LABORATORY MANUAL

PHYS 212
MODERN PHYSICS LAB
[PHYSICS]

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EXPERIMENT 0

Instructions

- The Course:

  The modern physics laboratory contains experiments related to the physical phenomena which led to the development of "quantum physics" and the "theory of relativity" includes experiments on the nature of electromagnetic waves, atomic spectra, radioactivity and other quantum effects.

  The student should find this laboratory most interesting. First, the laboratory offers Independent experiments deal with different physical ideas Therefore the feeling of monotony caused my employing similar techniques each Second, the student have the opportunity witness the conversion of some quantum effects, which may have seemed so abstract and unusual, Into measurable quantities, using ordinary laboratory equipment. To achieve that, however, the student must come prepared, in which case he or she be offered all the necessary assistance and cooperation to make the 3 hours stay in the laboratory most useful, constructive and enjoyable. Otherwise, the Laboratory work would become more of a burden and a punishment and the instructor would rather ask the student to leave. The choice is yours.

- preparation:

  Although most of the theoretical background needed for an experiments is covered by the physics courses, 231 and 232, It is not always the case that the theory related to the experiment is taught in class before carrying the actual laboratory work. The student is thus expected to do some rending in order to understand the physical ideas behind the experiment. Each experiment in the laboratory manual is forwarded by a theoretical discussion which makes a good basis for preparation. If anything is not sufficiently explained, the student Is referred to the recommended references in the laboratory sheet. Physics students are encouraged to own the following books

1. Melissions "Experiments in Modern Physics".
2. Jenkins and White: "Fundamentals Of Optics".
3. A textbook on Modern physics ' (Tipler, Weidner and Sells,

preparing the experimental side could prove to be more difficult. It is almost impossible to find an experimental textbook which explains the experiments offered by our laboratory with exactly the Same experimental details. This is because the degree of experimental complexity was adjusted to match the knowledge acquired by the students the previous laboratory courses and also to be consistent With the general plan offered by the department. Nevertheless, the student be expected at least to draw a block diagram of the experimental scheme and realize the function of each piece of equipment involved in it.

The laboratory instructors are, of course, aware of the fact that students come to the laboratory mainly to learn things rather than to invent them. On the other hand there is a minimum amount of knowledge, which must be acquired by the student through preparation, for him or her to be able to conduct the experimental work properly. Lack of that might result in the student missing taking necessary data, damaging laboratory equipment and/or consuming much of the Instructor's time. How do you decide Whether you are sufficiently prepared? A good indication is your ability to understand and answer the preliminary questions to be found in each experimental sheet. These questions were chosen carefully to emphasize the basic ideas, both theoretical and experimental, related to the particular experiment. If you are not able to cope with them on your own, you may seek the advice of your laboratory Instructor during his office hours. To ensure taking maximum advantage of the course and smooth running of the laboratory work, the procedure will be followed. Early in the session you may be asked by the instructor to answer questions related to the experiment. If the instructor is not satisfied with your preparation, you will be disqualified from continuing the experiment. In that ease you be given 40% of the grade for that experiment and obviously lower your evaluation mark. If disqualified three times the student will automatically the course with a grade of 40% Please spare you Instructor and yourself all the inconvenience by coming prepared.

- Handling equipment:
  Most pieces of equipment found in this laboratory were imported from abroad. The purchasing process complicated and very expensive. Obtaining a piece of equipment could easily take more than a year obviously the department can only buy a limited number of each item, subject to budget constraints. Damaging a piece of equipment due to mishandling right cause a serious disruption of the working order; work in useful experiments might be discontinued for long periods. So, do treat the equipment with much care so that it can serve the following student generation. Read and understand all safety instructions and warnings given with each experiment. These will either be written in capital letters in the manual or posted the laboratory. If doubt consult your Instructor.

  Remember: You may be held responsible for any damage that you cause due to negligence .

- The experimental work:
  Unlike the introductory laboratory courses, the experiments in this laboratory are not prepared as ready to operate. At this level, it is expected that the student is able to show a reasonable degree of initiative experimental skill. The remarks listed below are made to assist you in your experimental work efficiently. They are the result of several years of experience running this laboratory.

  1. Read the precautions and warnings
  2. Take your time and think before starting. Identify every piece of equipment on your table. Make sure you know how to use them and how they fit the experimental scheme shown in the manual. Do not rush into taking data under the false impression of pressure. The three hours assigned to each are than enough
  3. Make a quick run of the experiment end sure that works properly over the whole range of measurement. Make a rough estimate of the expected results to see whether the readings shown by your instruments are reasonable. (Thus you should be able to judge if you are well prepared.) Now decide on the range, precision of measurement and proceed to take data.
  4. Never forget to estimate the error each measurement. As you should know by now, experimental results without error estimates are meaningless. You

may estimate measurement precision any way you see appropriate subject to the nature of measurement and the time available. Be reasonable and use your Judgment rather than strict rules.

5. When faced with a problem do not jump immediately and ask your instructor without even trying to find out what went wrong. Try first to sort out by yourself. Remember that the evaluation of your performance in the course depends on the quality of your work. If the instructor is busy, wait in your place and make yourself busy with things related to the experiment until he is available.
6. DO not switch off or dismantle the setup, with the intention of terminating the experiment, without the instructor's permission. You may be asked to repeat measurement or take extra ones, the thing Which may become impossible if the setting of the apparatus is changed.

- Honesty :

  Honesty in reporting experimental results was always a major factor in the progress of science. When you obtain a value of a physical quantity from your data, report It without any "adjustment". Do not attempt to "match" it to a value you think or you have been told is the "right" one. Other than being unscientific, this would be cheating and would be treated as such. Of course, you obtain a value of a constant several orders of magnitude from the accepted value found in the literature, then you check and recheck. If you cannot find the cause and remedy it yourself, consult your instructor. Honesty in this case is the best policy. The instructor have done all the experiments in laboratory themselves and are in possession of detailed data about each. Any "adjustment" in the data is not likely to pass unnoticed.

EXPERIMENT 1

Physical Optics

1.1 Objectives

1. To demonstrate the wave properties of light by observing interference and diffraction effects with single and double slits.
2. To study the properties of the diffraction grating.
3. To observe phenomena related to the polarization of light and to observe the phenomena of double refraction and optical activity.

1.2 References

1. Jenkins and White: Fundamentals of Optics, 4th cd., sections
2. Resnick and Hattiday: Fundamentals of Physics, 2nd cd., extended version, ch. 40, 41.
3. Berkeley Physics Course, volume 3: Waves.

1.3 Preliminary Exercises

1. Find an expression for the spatial separation of interference fringes in a double slit experiment. The distance between fringes on the screen should be given in terms of the wavelength λ the slit separation d and the distance to the screen, L.
2. Derive eq. 1, which gives the intensity of the diffraction pattern produced by a single slit.
3. Compute the width of the central maximum in a single slit diffraction pattern on a screen a distance $L = 1.5m$ from the slit for a wavelength $\lambda = 6328A^o$ and a slit width w = 0.1 mm.
4. In a diffraction grating experiment, the distance between the zero and fifth order maxima was measured to be 40 cm on a screen 1.5 m from the grating. Calculate the number of lines/mm on the grating. The wavelength of the light was $\lambda = 6328A^o$.

1.4 Theory

1.4.1 Diffraction

Suppose that we have a single slit of width w which is illuminated by a monochromatic, parallel beam of light. The light intensity transmitted through the slit is observed on a distant screen, which is parallel to the plane of the slit. If the width of the slit is large, we will simply observe a bright band, the image of the slit, on the screen. If the width of the slit is made sufficiently small, however, the phenomena of diffraction will be observed. Bright bands, separated by dark areas, will appear on the screen. There will be a very bright band in the center (the central maximum), with narrower, less intense bright bands on both sides. Moreover, the width of this pattern increases as the slit is made narrower. This is certainly not what we would expect on the basis of geometrical optics, which assumes that light rays travel in straight lines. The existence of diffraction demonstrates the wave nature of tight. To calculate the intensity of the single slit diffraction pattern, we will assume that the slit is infinitely long (much longer than its width). The problem can then be treated in two dimensions, which is a great simplification. The slit is treated as a row of point sources. The intensity due to this row of sources is computed at some distant observation point. Taking the limit as the number of sources goes to infinity and their separation goes to zero, arrive at an expression for the diffracted intensity:

$$I = I_o(\frac{sin\beta}{\beta})^2 \tag{1.1}$$

where β is:

$$\beta = \frac{\pi w}{\lambda} sin\theta \tag{1.2}$$

I_o = incident light intensity
λ = wavelength
w = slit width
θ = angle of observation (figure1.1) The intensity of the diffraction pattern as a function of observation angle θ is shown in figure 1.1. Note that the maximum intensity is obtained at $\theta = 0$ ($\beta = 0$). We obtain minima in intensity when:

$$sin\beta = 0 \text{ but } \beta \neq 0$$
$$\beta = m\pi \ , \ m = \pm 1, \pm 2, ...$$

We will take the width of the central maximum to be the distance (or angle) between the first two intensity minima, which occur when $\beta = \pm\pi$. For small diffraction angles, we can use the approximation $sin\theta = \theta$. The parameter β, defined by eq. 1.2, will be given by:

$$\beta \approx \frac{\pi w}{\lambda}\theta(\theta \text{ small})$$

The angular width of the central maximum will be approximately:

$$\Delta\theta = \theta(\beta = \pi) - \theta(\beta = -\pi) = \frac{2\lambda}{w}$$

On a screen a distance L from the slit, the spatial width of the central maximum will be:

$$D = L\Delta\theta = \frac{2\lambda L}{w} \tag{1.3}$$

From this expression, we see that the width of the central maximum will increase as the slit width, w, decreases.

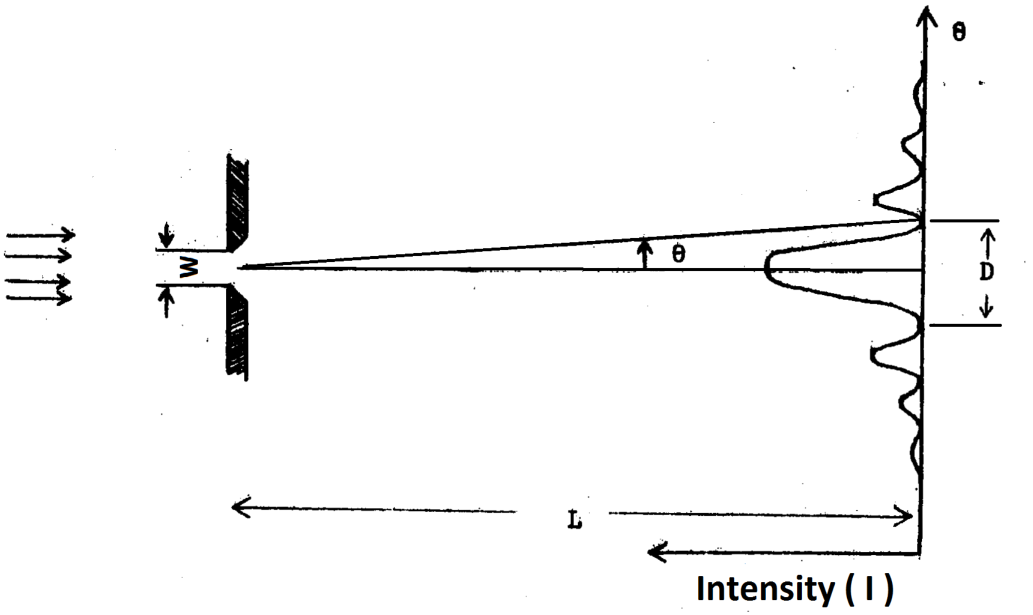

FIGURE 1.1: Intensity from a single slit of width w

1.4.2 Interference

If a double slit is illuminated by a spatially coherent beam of light (from a point source or a laser) and the transmitted intensity is observed on a distant screen, interference fringes will be observed. if the width of the slits is small compared to the distance between them, then the diffraction patterns of the two slits still overlap (to a good approximation) and the intensity of the pattern will be given by:

$$I = 4I_o(\frac{sin\beta}{\beta})^2 cos^2\gamma \tag{1.4}$$

where:

$$\gamma = \frac{\pi d}{\lambda} sin\theta \tag{1.5}$$

d = distance between slits
$\beta = \frac{\pi w}{\lambda}$
w = width of the slits
The pattern described by eq. 1.4 is shown in figure 1.2 With $w \ll d$ (slit width much less than slit separation) the $cos^2\gamma$ term will oscillate rapidly with θ, giving a series of

equally spaced interference fringes. The $(\frac{sin\beta}{\beta})^2$ term will change more slowly, giving the overall diffraction envelope which modulates the intensity of the fringe pattern.

