

Physics Department

Each question is 8 points

Phys333 Final exam duration 2.5 hours

Q1: Consider a plane pendulum that consists of a mass ***m,*** attached to one end of a very light rigid rod of length ***l***, that is attached to a pivot at its other end. The pendulum is constraint to move in a plane in a uniform gravitational field and it is free to swing all the way around the pivot.

1. Sketch the phase space portrait (diagram) showing all types of motion. (with no damping)
2. If a damping force of the form ***–βdθ/dt*** is added and considering only small angle displacement such that ***sinθ ≈ θ***, write the equation of motion.
3. Find the conditions for over damping, critical damping and under damping.

Solve the case of critical damping and sketch its phase space portrait.

Q2: A particle of mass *m* is sent directly toward a central point ***x=0*** with an initial ***x=***  and speed ***v0***. It is observed that as the particle approaches ***x=0***, it slows down gradually, and eventually turns back at ***x =x0 >0***. Its motion during the time *t < t0* ( where *t0* is the moment of turnabout ) is found to be fitted well by the Formula

***x – x0 = s ln{cosh[v0(t – t0)/s]}***

With a certain value for the length s, show that the observed motion can be understood as derivable from a potential

***U(x) = Be-2|x|/s***

Where B is predicted to be the least energy the particle would need to be given in order to bass through ***x=0*** without being turned back.

Q3: A pendulum consists of a mass m attached to the end of a massless rod of

length l. The pivot point of the pendulum slides without friction on a straight wire which

makes a fixed angle *α* with the horizontal. the pendulum is constrained to swing in the

plane of the page as in the following figure

1. Show that the Lagrangian can be written as

$$L=\frac{1}{2}m(\dot{x}^{2}+l^{2}\dot{θ}^{2}+2\dot{x}\dot{θ}l\cos(\left(α+θ\right))+ mg(x\sin(α)+l\cos(θ))$$

1. Write Lagrange equations of motion
2. Find the Hamiltonian
3. Write Hamilton equations of motion.
4. Determine whether H is a constant of motion and how is it related to the total energy.

Q4: A particle of mass m moves in three dimensions under influence of a force described by a potential which is expressed in terms of cylindrical coordinates ρ, φ, z and constant parameters, a, b, V0, as

$V\left(ρ,φ,z\right)=V\_{0}(\cosh(\frac{ρ}{b})+\cos(ρ-z/a))$.

1. Determine the hamiltonian of the system (= the particle).
2. Find Hamilton equation of motion.
3. Determine the constants of motion.

Q5: A point like mass *m* is undergoing a three-dimensional projectile motion in a uniform gravitational field *g*. Consider that the air resistance is negligible.

a) Write the Hamiltonian function for this problem *H* = *H (q, p),* choosing as generalized coordinates the Cartesian coordinates (*x,y,z*) with the *z* axis pointing in the vertical direction.

b) Show that these equations lead to the known equations of motion for Projectile motion and that the Hamiltonian function corresponds to the Total Mechanical energy of the particle.

c) Once the particle reaches the highest point in its trajectory, a retarding force proportional to the velocity of the particle starts acting. Assume the proportionality constant to be known and given by *k. and* the wind is has a constant force along the x direction ***F0***Calculate the velocity as a function of time for the descending particle, and find the terminal velocity.

***Good luck***