Motion in 2 and 3 Dimensions

• Position and Displacement

If the position vector changes from $\vec{r}_1$ to $\vec{r}_2$ during a certain time interval Then the displacement $\Delta\vec{r}$ during that time interval is:-

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Δx Δy Δz

In 2D: When an object reaches Max position in one direction Then its velocity in this direction = 0 component-

• Average velocity & Instantaneous velocity

If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt then it's average velocity $\vec{V}_{avg}$ is

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$V_{Instant} = \frac{d\vec{r}}{dt}$$

V is the same direction as r

V_{avg} takes the tangents (at r) direction

• Average Acceleration & instantaneous Acceleration

$$a_{avg} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\Delta\vec{V}}{\Delta t}$$

$$\vec{a}_{Instant} = \frac{d\vec{V}}{dt}$$

If velocity changes in Magnitude or direction or Both The particle must have $\vec{a}$

* If the velocity changes in direction or magnitude → particle must have an acceleration

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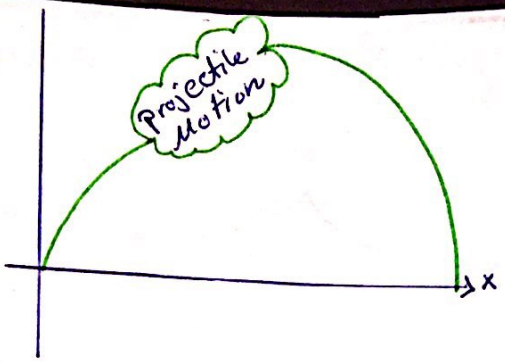
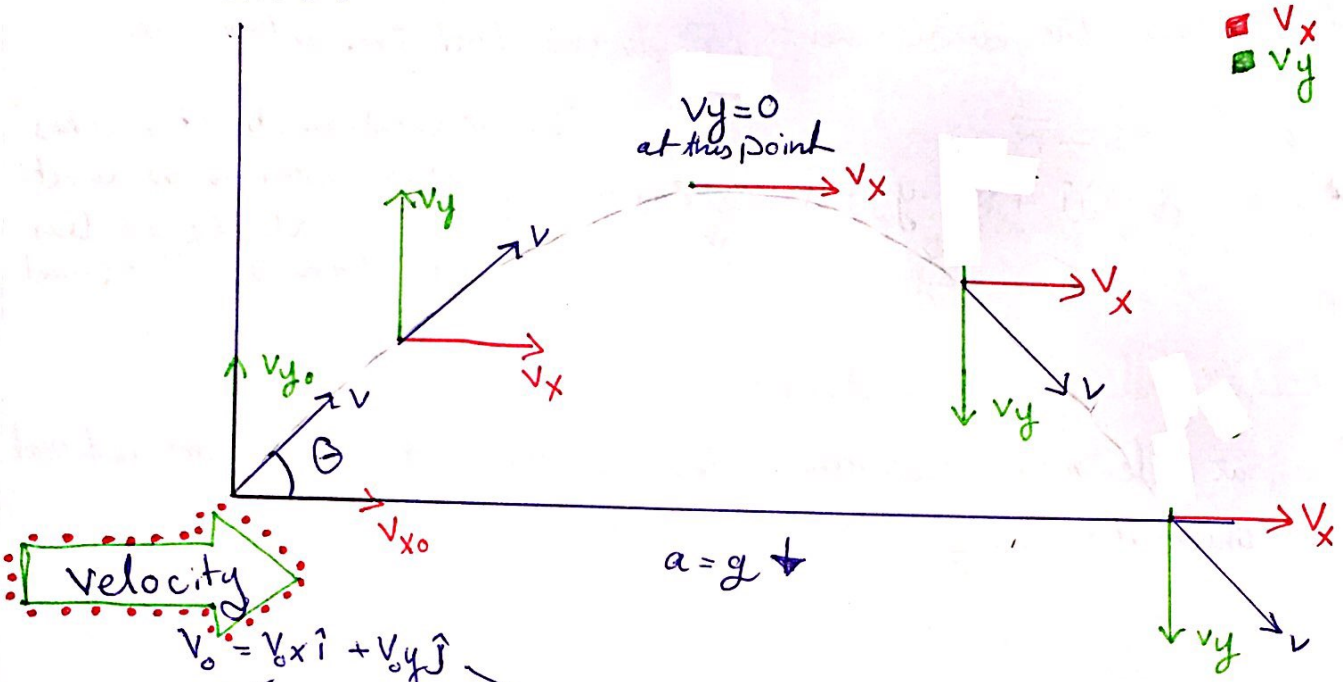


2D motion

» Horizontal and vertical motions are independent of each other



Velocity

$$\vec{V_0} = V_{0x}\hat{i} + V_{0y}\hat{j}$$

$V_{0x} = V_0 \cos\theta^\circ$ — Constant

$V_{0y} = V_0 \sin\theta^\circ$ — Variable

$$V = \sqrt{V_x^2 + V_y^2}$$

$V_x = V_{0x}$

- $V_y = V_{0y} - gt$ or - $V_y^2 = V_{0y}^2 - 2g(y - y_0)$ ($V_{0y} = V_0 \sin\theta$)