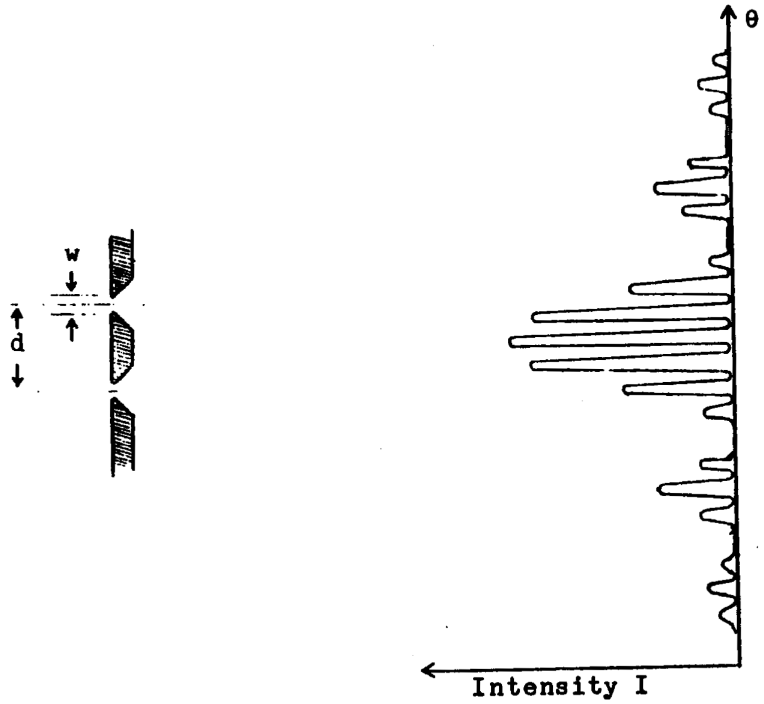

FIGURE 1.2: Intensity from a double slits each of width w and separated by distance d

Bright interference fringes will occur when $cos^2\gamma = 1$. The angles at which bright fringes will occur are given by:

$$\gamma \;=\; 0, \quad \pi, \quad 2\pi, \quad 3\pi, ...$$

or

$$dsin\theta \;=\; m\lambda \quad m = 0, \pm1, \pm2, ... \tag{1.6}$$

Fringes which coincide with a minimum in the diffraction pattern will not be visible. For small angles (such that $sin\theta = \theta$) the angular locations of the bright interference fringes will be given by:

$$\theta \;=\; \frac{m\lambda}{d} \qquad m = 0, \pm1, \pm2, ... \tag{1.7}$$

Note that the Spacing of the interference fringes is determined by the distance between the slits, d, white the width of the diffraction envelope is determined by the slit width w.

1.4.3 The Diffraction Grating

The diffraction grating is a multiple slit arrangement containing many parallel slits. It is not difficult to produce gratings which contain as many as 1000 lines/mm. The intensity of the diffraction pattern produced by a grating is given by:

$$I = I_o(\frac{sin\beta}{\beta})^2(\frac{sinN\gamma}{\gamma})^2 \tag{1.8}$$

$$\gamma = \frac{\pi h}{\lambda}sin\theta$$
$$\beta = \frac{\pi w}{\lambda}sin\theta$$

h = separation of slits
w = width of slits
N = number of slits illuminated
The resulting intensity pattern is shown in figure 1.3. This figure has been drawn f or a small value of N to make the details clearer. Strong intensity maxima (principle maxima) will occur when $sin\gamma = 0$. The angles at which the principle maxima are observed will be given by:

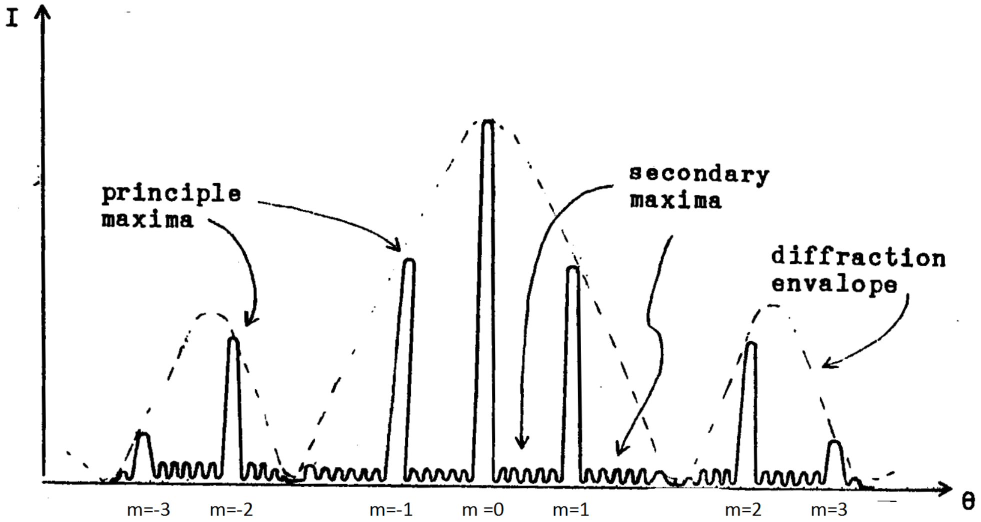

FIGURE 1.3: Intensity from diffraction grating

$$\gamma = m\pi, \qquad m = 0, \pm1, \pm2, ...$$

or

$$hsin\theta \quad = \quad m\lambda \tag{1.9}$$

The value of the integer m is called the order of the principle maximum. Note that when $sin\gamma = 0$, we will also have $sinN\gamma = 0$. Eq. 1.8 is indeterminant at the principle maxima, and must be evaluated using $L'Hospital's$ rule. There will be many weaker secondary maxima occurring when $sinN\gamma = 1$. These secondary maxima are normally far too weak to be observed. To find the width of the principle maxima, we isc find the locations of the zeros in intensely. Minima ($I = 0$) will occur when:

$$sin(N\gamma) \quad = \quad 0 \qquad \text{or } N\gamma = nn = \pm 1, \pm 2, ...$$

but

$$sin\gamma \neq 0 \text{ but } n \neq mN$$

The condition $sin\gamma = 0$ is necessary because we must exclude the zeros of $sinN\gamma$ which correspond to principle maxima. The angles at which zeros in intensity occur are given by:

$$\begin{aligned} Nhsin\theta = n\lambda \qquad & n = \pm 1, \pm 2, ... \\ & n \neq mN \end{aligned}$$

At principle maximum m, we will have $n = mN$. The minima on either side of this principle maximum correspond to $n = mN - 1$ and $n = mN + 1$. These minima occur at angles:

$$\begin{aligned} sin\theta_1 &= \frac{mN-1}{N}(\frac{\lambda}{h}) \\ sin\theta_2 &= \frac{mN+1}{N}(\frac{\lambda}{h}) \end{aligned}$$

For $\Delta\theta = \theta_2 - \theta_1$ small we use the approximation: $sin\theta_2 - sin\theta_1 \approx cos\theta\Delta\theta$ to obtain:

$$\Delta\theta \quad = \quad \frac{2\lambda}{Nhcos\theta} \tag{1.10}$$

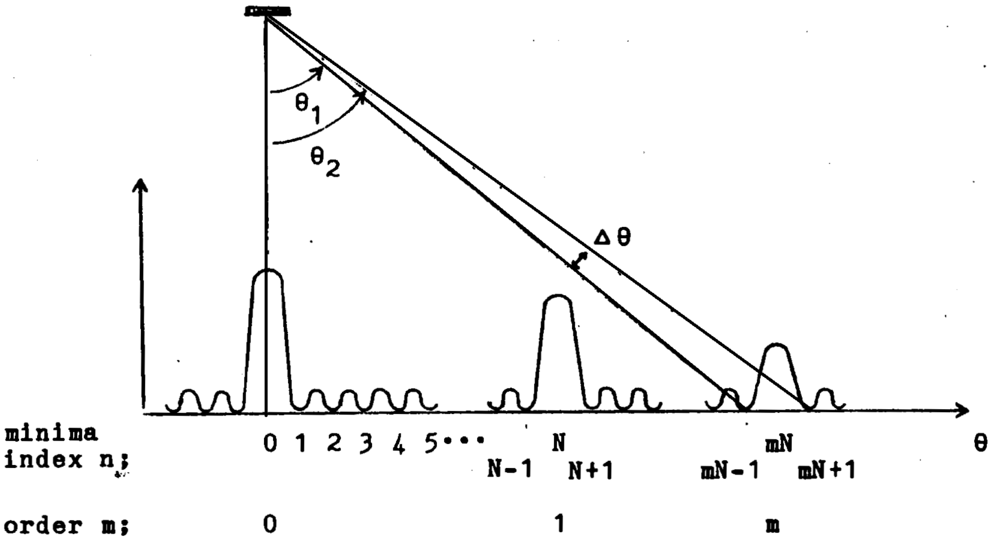

FIGURE 1.4

Notice that the angular width of the principle maxima is proportional to 1/N. The width of the principle maxima will be small for large values of N. This is essentially due to the rapidly oscillating $sinN\gamma$ term in the numerator of eq.1.8 . The fact that the principle maxima are narrow is an important factor when the grating is used to disperse light of different wavelengths in a spectroscope. You will not need to deal with dispersion of different wavelengths or the resolving power of a grating in this experiment, however, these topics will be important in a later experiment (Atomic Spectra) so we will discuss them here.

1.4.4 Dispersion and Resolving Power of Grating

Suppose that a diffraction grating is illuminated by light containing two wavelengths λ_1 and λ_2 separated by some amount $\Delta\lambda$. We will obtain two sets of principle maxima; one for each wavelength. The angular locations of these principle maxima are given by:

$$hsin\theta = m\lambda_1 \qquad m = 0, \pm1, \pm2, ...$$
$$hsin\theta = m\lambda_2 \qquad m = 0, \pm1, \pm2, ...$$

If the separation in wavelength $\Delta\lambda$ of the two wavelengths is small, then the angular separation of the principle maxima, for a given order m, will also be small:

$$sin\theta_2 - sin\theta_1 = \frac{m\Delta\lambda}{h}$$

$$\Delta\theta \approx \frac{m\Delta\lambda}{hcos\theta} \tag{1.11}$$

If $\Delta\lambda$ is made sufficiently small, then the principle maxima of the two wavelengths will overlap. As long as we are able to detect two distinct principle maxima, then we would say that the two wavelengths are resolved by the grating, if, however, the two principle maxima overlap to such an extent that they can not be distinguished from a single intensity maximum, then the two wavelengths are not resolved.

