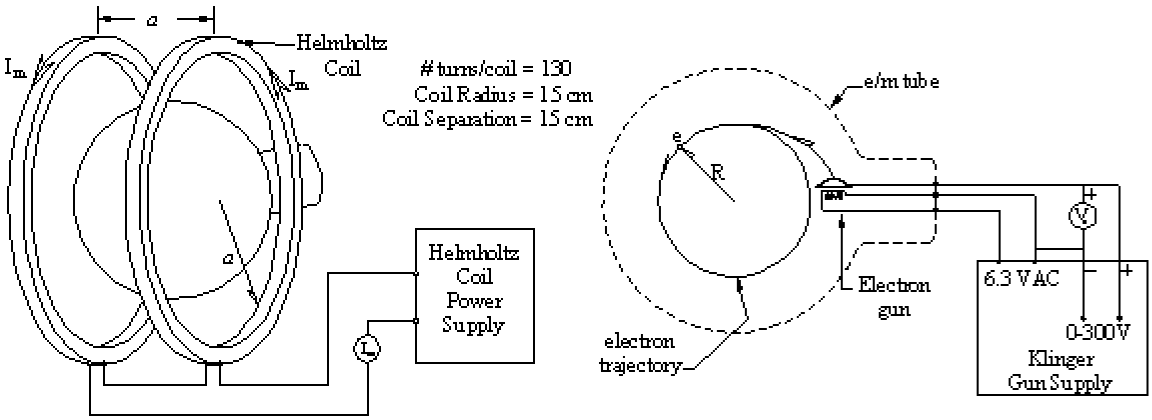


Figure 2: e/m apparatus.

Prepare the apparatus: e/m tube Klinger, Helmholtz coils, Projection scale to measure electron trajectory diameter, Stand with cm scale to check calibration of projection scale, Power supply: 6.3 VAC for gun filament of e/m tube plus an adjustable 0 - 300 V DC supply to accelerate the electrons in the e/m tube, Power supply for Helmholtz coils, Voltmeter to measure accelerating voltage. Ammeter to measure current to Helmholtz coils.

1. Measure the diameter of the electron trajectory.
2. Do several combinations of accelerating voltage and Helmholtz coil current.
3. Repeat the previous two steps to take another value for e/m.

– Data:

$D_1 = 7.00cm$		$D_2 = 8.50cm$	
V(V)	I(A)	V(V)	I(A)
152.6	1.41	150.1	1.13
191.6	1.61	190.2	1.26
230.4	1.77	230.3	1.38
271.4	1.90	269.5	1.49
310.1	2.03	311.3	1.55

Table 1: Data of the experiment.

$N = 130\ turns \quad , \quad a = 15.00cm$

– Calculations:

	$Slope$	y_{int}
Value	255.4781652	-214.3339201
Error	15.1915461	26.69911526

Table 2: The slope and the y-int from graph 1.

$R_1 = 3.50cm$

$$\delta_1 = \frac{125\mu_0^2 N^2 R_1^2}{128a^2} = \frac{125(4\pi \times 10^{-7})^2(130)^2(3.50 \times 10^{-2})^2}{128(15.00 \times 10^{-2})^2}$$

$$= 1.41893 \times 10^{-9} kg^2m^2/s^4A^3$$

$$R_{em1} = \frac{Slope}{\delta_1} = \frac{255.4781652}{1.41893 \times 10^{-9}} = 1.8005 \times 10^{11} C/kg$$

$$\Delta R_{em1} = \frac{\Delta Slope}{\delta_1} = \frac{15.1915461}{1.41893 \times 10^{-9}} = 0.11 \times 10^{11} C/kg$$

$$R_{em1} \pm \Delta R_{em1} = (1.80 \pm 0.11) \times 10^{11} C/kg$$

	$Slope$	y_{int}
Value	369.2999656	-272.7065531
Error	28.2874645	38.76827712

Table 3: The slope and the y-int from graph 2.

