Ch 16: Waves I

10- The equation of a transverse wave traveling along a very long string is $y = 6.0 \sin(0.020\pi x + 4.0\pi t)$, where x and y are expressed in centimeters and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the speed, (e) the direction of propagation of the wave, and (f) the maximum transverse speed of a particle in the string. (g) What is the transverse displacement at $x = 3.5$ cm when $t = 0.26$ s?

(10) $y(x,t) = 6\,\text{Sin}(0.02\pi x + 4\pi t)$, x, y in cm, t in sec.

(a) $y_m = 6$ cm, (b) $\lambda = \frac{2\pi}{k} = \frac{2\cancel{\pi}}{0.02\cancel{\pi}}$

$\Rightarrow \lambda = 100$ cm

(c) $f = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2$ Hz

(d) $V = \frac{\omega}{k} = \frac{4\pi}{0.02\pi} = 200$ cm/s $= 2$ m/s

(e) along negative x axis.

(f) $u(x,t) = \frac{dy}{dt} = 4\pi * 6 \cos(0.02\pi x + 4\pi t)$

Maximum $u = 4\pi * 6 = 75.4$ cm/s

(g) $y(x,t)\Big|_{\substack{x = 3.5\text{ cm} \\ t = 0.26\text{ s}}} = 6\,\text{Sin}(0.02\pi * 3.5 + 4\pi * 0.26)$

$= 6\,\text{Sin}(1.11\pi)$

$= -6 * 0.3387\ldots$

$= -2.03$ cm

15- A stretched string has a mass per unit length of 5.00 g/cm and a tension of 10.0 N. A sinusoidal wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is traveling in the negative direction of an *x* axis. If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m, (b) k, (c) ω, and (d) the correct choice of sign in front of ω?

(15) $\mu = 5\, g/cm$, $\tau = 10\, N$, $y_m = 0.12\, mm$, $f = 100\, Hz$

along − x axis.

$$y(x,t) = y_m \sin(kx \mp \omega t)$$

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{10}{\frac{5\times 10^{-3}}{10^{-2}}}} = 4.47\ m/s$$

$$\omega = 2\pi f = 2\pi * 100 = 200\pi = 628.3\ rad/s$$

$$v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{v} = \frac{628.3}{4.47} \cong 141\ m^{-1}$$

$$y(x,t) = 0.12\times 10^{-3} \sin(141x + 628t)$$

x, y in meters

t in sec.

•26 A string along which waves can travel is 2.70 m long and has a mass of 260 g. The tension in the string is 36.0 N. What must be the frequency of traveling waves of amplitude 7.70 mm for the average power to be 85.0 W?

26 $L = 2.7\,m$, $m = 260\,g \Rightarrow \mu = \frac{m}{L} = 9.63 \times 10^{-2}\ kg/m$

$\tau = 36\,N$, $y_m = 7.7\,mm$, $P_{avg.} = 85\,W$, $f = ??$

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 \qquad \left\{ v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{36}{9.63 \times 10^{-2}}} \right.$$

$$\boxed{v = 19.3\ m/s}$$

$$85 = \frac{1}{2} * 9.63 \times 10^{-2} * 19.3 * (2\pi f)^2 * (7.7 \times 10^{-3})^2$$

$$85 = 2.18 \times 10^{-3} f^2$$

$$\Rightarrow \boxed{f = 198\ Hz}$$

•32 What phase difference between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.50 times that of the common amplitude of the two combining waves? Express your answer in (a) degrees, (b) radians, and (c) wavelengths.

(32)

$$2\cancel{y_m}\cos\left(\tfrac{1}{2}\phi\right) = 1.5\,\cancel{y_m}$$

$$\cos\left(\tfrac{1}{2}\phi\right) = \frac{1.5}{2}$$

$$\Rightarrow \tfrac{1}{2}\phi = 41.4^\circ \Rightarrow \boxed{\phi = 82.8^\circ}$$

in rad

$$\pi \rightarrow 180^\circ$$
$$\phi\,? \leftarrow 82.8^\circ \Rightarrow 1.45\ \text{rad.}$$

in wavelength

$$\lambda \rightarrow 2\pi\ \text{rad}$$
$$? \leftarrow 1.45\ \text{rad}$$

or

$$\lambda \rightarrow 360^\circ$$
$$?? \leftarrow 82.8^\circ$$
$$\Rightarrow \phi = 0.23\lambda$$

$$\boxed{\phi = 0.23\lambda}$$

43- What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g, and is stretched under a tension of 250 N?

(43)

$$L = 10\ \text{m},\quad m = 100\ \text{g} \Rightarrow \mu = \frac{m}{L} = 0.01\ \text{kg/m}$$

$$\tau = 250\ \text{N}. \qquad v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{250}{0.01}} = 158.1\ \text{m/s}$$

$$f_n = n\frac{v}{2L}, \quad n = 1, 2, 3.$$

$$f_1 = \frac{v}{2L} = \frac{158.1}{2*10} = 7.9\ \text{Hz}$$

$$f_2 = \frac{2v}{2L} = 15.8\ \text{Hz}$$

$$f_3 = \frac{3v}{2L} = 23.7\ \text{Hz}$$

76- A standing wave results from the sum of two transverse traveling waves given by

$$y_1 = 0.050\ \cos(\pi x - 4\pi t)$$
$$\text{and} \quad y_2 = 0.050\ \cos(\pi x + 4\pi t),$$

where x, y_1, and y_2 are in meters and t is in seconds. (a) What is the smallest positive value of x that corresponds to a node? Beginning at $t = 0$, what is the value of the (b) first, (c) second, and (d) third time the particle at $x = 0$ has zero velocity?

76) $y_1(x,t) = 0.05\ \cos(\pi x - 4\pi t)$

$y_2(x,t) = 0.05\ \cos(\pi x + 4\pi t)$

$y_1 + y_2 = 0.1 \cos(\pi x)\cos(4\pi t)$

$\cos A + \cos B = 2\left\{\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right\}$

a) node $\Rightarrow \cos(\pi x) = 0$

$\pi x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

smallest

smallest $\pi x = \frac{\pi}{2} \Rightarrow \boxed{x = \frac{1}{2} \text{ meter}}$

b) $u(x,t) = 0$ at $t = ??$

$u(x,t) = \frac{dy}{dt} = -0.1 * 4\pi \cos(\pi x)\sin(4\pi t)$

$\sin(4\pi t) = 0$

$\Rightarrow 4\pi t = 0, \pi, 2\pi, 3\pi, \ldots$

1st 2nd 3rd

① $t = 0$

② $4\pi t = \pi \Rightarrow \boxed{t = \frac{1}{4} \text{ sec}}$

③ $4\pi t = 2\pi \Rightarrow t = \frac{1}{2}$ sec.