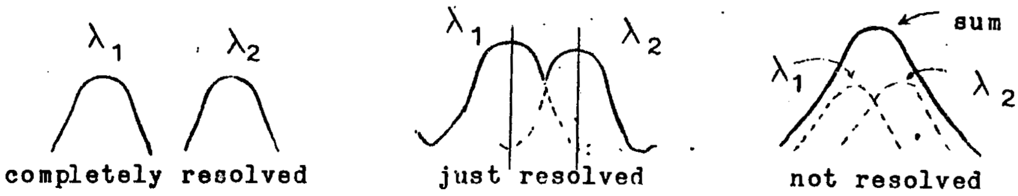

FIGURE 1.5

As the angular separation between the two principle maxima decreases, they will gradually merge into one intensity maximum. At what point would we say that the maxima are "just resolved"? that is, what is the minimum separation that we would require between the two peaks in order to consider them resolved? This is a somewhat arbitrary choice. A common criteria is the Rayleigh criteria, which states that two maxima will be considered "just resolved" when the peak of one principle maximum falls on the first intensity zero of the other.

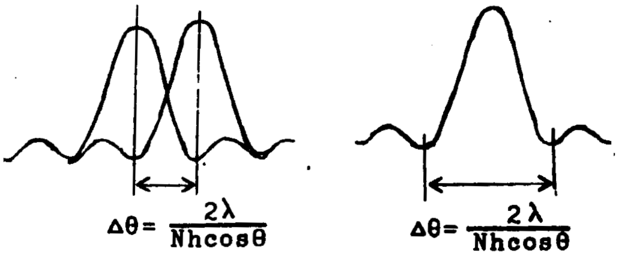

FIGURE 1.6

In the previous section, we found that angular width of a principle maximum, as measured from the minimum on one side to the minimum on the other, is $\frac{2\lambda}{Nhcos\theta}$. In order for two principle maxima to be resolved, their angular separation must be at least one half this amount, or $\frac{\lambda}{Nhcos\theta}$. Eq. 1.11 relates the angular separation between to principle maxima to their difference in wavelength, $\Delta\lambda$. For two principle maxima to be just resolved, we must have:

$$\frac{m\Delta\lambda}{hcos\theta} = \frac{\lambda}{Nhcos\theta}$$

or

$$\frac{\lambda}{\Delta\lambda} = mN$$

In this expression, λ may be taken as the average of the two wavelengths; $\lambda = \frac{1}{2}(\lambda_1 + \lambda_2)$. The quantity $\frac{\lambda}{\Delta\lambda}$ is called the resolving power of the grating. Here, $\Delta\lambda$ is the minimum wavelength separation that can be resolved. Note that the resolving power is larger for higher orders, and that it depends on the total number of slits illuminated, and not the number of lines/mm of the grating.

1.4.5 Polarization

The polarization of a tight wave refers to the direction of the electric field vector. For a plane wave in a nonconducting mediums the oscillating field will always point in a direction that is perpendicular to the direction of propagation. If the direction of the $\vec{E}$ field, in a plane perpendicular to the direction of propagation, changes rapidly and randomly in time, then the light is unpolarized. If the $\vec{E}$ field oscillates along one direction only, then the light is linearly polarized. If the $\vec{E}$ field vector remains constant in amplitude and rotates about the direction of propagation, sweeping out a circle as the light wave propagates then the light is circularly polarized. Looking toward the advancing light wave, if the $\vec{E}$ field vector rotates in the clockwise direction, then we have right hand circular polarization. If the $\vec{E}$ field vector rotates in the counter clockwise direction, then the light is left hand circularly polarized. Circular polarization is a special case of elliptical polarization, in which the magnitude of $\vec{E}$ changes as the vector rotates so that the tip of the $\vec{E}$ field vector sweeps out an ellipse. To describe polarized light, It is convenient to break the field into linearly polarized

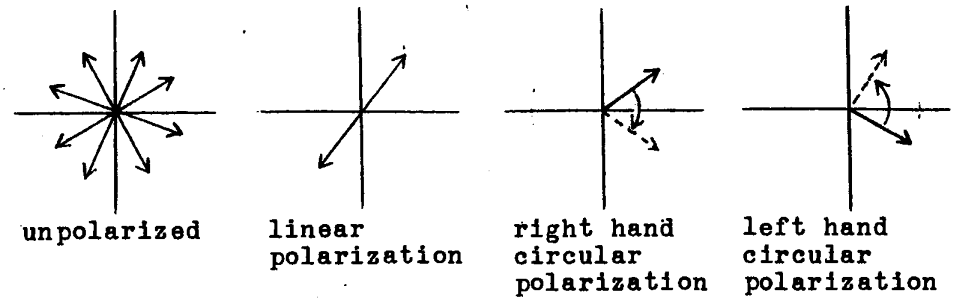

FIGURE 1.7: Different types of light polarization

components along the x and y directions. Writing the time dependence of the fields explicitly,we have:

1. Linear

$$\vec{E} = Asin(\omega t)\hat{x} + Bsin(\omega t)\hat{y}$$

2. R.H. Elliptical Polarization

$$\vec{E} = Asin(\omega t)\hat{x} + Bsin(\omega t + \phi)\hat{y}$$

becomes circular if A = B and $\phi = \frac{\pi}{2}$

3. L.H. Elliptical Polarization

$$\vec{E} = Asin(\omega t)\hat{x} + Bsin(\omega t - \phi)\hat{y}$$

becomes circular if A = B and $\phi = \frac{\pi}{2}$

4. Unpolarized

$$\vec{E} = Asin(\omega t)\hat{x} + Bsin(\omega t + \phi(t))\hat{y}$$

$\phi(t)$ changes rapidly and randomly in time

As an example, consider the representation of right hand circularly polarized light. We have $\vec{E}$ fields of equal amplitude oscillating along the x and y axis. The $\vec{E}$ field oscillating in the y direction is ahead of the $\vec{E}$ field oscillating in the x direction by a phase of $\pi/2$. As shown in Fig. 1.8, the resultant $\vec{E}$ field vector rotates about the

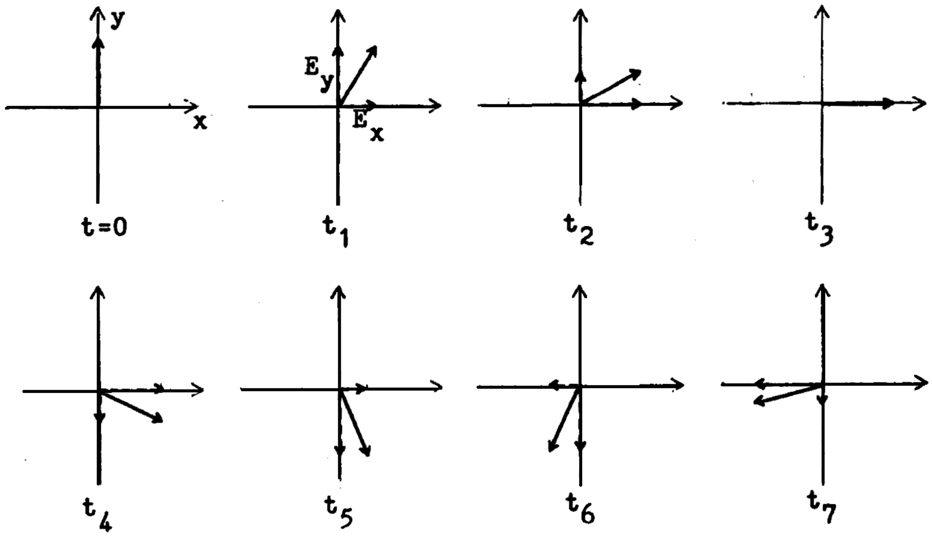

FIGURE 1.8

origin while remaining constant in magnitude, If the peak amplitudes of E_x and E_y are different, or phase difference between them is not $\pi/2$, elliptical polarization would be obtained.

One method of producing linearly polarized light i to use sheets of a material called polaroid. A polaroid sheet will transmit only the component of the incident light that is linearly polarized in the direction of the axis of the sheet. If un polarized light falls on a polaroid sheet the transmitted light will be linearly polarized along the axis of the sheet. if a second polaroid sheet Is placed behind the first, only the component of E along the axis of the second sheet will be transmitted. The light will emerge from the second sheet reduced in amplitude and linearly polarized along the axis of the second sheet. If the axis of the two polarizers are made perpendicular, no light will be transmitted. Another method of producing polarized light is by reflection from a dielectric (nonconducting material). When light is Incident on the surface of a dielectric, such as glass, at some angle of incidence ϕ, the refracted and reflected rays are found to be partially polarized. At a particular angle of incidence called Brewsters angle, the reflected light is completely polarized in a direction perpendicular to the plane of incidence. Brewaters angle is the angle at which the reflected and refracted rays make an angle of 90^o. If light is incident from air (n=1) to a medium having index of refraction n, then Brewsters angle will be given by $n = tan(\phi_B)$

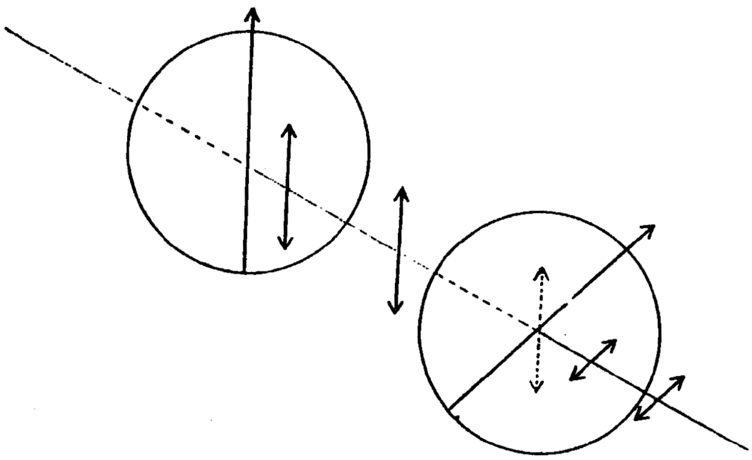

FIGURE 1.9

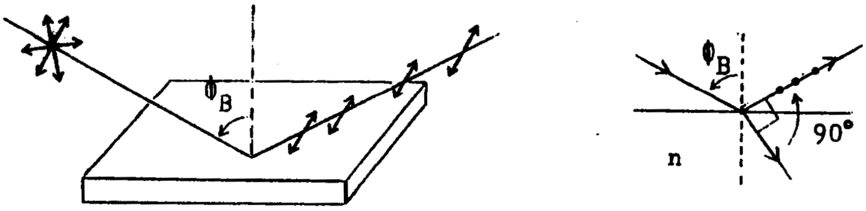

FIGURE 1.10

1.4.6 Half and Quarter Wave Plates

Many materials are not isotropic in their optical properties. In such materials, the index of refraction may depend on which direction the $\vec{E}$ field of the light is pointing in. This property enables one to make wave plates. Wave plates are used to introduce a phase change between the two perpendicular, linearly polarized components of polarized light. They may be made from thin sheets of certain minerals, such as mica or calcite or they may be made from certain plastics.