$R_2 = 4.25cm$

$$\delta_2 = \frac{125\mu_0^2 N^2 R_2^2}{128a^2} = \frac{125(4\pi \times 10^{-7})^2(130)^2(4.25 \times 10^{-2})^2}{128(15.00 \times 10^{-2})^2}$$

$$= 2.09219 \times 10^{-9} kg^2 m^2/s^4 A^3$$

$$R_{em2} = \frac{Slope}{\delta_2} = \frac{369.2999656}{2.09219 \times 10^{-9}} = 1.76513 \times 10^{11} C/kg$$

$$\Delta R_{em1} = \frac{\Delta Slope}{\delta_1} = \frac{28.2874645}{2.09219 \times 10^{-9}} = 0.14 \times 10^{11} C/kg$$

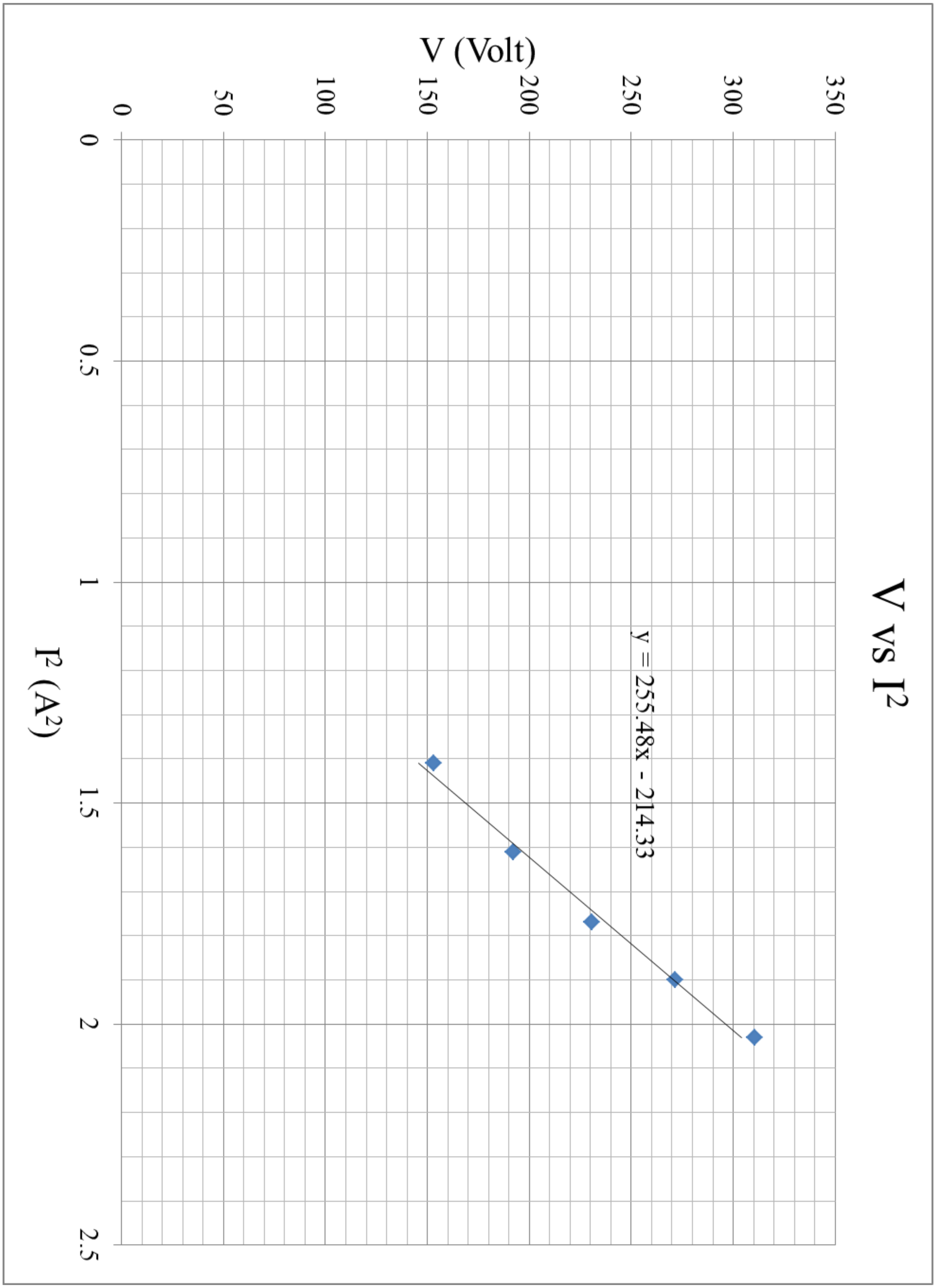
$$R_{em1} \pm \Delta R_{em1} = (1.76 \pm 0.14) \times 10^{11} C/kg$$

