

1. (10 points) The phase velocity  $v$  of transverse wave in a crystal of atomic separation  $a$  is given by:

$$v = c \left( \frac{\sin(ka/2)}{(ka/2)} \right)$$

where  $k$  is the wave number and  $c$  is a constant. Find the group velocity and discuss the limits for long wavelength.

2. An object of mass  $m$  is connected to a harmonic oscillator with stiffness constant  $s$ , natural angular frequency  $\omega_0$ , and damping constant  $r$  is driven by an external force  $F(t) = F_0 \cos(\omega t)$ .

- (a) (10 points) Show that the displacement amplitude is given by:

$$x = \frac{F_0}{\omega \sqrt{r^2 + (m\omega - s/\omega)^2}}$$

- (b) (5 points) Show that the displacement amplitude is independent of  $\omega$  at low frequencies

- (c) (5 points) Show that the velocity amplitude at velocity resonance is independent of  $\omega$

- (d) (5 points) Show that the frequency amplitude is independent of  $\omega$  at high frequencies

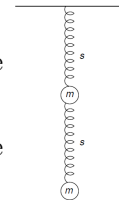
3. (20 points) Find the Fourier transformation of the function  $f(x) = 1 - x^2$  in the interval  $[-1, 1]$

4. (20 points) A ring of radius  $R$  carries a charge  $q$  that is uniformly distributed on its circumference. If an electron is released at point  $z$  above its center where  $z \ll R$ , show that the electron will exhibit a simple harmonic motion and find its period.

5. The equal masses in the figure oscillate in the vertical direction.

- (a) (15 points) Find the frequencies of the normal modes of oscillation and discuss the relative amplitude of the two masses

- (b) (5 points) What happens when either of the two masses is much larger than the other.



6. For three identical masses that are loaded on a string (the distance between the masses is  $a$ ) answer the following:

- (a) (8 points) Find the normal frequencies  $\omega_j^2 = 2\omega_0^2(1 - \cos(\frac{j\pi}{n+1}))$

- (b) (12 points) Find the relative displacement of the three masses for each normal mode frequency

$$-A_{r-1} + (2 - \frac{ma\omega^2}{T})A_r - A_{r+1} = 0$$

7. (25 points) Two acoustic mediums of impedance  $Z_1$  and  $Z_2$  show by applying the boundary conditions that if we insert a quarter wavelength medium of impedance  $Z_2 = \sqrt{(Z_1 Z_3)}$  then the coefficient of sound transition is 1

Question:	1	2	3	4	5	6	7	Total
Points:	10	25	20	20	20	20	25	140
Score:								

Good Luck