A wave plate has two axis, a fast axis and a slow axis. If linearly polarized light, with polarization along the direction of the fast axis, passes through a wave plat, it will propagate through the plate with some velocity v, and will emerge from the plate linearly polarized. Similarly, light that is linearly polarized along the direction of the slow axis will pass through the plate, propagating with some velocity v, and will emerge with its direction of polarization unchanged.

Suppose that linearly polarized light is incident on the wave plate with the polarization vector making an angle of 45^o with respect to the fast and slow axis. We resolve the wave into linearly polarized components along the fast and slow axis. The magnitudes

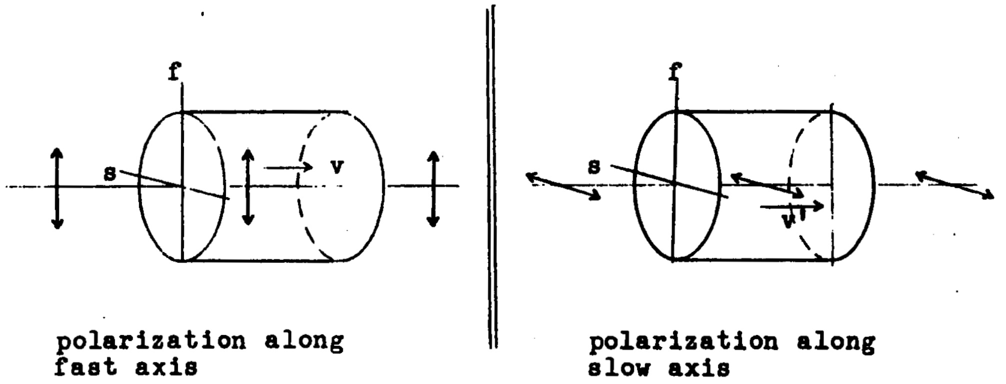

FIGURE 1.11

of these components will be equal (since we chose the angle to be 45^o) The component with polarization along the fast axis will propagate with velocity v in the plate. The other component, with polarization along the slow axis, will propagate with a slower velocity, v'. The two components will get out of phase as they propagate through the plate.

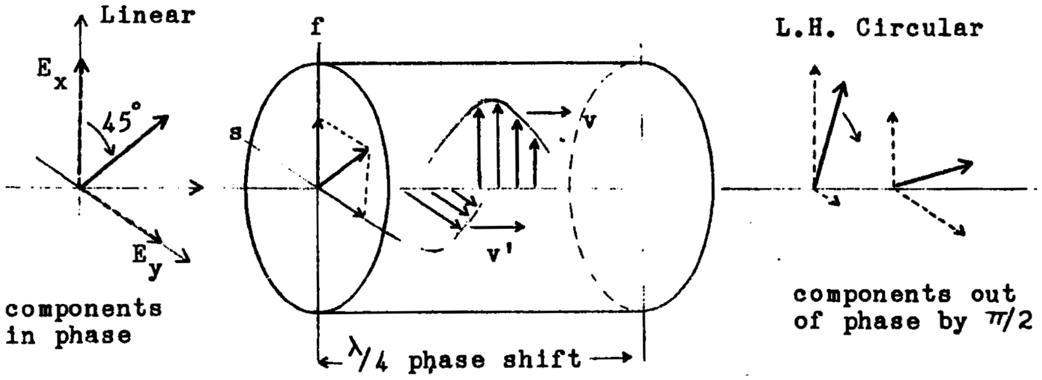

FIGURE 1.12

The two linearly polarized components will emerge from the plate with a phase difference ϕ, which depends op the thickness of the plate and the velocities v and v'. If we choose thickness of the plate to make this phase difference equal to $\pi/2$, then we will have a quarter plate. The linearly polarized component along the fast axis will be ahead of the component along the slow axis by a phase factor of $\pi/2$ at the point where the waves leave the plate. Since the amplitudes of these two components are equal, the merging light will be circularly polarized.

If linearly polarized tight is incident on the quarter wave plate with the polarization vector making some angle other than 45^o with the fast axis, then the amplitudes of the fast and slow components will not be equal. The light emerging from the quarter wave plate will be elliptically polarized. If the thickness of the wave plate is adjusted to give a phase difference of π between the emerging components, then we will have a half wave plate. If Linearly polarized light is incident on a half wave plate, it will emerge linearly polarized, but with the direction of polarization rotated.

1.4.7 Double Refraction and Optical Activity

Many crystals are not isotropic in their optical properties due to the arrangement of the atoms in the crystal. We will consider the example of calcite because it is relatively simple. In general, when unpolarized Light enters a calcite crystal, there will be two refracted rays. One component of polarization will be refracted through one angle, while the other component will be refracted through a different angle. In general, only one of the refracted rays (called the O or ordinary ray) wilt lie In the plane of incidence. The angle between the other ray (called the E or extraordinary ray) and the plane of incidence depends on the orientation of the crystal. The O and E rays travel the same path if the Light travels either along the optic axis or perpendicular to it. (To make a quarter wave plate, the calcite must be cut so that the optic axis lies along the surface of the place, so that light entering the plate normally will be traveling perpendicular co the optic axis). The Nicol prism is a high quality polarizer which is made by cutting and cementing together a calcite crystal in such a way that one of the refracted rays is eliminated by total internal reflection.

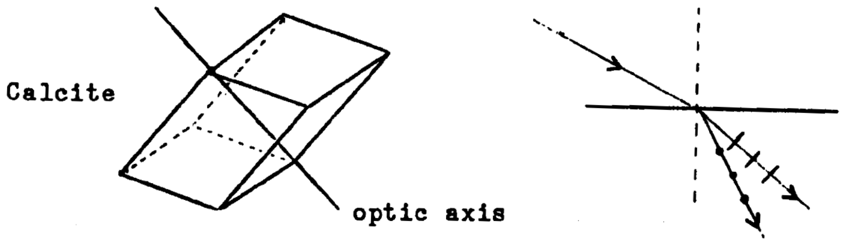

Figure 1.13

In some substances, the direction of polarization of linearly polarized Light wilt be rotated as the light passes through the substances. These substances are said to display activity. Some birefringent crystals display optical activity he linearly polarized tight propagates along the optic axis. Soma organic materials display optical activity both in crystaline form and in solution. Optical activity, like birefringence, is due to the molecular structure of the substance. through which the polarization vector rotates is strongly dependent on the wavelength of the light.

When materials which display either optical activity or double refraction are placed between two polarizers, colors or patterns of colors may be observed. In the case of a material which displays optical activity, this is caused by the fact that the plane of polarization will be rotated through different angles for different wavelengths. One color may be blocked by the second polarizer, and we will observe the complement of that color when viewing the transmitted tight. In a material that is birefringent, the colors are due to light passing through the crystal in a direction perpendicular to the optic axis, the crystal then acts as a waveplate. The phase difference, between the two linearly polarized components depends on wave length because the propagation velocities v and v' depend on wavelength. it one wavelength emerges linearly polarized, it may be blocked by the second polarizer. Because the samples are usually irregularly shaped, we see color patterns. Birefringence is induced in glass and in many plastics by mechanical stress. Color patterns are therefore observed in these materials when mechanical stress is applied, and the color patterns indicate areas of mechanical strain. Glassblowers often check their work between two crossed polarizers to check for undue stress in the glass.

1.5 Procedure

Caution: DO NOT LOOK DIRECTLY INTO THE LASER BEAM

1. You will Use a He-Ne laser ($\lambda = 6328A^o, P = 0.5mW$)as the light source for the single slit, the double slit and the diffraction grating parts of this experiment. Switch on the laser. Adjust the direction of the beam to be parallel to the optical bench. Slits, gratings, etc. may be mounted in holders on the optical bench. Note that the holders have a tilt adjust screw to make small changes in the position (from side to side) of the clamp.

2. **Observe single slit diffraction as a functions of slit width** Clamp a variable slit into the holder. Observe the transmitted intensity on a distant screen (the wall). Note the onset of diffraction when the slit width is made sufficiently small. Observe the manner in which the diffraction pattern changes as the slit width varies.

3. **Observe double slit patterns for various slit width /separation ratios** You are provided with a plate containing several double slits with different ratios of slit width (a) to slit separation (b). Observe the interference / diffraction patterns of these slits. Note the changes which occur in the pattern as the ratio of slit separation I slit width decreases.

4. **Measure width of central maximum for single slits of unknown width 3** Clamp a fixed slit of unknown width into the holder and place it a few cm from the laser. Adjust the position of the holder so that the beam passes through the slit. Place a screen at the other end of the optical bench. Record the distance between the slit and the screen. (You may wish to use the wall as a screen). Measure the width of the central maximum of the diffraction pattern. Perform this measurement for the three single slits of unknown width. You may wish to measure the width of the pattern by placing a piece of paper against the wall, or the screen and marking the position of the central maximum. Do not, however, write on the vail or the screen. Keep in mind that when we derived eq. 1.3 for the width of the central maximum, we took this width to be the distance between the centers of the minima on either side of the central maximum.

5. **Measure the width of a hair using a ruler** - This might sound difficult, but actually it is not. Hold a single strand of hair is the Laser beam. Note that a

single barrier produces the same diffraction pattern as a single slit. Use the ruler to measure the width of the central maximum of this pattern and the distance to the screen. The diameter of the strand of hair may then be calculated using eq.1.3.

6. **Measure the spacing of fringe, and width of the central maximum for double slits of unknown dimensions** - Clamp a double slit in the holder. You should observe the combination of diffraction and interference effects in the resulting two slit pattern. For each of the unknown double slits, measure the spacing of the interference fringe, and the width of the central maximum of the diffraction envelope. Record the distance between the slits and the screen.

7. **Measure diffraction patterns produced by diffraction gratings** Place the diffraction grating having 80 lines/mm in the holder. You should observe spots of light spreading horizontally across the wall. The central spot corresponds to the zero order (m = 0) principle maximum, the next spot is the first order (m = +1 on one side, m = -1 on the other) and so on. Measure the distance train the m^{th} order to the center for as many orders as possible. Record the distance from the grating to the wall. Repeat this measurement using the unknown grating.