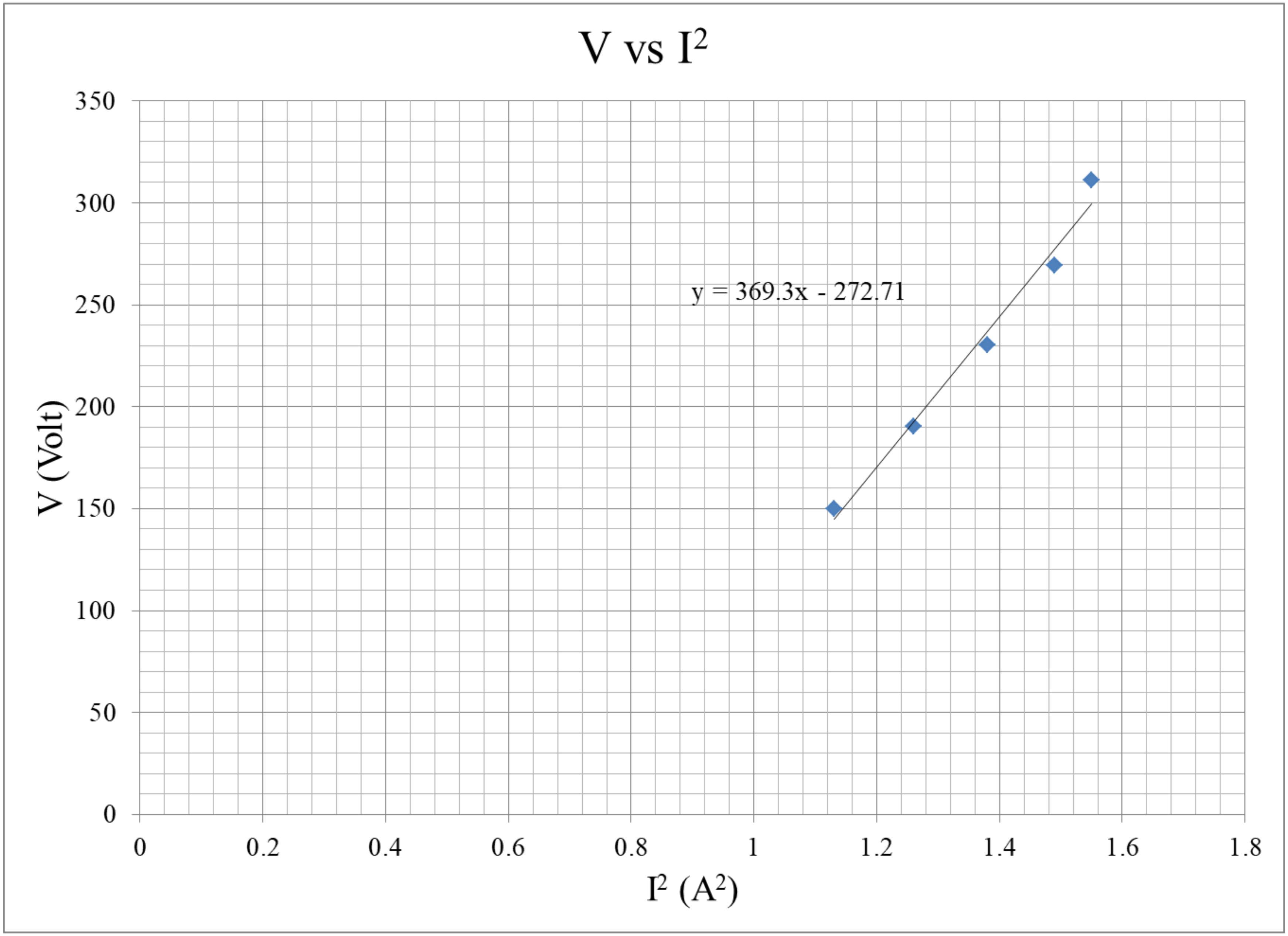
Average Value:

$$\bar{R}_{em} = \frac{R_{em1} + R_{em2}}{2} = \frac{(1.80 + 1.76) \times 10^{11}}{2} = 1.78 \times 10^{11} C/kg$$

$$\Delta \bar{R}_{em} = \frac{\Delta R_{em1} + \Delta R_{em2}}{2} = \frac{(0.11 + 0.14) \times 10^{11}}{2} = 0.12 \times 10^{11} C/kg$$

$$\bar{R}_{em} \pm \Delta \bar{R}_{em} = (1.78 \pm 0.12) \times 10^{11} C/kg$$


V vs I^2
V (Volt)
I^2 (A^2)
y = 255.48x - 214.33
0
50
100
150
200
250
300
350
0
0.5
1
1.5
2
2.5


V vs I²
y = 369.3x - 272.71
V (Volt)
I² (A²)
0
50
100
150
200
250
300
350
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

– Results:

$$\bar{R}_{em} \pm \Delta\bar{R}_{em} = (1.78 \pm 0.12) \times 10^{11} C/kg$$

– Discussion:

The 2014 CODATA recommended value for an electron's e/m is $1.76 \times 10^{11} C/kg$. The experimental value was $(1.78 \pm 0.12) \times 10^{11} C/kg$. Both values are close to each other. So, this result is accepted.

For each value of V, the electron has a velocity as seems in equation (3):

$R_1 = 3.50cm$		$R_2 = 4.25cm$	
V(V)	v_e(m/s)	V(V)	v_e(m/s)
152.6	7,412,917	150.1	7,279,375
191.6	8,306,338	190.2	8,194,246
230.4	9,108,631	230.3	9,016,764
271.4	9,885,913	269.5	9,754,008
310.1	10,567,269	311.3	10,483,186

Table 4: V vs v_e.

We can see that the velocity is increasing as the voltage accelerating increasing.

For each value of I, the current produce a magnetic field as seems in equation (9):

$R_1 = 3.50cm$		$R_2 = 4.25cm$	
I(A)	B_1(T)	I(A)	B_2(T)
1.41	0.002146081	1.13	0.001719909
1.61	0.00245049	1.26	0.001917774
1.77	0.002694016	1.38	0.00210042
1.90	0.002891882	1.49	0.002267844
2.03	0.003089748	1.55	0.002359167

Table 5: I vs B.

We can see that the magnetic field is increasing as the current in the coils increasing.

There are many sources for systematic errors in this experiment. For example, the light of ionized gas which forms as a result of collisions with the electrons is too dim. Moreover, the systematic errors from the apparatus itself change the results of e/m.

<u>The error of helical path:</u>

The experimenter must take care that the electrons are in a uniform circularity motion without a helical path. Since the helical path means that there is a nonperpendicular angle between the velocity and the magnetic field.

$$e\left\|\overrightarrow{v_e} \times \vec{B}\right\| = m_e \alpha \tag{16}$$

$$e v_e B \sin\theta = m_e \frac{v_e^2 \cos^2\varphi}{R} \tag{17}$$

$$\theta = \frac{\pi}{2} + \varphi \tag{18}$$

$$\sin\theta = \cos\varphi \tag{19}$$

Rearrange the equation (17) and substitute equation (18) to get:

$$v_e = \frac{BR}{\cos\varphi}\left(\frac{e}{m_e}\right) \tag{20}$$

From equations (5) & (18) we get:

$$\left(\frac{e}{m_e}\right) = \frac{2V\cos^2\varphi}{B^2R^2} \tag{21}$$

Substitute the equation (9) in equation (19):

$$V = \left(\frac{e}{m_e}\right)\frac{125\mu_0^2 N^2 R^2}{128a^2\cos^2\varphi} I^2 \tag{22}$$

Therefore, there will be an error in calculations of the e/m for the electron.

– References:

1. H. Abusara, & A. Shawabkeh (2016, November). *Laboratory Manual: Modern Physics Lab* (Second Edition). *Measurement of the specific charge of the electron e/m* (pp. 49-57). Birzeit University: Faculty of Science
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4. Werner U. Boeglin (2019, February 14). *Modern Lab Experiments, The e/m Ratio*. http://wanda.fiu.edu.