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Displacement

$$X = V_0 t \cos\theta$$
$$y = V_0 t \sin\theta - \tfrac{1}{2} g t^2$$

$$\Delta r = \sqrt{x^2 + y^2}$$

* throughout the path $a_x = 0$, $a_y = g$
* Path is parabolic
* air has a large effect on R or the path in general

• the Horizontal Range

$$R = V_{0x} \, t_p \quad \text{---} \; (1)$$

$$\Delta y = V_{0y} t - \tfrac{1}{2} g t^2$$

$\Delta y = 0$ لأنه انطلق من مستوى والمستوى نفسه

$$0 = V_{0y} t - \tfrac{1}{2} g t^2$$

$$t_p = 2 V_{0y} / g$$

نعوض في (1) فنستنتج:

Range: تقاس على نفس المستوى

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

$$R_{max} = V_0^2 / g \quad \text{when } \theta = 45^\circ$$

• the height

$$h = \frac{V_0^2 \sin^2(\theta)}{2g}$$

• Relation between R and h

$$\frac{h}{R} = \frac{\tan\theta}{4} \Rightarrow h = \frac{R \tan\theta}{4}$$

• The equation of the path

$$y = \tan\theta \cdot x - \frac{g \cdot x^2}{2 V_0^2 \cos^2\theta}$$

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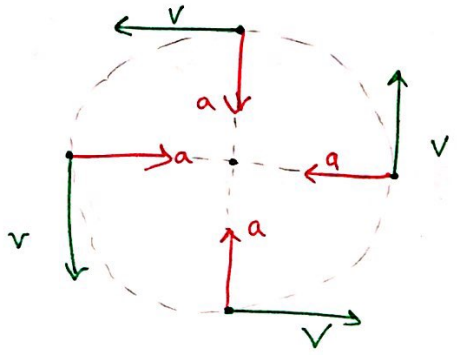
Uniform Circular Motion

- a particle travelling around a circle in a constant speed
- acceleration : centripetal (uniform circular motion)

$\vec{a}_r = \frac{v^2}{r}\hat{y}$ → speed of the particle, $\hat{y}$: vector
r → radius

$T = \frac{2\pi r}{v}$ T: period
$\pi = 3.14$



$a_t = 0$

$a_{tangent} = \frac{dv}{dt}$ (non-uniform circular motion)

- v is not constant
- so accelerating with a tangential acceleration

$a_t \neq 0$

$a_r = \frac{v^2}{2\pi r}$

- Net force : حاصل القوى

$\sum \vec{F} = m\vec{a}$

$\vec{a} = \vec{a}_t + \vec{a}_c$ ~~(perpendicular vectors)~~

$F = |\vec{F}| = |m\vec{a}|$
$= m|\vec{a}|$
$= m\sqrt{a_t^2 + a_c^2}$

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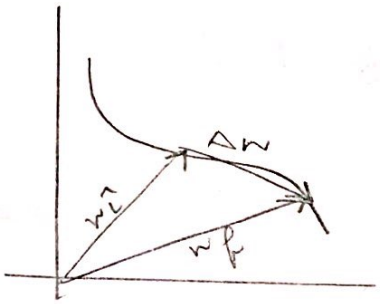
vectors

Displacement vector

$\Delta r = r_f - r_i$



Velocity vectors

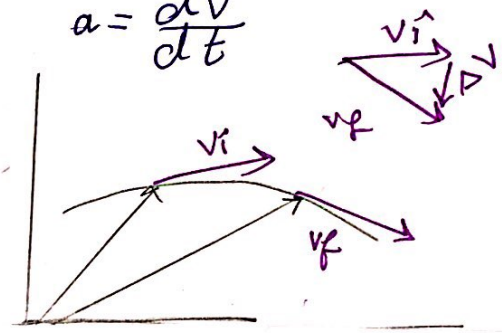
avg: $\bar{V} = \frac{\Delta r}{\Delta t}$

Ins: $V = \frac{dr}{dt}$

Accelaration vectors

$\bar{a} = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i}$

$a = \frac{dV}{dt}$



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Motion with Constant Acceleration

$$r = xi + yj$$

$$v = v_x\hat{i} + v_y\hat{j}$$

$$v_f = v_{xf}\hat{i} + v_{yf}\hat{j} \longrightarrow \underline{v_f = v_i + at}$$

$$r_f = x_f\hat{i} + y_f\hat{j} \longrightarrow \underline{r_f = r_i + v_i t + \tfrac{1}{2}at^2}$$

- Difference between Motion in 1D, 2D and 3D

1D: The Motion is in a straight line

2D: The Motion is in a curved path but in a single plane

3D: The Motion is throughout the space / not in a plane but in a complete space

Ex: a paper moving freely in the air

Note: • speed is the Magnitude of velocity

• if we have $\Delta \vec{V} \neq 0$ Then There is a { Change in speed and/or Change in direction

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