8. **Measure the wavelength of the laser using a ruler** This is somewhat more difficult than measuring the diameter of a hair with a ruler. In fact, you will require two rulers. Lay a steel rule with mm rulings on the table. Tilt the laser so that the beam is spread over 1 to 2 cm across the mm rulings, as shown in fig. ?? You should observe a series of spots spreading vertically across the wall. The ruler, with its mm markings, is acting as a reflection grating. By measuring

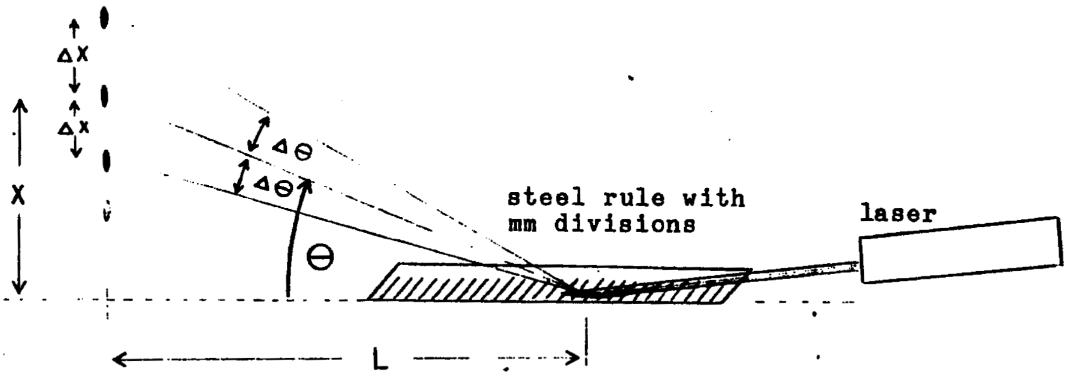

FIGURE 1.14

the angle θ to one of the orders and the spacing of the adjacent order, $\Delta\theta$, we can determine the wavelength of the laser. Choose an order that is a distance x

= 20 cm up the wall. Measure the distance x and the spacing of the adjacent orders, Δx. Record the distance L from the area at which the laser beam strikes the ruler to the wall. You must be quite careful in your measurements of x and Δx to achieve reasonable accuracy.

9. **Observe the variations in light intensity transmitted by two polarizes as their axis are rotated**- You should observe zero transmitted intensity when the axis are perpendicular and maximum transmitted intensity when they are parallel. Suppose that we have two polarizers with their axis perpendicular, and zero transmitted light intensity. Will placing a third linear polarizer between them, with its axis oriented at some arbitrary angle, cause light to be transmitted? Try this, and record your observation.

10. **Observe the polarization of light reflected from a glass plate**- Place a piece of glass near the window or near a lamp, so that reflected light from the surface of the glass is clearly visible. Observe the reflected light through a polarizer. Do you observe variations in the light intensity as the polarizer is rotated? Find the angle of reflection at which you can completely extinguish the reflected light by turning the polarizer. This is Brewsters angle.

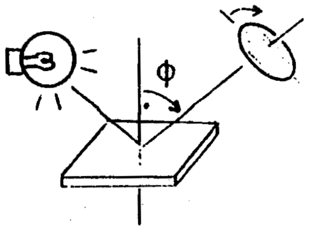

Figure 1.15

11. **Examine a quarter wave plate and a circular polarizer**- These experiments are best conducted using a sodium lamp with a diffusing screen because half wave plates (and even polarizers) may show some wavelength dependence in their effects. Place a quarter wave plate in front of a linear polarizer. Treat the transmitted light using a second linear polarizer. Locate the axis of the quarter wave plate. (When linearly polarized light enters the plate with the polarization vector along an axis, it will emerge linearly polarized and you will be able to obtain zero transmitted intensity by turning the second polarizer to the appropriate orientation). Once you have identified the axis, turn the first polarizer to place the polarization vector of the incoming light at 45^o with

respect to the axis. The light emerging from the quarter wave plate should then be circularly polarized. Rotating the analyser should not cause any change in the transmitted intensity. If the polarization vector of the incoming light is not making an angle of exactly 45^o with respect to the axis (or if the wave plate is not exactly a quarter wave plate, but introduces a phase difference that differs slightly from $\pi/2$) then the light emerging from the wave plate will be elliptically polarized. Rotating the analyser will then cause variations in the transmitted light intensity.

A circular polarizer is made by cementing together a linear polarizer and a quarter wave plate. To use this arrangement as a polarizer (to create circularly polarized from unpolarized light) the unpolarized light must enter from the polarizer side, and not from the wave plate side. If unpolarized light enters from the wave plate side, then this arrangement will act as a linear polarizer, since the wave plate has no observable effect on unpolarized light. Place your circular polarizer in front of the light source and test the transmitted light using a linear polarizer. With the circular polarizer in the correct orientation, you should find that rotating the analyser produce no variations in the light intensity. If the circular polarizer seems to act as a linear polarizer, then you have it flipped over.

To use the circular polarizer as an analyser, light must enter from the wave plate side. The circular polarizer used in this tab is a left hand circular polarizer. Left hand circularly polarized light will be transmitted, while right hand circularly polarized light, entering from the wave plate side, will be blocked by this polarizer. Note that rotating circular polarizers will, have no effect on their operation (unless they are illuminated by linearly or elliptically polarized light). Use one of your circular polarizers to produce left hand circularly polarized light. Use the second circular polarizer as an analyser. You should find that light is transmitted by this arrangement, and that rotating the circular polarizers has no effect on the transmitted intensity. Place a half wave plate between the two circular

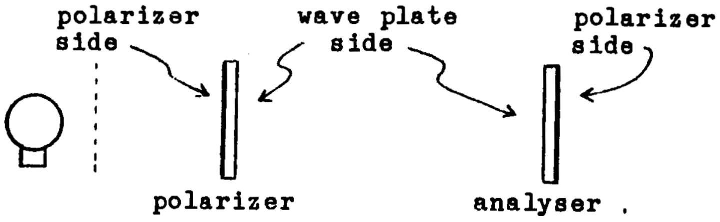

FIGURE 1.16

polarizers. What do you observe? What effect does the half wave plate have on left hand circularly polarized light?

12. **Use a polarizer to check for polarization of light from the sky**. If the sky is clear, go outside and observe various parts of the sky through a polarizer. You should find that light from the sky is partially polarized, and that this effect is maximum at an observation angle that is 90^o from the angle of the sun. You know that the blue color of the sky is due to the fact that we are observing sunlight scattered by fine particles. The scattered tight is also partially polarized.

13. **Observe birefringence in calcite**- Make a dot on a piece of paper and view the dot through a calcite crystal. You should observe two images, one due to the E ray and one due to the wave. As you rotate the crystal, one image should remain fixed, while the other revolves around it. The fixed image is due to the ray. Place a polarizer between the crystal and your eye. What happens as you rotate the polarizer? .

14. **Observe the photo-elastic effect, optical activity and the effects of birefringence on polarized light**- Set up two polarizers with a diffuse light source behind them. (A well illuminated, white piece of paper will work fine). Attach a C clamp or a rod clamp to a small piece of plastic. Place the piece of plastic between the two polarizers and rotate one of the polarizers to make the lines of stress in the plastic visible. Vary the tension of the clamp and observe the changing stress patterns. These patterns are due to birefringence induced by mechanical strain. This effect can often be observed in pieces of cast plastic and in cellophane. Place several pieces of cellophane tape across a piece of glass and place it between the polarizers. A similar effect can be observed in a wrinkled piece of cellophane from a cellophane bag. Place a thin flake of mica between the polarizers. Mica is a birefringent crystal. Orient the polarizers vertically with the light source below and place a test tube containing sugar (sucrose) dissolved

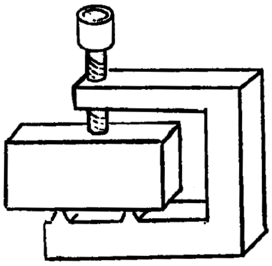

Figure 1.17

in water between the polarizers. (The solution should be saturated with sucrose, and should be at least 8cm in depth). As you slowly rotate one polarizer, note the color of the transmitted light. This effect is due to optical activity. (Sucrose and many other molecules come in two form, a right-handed form and a mirror image left-handed form. These two forms of the same molecule rotate polarized light in opposite directions. Molecules from biological sources, such as the sugar extracted from sugar cane, are always the right handed version. If our solution contained equal amounts of right and left handed sucrose molecules no net optical activity would be observed). Which direction is the plane of polarization being rotated by your sugar solution? Which wavelengths are being rotated through a larger angle? (Remember, the color that you see is the complement of the color being blocked by the polarizer).

1.6 Calculations

1. From the data obtained in part 4, calculate the widths of the three unknown single slits.
2. Calculate e width of the strand of hair from the diffraction pattern observed in part 5. This pattern is described by eq. 1 so you may use eq. 1.3 to find w, which in this case is the width of the barrier rather than the width of the slit. You need not find the uncertainty in this result.
3. Calculate (i) the separation of the slits and (ii) the slit width for each of the unknown double silts examined in part 6.
4. From your measurements on the principle maxima of the 80 ruling/mm grating, calculate the sine of the angle at which each principle maximum is observed. Plot a graph of $sin\theta_m$ v.s. the order m. If your data includes orders on both sides of the central maximum then your graph should include both positive and negative values of $sin\theta_m$ and m. Determine the wavelength of the laser from the slope of this graph.
5. Calculate the number of rulings/mm for the unknown grating. Use the value of for the laser that is given in the lab sheet.
6. Calculate the wavelength of the laser from your measurements of part 8, in which a steel ruler was used as a reflection grating. The equation governing the reflection grating is:

$$\pi\lambda = h(sin\theta + sin\phi)$$

where ϕ is the angle of incidence.
For two successive diffracted orders at angles θ_1 and θ_2, we have:

$$\begin{aligned}(n+1)\lambda &= h(sin\theta_1 + sin\phi)\\ n\lambda &= h(sin\theta_2 + sin\phi)\\ \lambda &= h(sin\theta_1 - sin\theta_2)\\ \lambda &= hcos\theta\Delta\theta\end{aligned}$$

Use the above expression to calculate λ from the values of θ and $\Delta\theta$ measured in part 8. (Average the two values of $\Delta\theta$). You need not find the uncertainty in this result and you should not be disappointed if your result ta not very close to the expected value since this is a relatively crude measurement.

1.7 Questions

1. When two linear polarizers are crossed (axes perpendicular) so that no light is transmitted, can introducing a third linear polarizer between them cause light to be transmitted?
2. In which direction did the sugar solution rotate the plane et polarized light? Which wavelengths were rotated through a larger angle?
3. What effect does a half wave plate have on left hand circularly polarized light? .

EXPERIMENT 2

Harmonic Analysis

2.1 Objectives

1. To determine the frequencies and amplitudes of the fourier components of a square wave, or any other periodic waveform.

2.2 References

2.3 Preliminary Exercises

1. What is an odd function? What is an even function? Give examples of each.
2. Show that equations 2.1 and 2.2 are equivalent and find expressions for the constants C_n, and ϕ_n in terms of the constants A_n, B_n.
3. Derive equation 2.7 starting from equation 2.6.
4. On the interval $-T/2 \leq t \leq T/2$, the expression for a square wave of period T is:

$$V(t) = \begin{cases} -V_0 & \text{if } -T/2 < t < 0 \\ +V_0 & \text{if } 0 < t < T/2 \end{cases}$$

(a) Derive the fourier series expression for a square wave of amplitude V_0 and frequency F = $1/T$.

(b) Calculate the amplitude and frequencies of the first five fourier components for a square wave of amplitude $V_0 = 1V$ and frequency F = 600 Hz.

2.4 Theory

2.4.1 The Fourier Series

Fourier's theorem states that any periodic waveform is equivalent to a sum of sinusoidal waves. These sinusoidal waves are called the Fourier components or harmonic components of the waveform. The frequency of the waveform, ω_0 , is called the fundamental frequency. The harmonic components will occur at frequencies ω_n = nω_0. , which are integer multiples of the fundamental frequency.

Let F(t) be some periodic function with angular frequency $\omega_0$and period T.

Fouriers theorem states that we can write:

$$F(t) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n sin(n\omega_0 t) \tag{2.1}$$

The additive constant $\frac{1}{2}A_0$ may be written as the n=0 term of the cos series in some texts. (The factor of $\frac{1}{2}$ is included to simplify other equations). For each value of n, we obtain sin and cos terms with frequency ω_n = nω_0 . We can write the fourier series in the form

$$F(t) = \frac{1}{2}C_0 + \sum_{n=1}^{\infty} C_n cos(\omega t + \phi_n) \tag{2.2}$$

or

$$F(t) = \frac{1}{2}C_0 + \sum_{n=1}^{\infty} C_n sin(\omega t + \theta_n) \tag{2.3}$$

Equations 2.1,2.2, 2.3 are equivalent and express the fact that a complex waveform may be regarded as a superposition of sinusoidal waves. For our purposes, It will, be convenient to use equation 2.1.

To find the coefficients A and B in the Fourier series, we make use of the following orthogonality relations:

$$\frac{1}{L}\int_{-L}^{L} cos(\frac{m\pi}{L}t)cos(\frac{n\pi}{L}t)dt \quad = \quad \begin{cases} 1 & \text{if n = m} \\ 0 & \text{if } n \neq m \end{cases}$$

$$\frac{1}{L}\int_{-L}^{L} cos(\frac{m\pi}{L}t)sin(\frac{n\pi}{L}t)dt \quad = \quad 0$$

$$\frac{1}{L}\int_{-L}^{L} sin(\frac{m\pi}{L}t)sin(\frac{n\pi}{L}t)dt \quad = \quad \begin{cases} 1 & \text{if n = m} \\ 0 & \text{if } n \neq m \end{cases}$$

for n, m nonzero integers.

The functions $sin(\frac{n\pi}{L}t)$ and $cos(\frac{n\pi}{L}t)$ are said to be orthogonal on the interval $[L, L]$. We will take $L = T/2$.

We start with the equation 2.1 Multiplying both sides by $cos(m\omega_0 t)$, where n can be any nonzero integer, and integrating the resulting expression from $t = -T/2$ to $t = T/2$, we obtain:

$$\begin{aligned} &\int_{-T/2}^{T/2} F(t)cos(m\omega_0 t)dt \\ &\quad = \frac{1}{2}A_0\int_{-T/2}^{T/2} cos(m\omega_0 t)dt \\ &\quad + \sum_{n=1}^{\infty} A_n \int_{-T/2}^{T/2} cos(m\omega_0 t)cos(n\omega_0 t)dt \\ &\quad + \sum_{n=1}^{\infty} B_n \int_{-T/2}^{T/2} cos(m\omega_0 t)sin(n\omega_0 t)dt \end{aligned}$$

The first integral on the right hand side is zero. Using the orthogonality relations with $L = T/2$ and $T = \frac{2\pi}{\omega}$, we see that all of the integrals in the third summation will be zero. All of the integrals in the second summation will be zero except in the term with n = m.

$$\begin{aligned}\int_{-T/2}^{T/2} F(t)cos(m\omega_0 t)dt &= A_m(T/2) \\ A_m &= \frac{2}{T}\int_{-T/2}^{T/2} F(t)cos(m\omega_0 t)dt\end{aligned}$$

Similarly, if we multiply equation 2.1 by $sin(m\omega_0 t)$, and integrating the resulting expression from $t = -T/2$ to $t = T/2$, we find:

$$\begin{aligned}\int_{-T/2}^{T/2} F(t)sin(m\omega_0 t)dt &= B_m(T/2) \\ B_m &= \frac{2}{T}\int_{-T/2}^{T/2} F(t)sin(m\omega_0 t)dt\end{aligned}$$

Finally, if we simply integrate equation 2.1 from $t = -T/2$ to $t = T/2$, we find .

$$A_0 = \frac{2}{T}\int_{-T/2}^{T/2} F(t)dt$$

In summary, we have:

$$A_m = \frac{2}{T}\int_{-T/2}^{T/2} F(t)cos(m\omega_0 t)dt \tag{2.4}$$

$$B_m = \frac{2}{T}\int_{-T/2}^{T/2} F(t)sin(m\omega_0 t)dt \tag{2.5}$$

Equations 2.4 and 2.5 enable us to calculate the coefficients A_n and B_n for any function F(t) as in eq. 2.1

2.4.2 Example

Consider a triangular wave with amplitude V_0 and period T. With the choice of origin as indicated below we have:

$$F(t) = \begin{cases} (\frac{4V_0}{T})t + V_0 & \text{if } -T/2 < t < 0 \\ -(\frac{4V_0}{T})t + V_0 & \text{if } 0 < t < T/2 \end{cases}$$

For this function F(t), equations 2.4 and 2.5 yield

$$\begin{aligned}
A_0 &= \frac{2}{T}[\int_{-T/2}^{0}(\frac{4V_0}{T}t+V_0)dt+\int_{0}^{T/2}(\frac{-4V_0}{T}t+V_0)dt] \\
A_n &= \frac{2}{T}[\int_{-T/2}^{0}(\frac{4V_0}{T}t+V_0)cos(n\omega_0 t)dt+\int_{0}^{T/2}(\frac{-4V_0}{T}t+V_0)cos(n\omega_0 t)dt] \\
B_n &= \frac{2}{T}[\int_{-T/2}^{0}(\frac{4V_0}{T}t+V_0)sin(n\omega_0 t)dt+\int_{0}^{T/2}(\frac{-4V_0}{T}t+V_0)sin(n\omega_0 t)dt]
\end{aligned}$$

Evaluating these integrals, we find

$$\begin{aligned}
A_0 &= 0 \\
A_n &= \begin{cases} 0 & \text{if n = even} \\ \frac{8V_0}{n^2\pi^2} & \text{if n = odd} \end{cases} \\
B_n &= 0
\end{aligned}$$

With these values for the coefficients, we can write the Fourier series expansion for the triangular wave:

$$\begin{aligned}
F(t) &= \sum_{n=odd}^{\infty}\frac{8V_0}{n^2\pi^2}cos(n\omega_0 t) \\
&= \frac{8V_0}{\pi^2}cos(\omega_0 t)+\frac{8V_0}{n9\pi^2}cos(3\omega_0 t)+\frac{8V_0}{25\pi^2}cos(5\omega_0 t)
\end{aligned}$$

Although the Fourier series is an infinite series, we can usually obtain a good approximation to the waveform by taking only the first few terms. For example, in our Fourier series expansion for the triangular wave, the coefficients contain a factor of $\frac{1}{m^2}$, so the amplitude of the higher order terms will be small. The effect of dropping all but the first three terms in the triangular wave expansion is shown in figure 2.1. Here, we have plotted the first three terms and their sum.

In general, a more complex waveform will require more terms in the expansion to obtain an accurate approximation. Also, waveforms with sharp corners or discontinuities (such as the square wave) will require more terms for a good approximation.

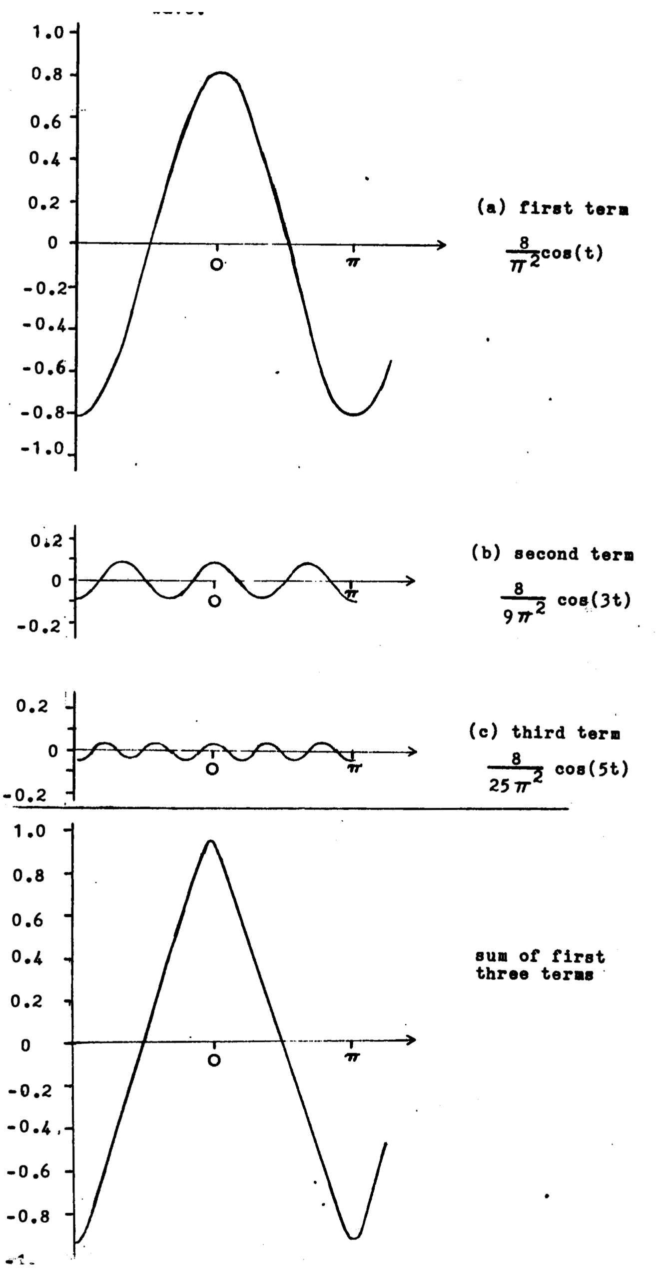

FIGURE 2.1: Fourier series expansion for a triangle

2.4.3 Beats

In order to measure the amplitudes and frequencies of the Fourier components of the waveform, we will use the phenomenon of beats. Consider the superposition of two sinusoidal waves, one with frequency ω_n and the other with frequency ω. We will assume that the waves have some phase difference and that both have amplitude A.

$$S(t) \quad = \quad Asin(\omega_n t + \phi_n) + Asin(\omega t)$$

Using the trigonometric identity

$$sina + sinb \quad = \quad 2sin(\frac{a+b}{2})cos(\frac{a-b}{2})$$

we obtain

$$S(t) \quad = \quad 2Acos(\frac{\omega_n - \omega}{2}t + \frac{\phi_n}{2})sin(\frac{\omega_n - \omega}{2}t + \frac{\phi_n}{2})$$

The sum of two sinusoidal waves of equal magnitude is equivalent to a product of two sinusoidal waves, one with frequency $\frac{\omega_n - \omega}{2}$ and the other with frequency $\frac{\omega_n + \omega}{2}$. In general, this will be some complicated waveform; however, consider the case in which ω_n and ω differ by a relatively small amount. The term $cos(\frac{\omega_n - \omega}{2}t + \frac{\phi_n}{2})$ will be a low frequency oscillation, while the tern $sin(\frac{\omega_n + \omega}{2}t + \frac{\phi_n}{2})$ will be a high frequency oscillation. Their product is shown in figure 2.2. The result is a rapidly oscillating function with relatively slow variations in the overall amplitude. These low frequency variations in the amplitude are called beats. As the difference between ω_n and ω becomes small, the period of the beats becomes long.

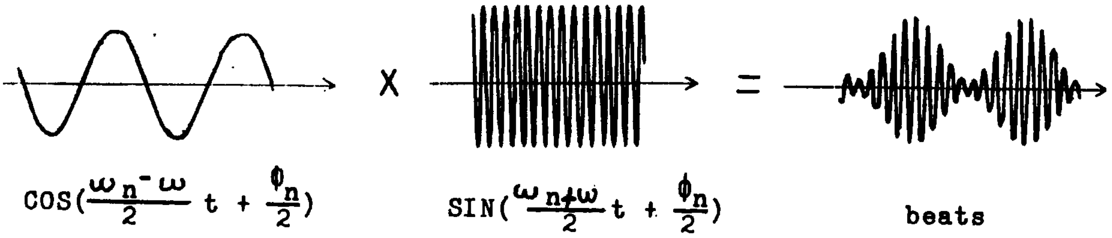

FIGURE 2.2

2.5 Apparatus

The frequencies and amplitudes of the Fourier components of a wave form are measured by mixing the waveform with a ”standard” sinusoidal signal of known frequency ω . The frequency of the standard signal is varied until a beat with one of the Fourier components of the waveform is obtained. The frequency of the standard signal is carefully adjusted to make the period of the beat very long. At this point, we know that $\omega_n \approx \omega$. The amplitude of this Fourier component, A_n, is determined by measuring the amplitude of the beat. By starting ω at some low value (below the fundamental frequency) and increasing it, we can obtain beats with each successive Fourier component of the waveform. Mixing of the waveform and the standard signal and detection of the beats is accomplished using the circuit shown in Figure. 2.3. First, we discuss

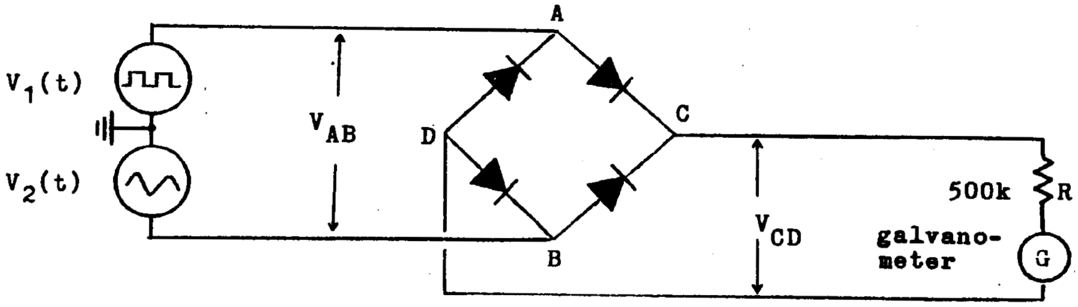

FIGURE 2.3

the operation of the diode bridge rectifier. Recall that a diode will only conduct when a positive potential difference is applied. If a potential difference is applied in the reverse direction, no current will flow; the diode behaves like an open circuit. Consider the bridge rectifier. When $V_A > V_B$, D_2 and D_4 conduct. D_1 and D_3 do not conduct. In effect, they are open circuits. Therefore, if $V_{AB} > 0$, ,then $V_{CD} = V_{AB}$

$V_B > V_A$, D_1 and D_3 conduct. D_2 and D_4 do not conduct. In effect, they are open circuits. Therefore, if $V_{AB} < 0$, ,then $V_{CD} = -V_{AB}$

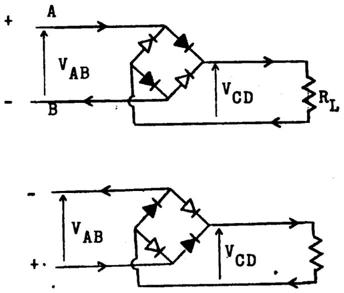

FIGURE 2.4

Current flows through the load in the same direction, regardless of the sign of the input voltage. In effect, the fullwave rectifier takes the absolute value of the input voltage. The action of a full-wave rectifier on a sinusoidal input voltage is shown in figure 2.5.

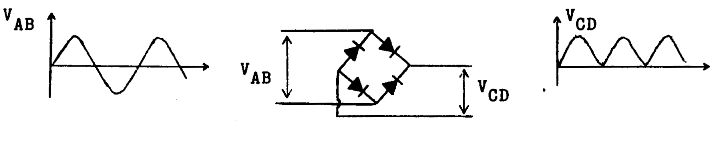

FIGURE 2.5

The circuit also includes a galvanometer, which is a very sensitive DC current meter. We do not actually require an instrument with very high sensitivity in this experiment, so resistor R is included to limit the current flowing through the galvanometer. The galvanometer uses a bean of light reflected from a mirror, rather than a pointer. The mirror is connected to a critically damped spring (which returns it to zero whey current flows). This means that the galvanometer can only to slowly varying currents. In effect, the galvanometer averages out rapidly varying currents (frequency more than a few Hz).

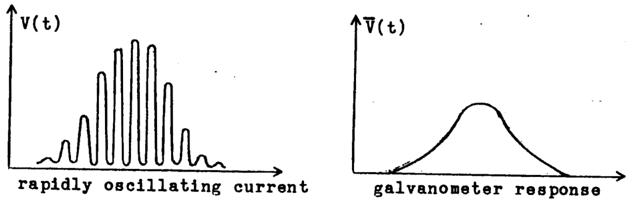

FIGURE 2.6

This makes the galvanometer useful for detecting beats, since it responds only to the low frequency variations in the overall amplitude of the signal. Returning to the circuit of figure 2.3, signal generator S_1 provides the waveform to be analysed; in this case, a square wave of frequency ω_0. Signal generator S_2 provides the sine wave of frequency ω. The voltage applied to the bridge, V_{AB}, is:

$$V_{AB} = V_A - V_B = V_1(t) - V_2(t)$$

Expressing $V_1(t)$ as a Fourier series, we have:

$$\begin{aligned} V_1(t) &= \frac{1}{2}C_0 + \sum_{n=1}^{\infty} C_n sin(\omega_n t + \phi_n) \\ \omega_n &= n\omega_0 \\ V_2(t) &= Asin(\omega t) \end{aligned}$$

$$V_{AB} = \frac{1}{2}C_0 + \sum_{n=1}^{\infty} C_n sin(\omega_n t + \phi_n) - Asin(\omega t) \tag{2.6}$$

in general, this will be a complicated, rapidly oscillating wave form. If the signal frequency ω is not close to any of the harmonic frequencies ω_n, we will not have any slow beats. The rectifier will produce a rapidly oscillating, positivegoing signal. Because the signal is oscillating rapidly, however, the galvanoiaeter will "see" a constant DC level, corresponding to the average of the signal.

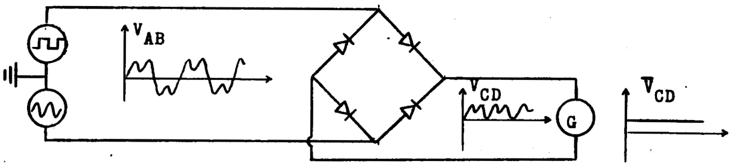

FIGURE 2.7

Now, suppose that we vary the sine wave frequency ω until it becomes near one of the harmonic frequencies, ω_m. We can rewrite equation 2.6 as:

$$\begin{aligned}
V_{AB} &= \frac{1}{2}C_0 + \sum_{n=1,n\neq m}^{\infty} C_n sin(\omega_n t + \phi_n) \\
&- (A - C_m)sin(\omega t) + C_m sin(\omega t) + C_m sin(\omega_m t + \phi_m) \\
&= \frac{1}{2}C_0 + \sum_{n=1,n\neq m}^{\infty} C_n sin(\omega_n t + \phi_n) - (A - C_m)sin(\omega t) \\
&+ C_m sin(\omega t) + C_m sin(\omega_m t + \phi_m) \\
&+ 2C_m cos(\frac{\omega_m - \omega}{2}t + \frac{\phi_m}{2})sin(\frac{\omega_m + \omega}{2}t + \frac{\phi_m}{2})
\end{aligned} \tag{2.7}$$

We have separated the n = m term from the series, and have divided the sinusoidal, signal into a part with amplitude $(A - C_m)$ and a part with amplitude C_m. We will obtain beats between these two signals. if the difference between ω and ω_m is small, the beats will, have a long period. Our signal will consist of the beat signal, plus rapidly oscillating signals corresponding to the first two terms

The slow response of the galvanometer averages out the rapidly oscillating part of the signal. We will, obtain a DC level, plus a slow variation in amplitude due to the beats. The signal that we finally detect is shown below. In order to observe the beats, the period of the beats must be at least 0.2 s. For an accurate measurement of the amplitude of the beats, the period must be much longer (2 s) to allow the relatively slow galvanometer to follow the signal. The generator frequency, gives us the frequency of this particular harmonic. From equation 2.7, we see that the amplitude of the beats is thus we obtain both the amplitude and the frequency of this harmonic component of the waveform.

The zero getting of the galvanometer is arbitrary, so in the actual experiment, the no-beat DC level can be set at zero. The beats will then appear as oscillations swinging

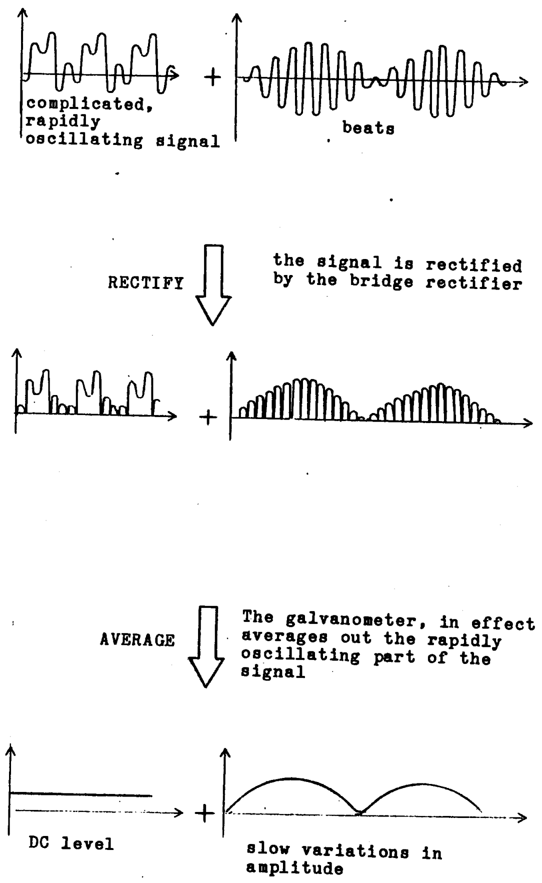

FIGURE 2.8

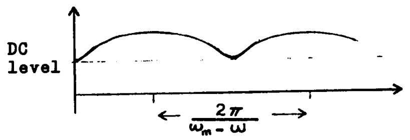

FIGURE 2.9

to either side of the zero. You might wonder why the rectifier is required. The answer is simple; without it, the signal would swing negative as often as it swings positive and the average would be zero. We would not observe the beats.

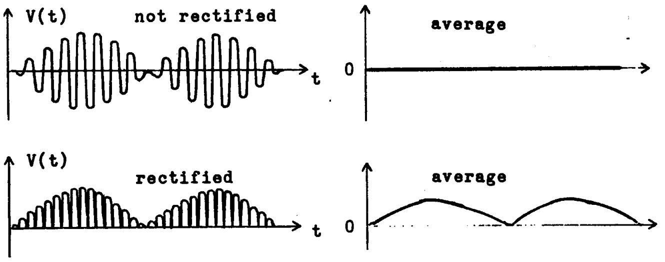

FIGURE 2.10

2.6 Procedure

1. Connect the circuit as shown in figure 2.3. Signal generator S_1 should be set to give a square wave at a frequency of approximately 60h Hz. Signal generator S_2 be used to provide the sine wave. Check the signals on an oscilloscope.

2. Set the galvanometer on the x.03 scale. Set the signal generators for a peak amplitude of $\approx$ 3V. Tune S_2 to obtain beats st the fundamental frequency (the frequency of the square wave). The period of the beats should be 2 sec.

3. Adjust the amplitudes of the two signals so that the galvanometer spot swings through almost the full scale. Keep the amplitudes of the two signals approximately equal.

4. Record the amplitude of the oscillation and the frequency of the sine wave. Measure the sine wave frequency from the oscilloscope trace. Do not rely on the dial setting of the generator, it is only approximate.

5. Increase the frequency of the sine wave. Check for beats at integral multiples of $F_0 = \frac{\omega_0}{2\pi}$. When a beat is located, carefully tune S_2 to make the bear frequency as small as possible. This will require some care. it is very important to have a long period for the beats in order to obtain an accurate value of C_m

6. Perform this measurement for as many harmonics as possible (at least 5 or 6).

2.7 Calculations

1. Calculate the Fourier series coefficients for the square wave. You should find that all A_n are zero and that the B_n; are zero for even n.

2. Compare the observed frequencies of the harmonic frequencies with the frequencies expected from the calculation performed above.

3. Divide the beat amplitudes by the amplitude of the beat at the fundamental frequency to obtain the ratio $\frac{A_2}{A_1}$, $\frac{A_4}{A_1}$,$\frac{A_3}{A_1}$ Compare these with the ratios expected from you calculation.

EXPERIMENT 3

The Millikan oil drop

3.1 Objectives

1. To determine the charge of a single electron by measuring the charges of several oil droplets.
2. To demonstrate the quantization of charge.

3.2 References

3.3 Preliminary Exercises

1. Estimate the time required for an oil droplet to reach its terminal velocity and the distance that it would fall before reaching the terminal velocity. The oil droplets observed in this experiment typically have a radius $R \approx 1O^{-4}cm$. The viscosity of air at 250c is approximately 1.8×10^{-4} poise. Assume that the density of oil is $1g/cm^3$
2. What effect could the finite size of the conducting plates and the hole in the upper plate have on the results of this experiment?

3. Why is oil, rather than same other fluid such as water or alcohol used in this experiment?

3.4 Theory

A droplet of oil moving through air may pick up a small, static charge due to friction. This static charge may result from the gain or toss of only a few electrons. To measure the charge on an . oil droplet, we allow it to fall (through a small hole) into the space between two parallel conducting plates (see figures 3.1 and 3.2).

A potential. difference V is applied across the plates. If the diameter of the plates is large compared to their separation d, then an approximately uniform electric field $E = V/d$ will be established between the plates. A droplet having charge q will experience a force $F = qE$. The potential difference V can be adjusted so that the electric force balances the gravitational force on the droplet. The droplet will, then be suspended between the plates. If no net forces are acting on the droplet, it could move with a constant velocity. Once the electric and gravitational forces are balanced, however, friction with the air will bring the droplet to rest). Let V_b be the value of the potential at which balance is achieved. The condition on the forces is:

$$q(\frac{V_b}{d}) = \frac{4}{3}\pi R^3(\rho - \sigma)g \tag{3.1}$$

where

ρ	density of the oil
σ	density of air
R	radius of the droplet
g	gravitational acceleration

The quantity $\frac{4}{3}\pi R^3(\rho - \sigma)g$ is the effective weight of the droplet, which takes into account the buoyancy of the droplet in air. (Recall Archimedes principle; a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body).

The direction of the electric force depends o the sign of q and the polarity of the potential difference applied to the plates. The potential V must be applied in the correct polarity to make the electric force oppose the gravitational force. In the experiment, a reversing switch is placed between the voltage source and the plates to make reversing the direction of the electric field easy.

In order to find the charge q using equation 3.1, the radius of the droplet, R must be determined. This is done by measuring the free-fall velocity of the droplet and applying Stokes law. A sphere falling through a medium experiences a viscous drag (frictional force) equal to $6\pi nRv$, where v is the velocity of the sphere relative to the medium and n is the viscosity of the medium, which in this case is air. Since the frictional force is proportional to the velocity of the droplet, the droplet will eventually reach a constant velocity u, which we call, the terminal velocity. The terminal velocity is reached when frictional force balances the gravitational force.

$$\frac{4}{3}\pi R^3(\rho - \sigma)g = 6\pi nRu \tag{3.2}$$

The droplets reach the terminal velocity within a very short period of time. In the experiment, the terminal velocity is determined by measuring the time taken for a droplet to fall through a known distance with the electric field off. Using the measured value of u in equation 3.2, we can solve for the radius of the droplet. Substituting this value for R into equation 3.1, we can determine q, the charge on the droplet.

in conclusion, the charge on an oil droplet can be determined by measuring the potential difference required to balance the droplet, V_b, and the terminal velocity of the droplet in freefall, u. The plate separation, d, and the density of the oil ρ must also be known. The values will be provided by the instructor. The viscosity of air may be obtained from reference 1 or from standard table of physical and chemical data. The viscosity of air depends on temperature, so the temperature of the room should be recorded when the experiment is performed.

In the references, you may find slightly different procedures for performing this experiment. For example, rather than measuring the potential required to balance the droplet, you could apply a potential greater than V_b and measure the terminal velocity of the droplet as it moves upward. The basic ideas of the experiment are the same.

3.5 apparatus

A diagram of the apparatus is shown in figure 3.1 a schematic diagram of the apparatus is shown figure 3.2. The major comments are:

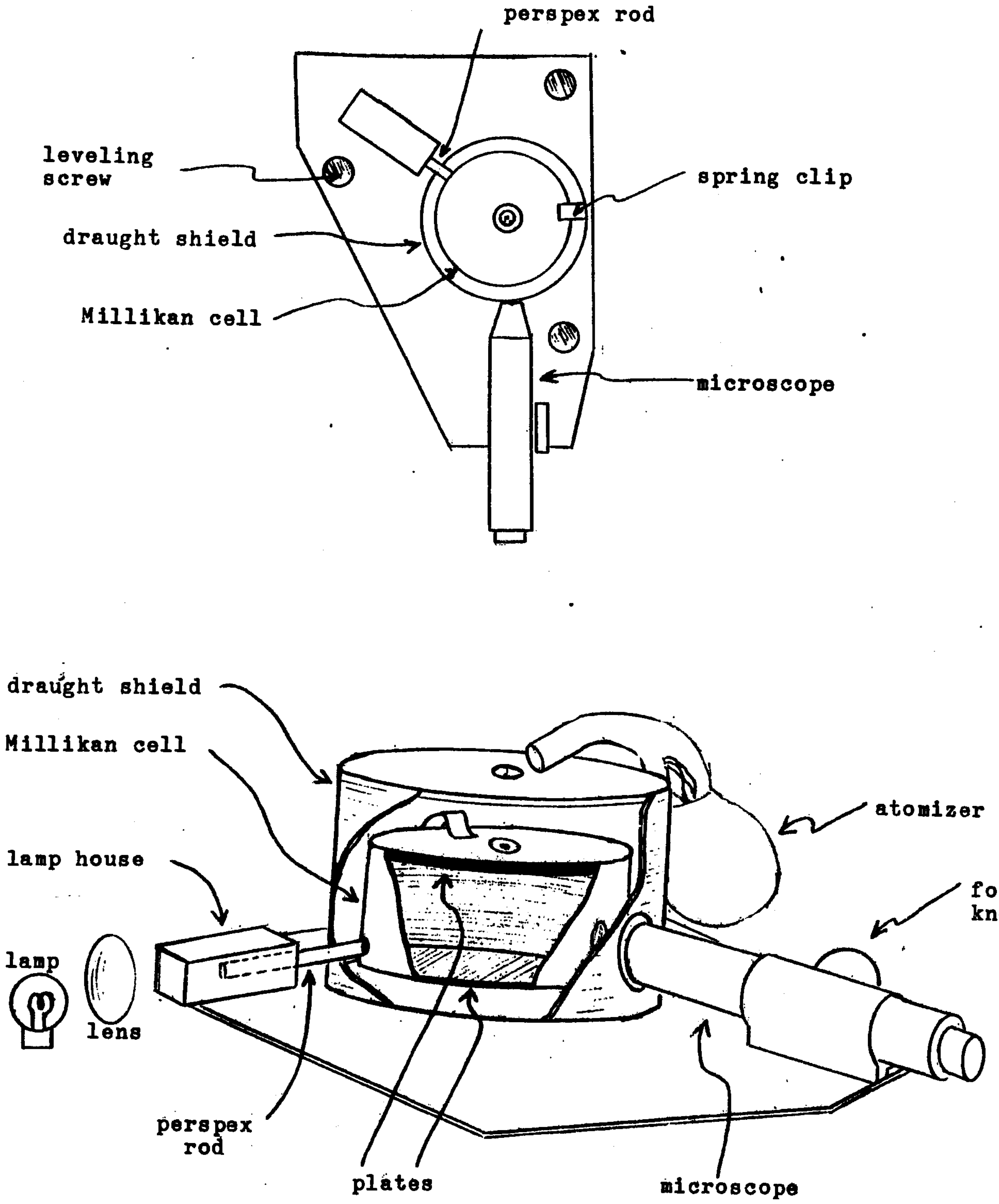

Figure 3.1

1. Millikan Cell - The cell is made up of parallel conducting disks separated by an insulating cylinder. The plates are connected to a DC power supply through a potentiometer and a reversing switch. The upper plate has e hole Ln the center to allow oil droplets to fall into the cell. Inside of the cell is blackened to reduce reflected light

2. A light source is needed to illuminate the droplets so that they can be observed. Light is directed into the cell by a Perspex rod. This arrangement prevents the lamp from heating the cell and causing air currents. A small lamp (6.3V lens fronted bulb) is provided with the apparatus, however, we find that this lamp is usually inadequate. Better illumination is obtained by using an external 12V/24W bulb with a lens to focus the onto the end of the perspex

3. Microscope - A microscope is used co observe the oil The microscope contains a ruled scale (graticule) which used to the distance that the droplets (to determine their velocity). The physical distance between the lines in the Image of the graticule depends on the focal length of the microscope, which change if the eyepiece moves in or out of the microscope tube. With the eyepiece in its present position, the scale is 0.43 ± 0.01 between the major lines of the grid. This calibration should be checked, either by you or by the instructor, the experiment performed. To focus the microscope on a droplet the entire microscope tube Is by using the "focus" knob.

These components are on a plate which is fitted with leveling screws. The cell must be level, since otherwise the electric field not be parallel to the gravitational force. The cell is surrounded by a plastic "draught shield" to reduce currents. A digital voltmeter should be used to the potential difference between the plates.

A glass jet rubber atomizer (spray bottle) is used to produce the oil droplets. A small wire probe, which can be inserted in the hole in the top Of the may be used to focus the microscope on the center of the cell to clear the hole if it becomes clogged. You also require a stopwatch and a thermometer.

3.6 Procedure

NOTE: DO POUR OIL INTO THE CELL! Do not ene the atomizer over so far that oil spills into the cell. HANDEL THE ATOMIZER CAREFULLY! breaks easily. Always return the atomizer to Its box not in use.

1. Level the cell: place a level (tilt indicator) on the lid of the cell (directly on the cell, on the plastic lid of the draught shield). Adjust the leveling screws to make the cell level.

2. Electrical Connections : Connect the lamp to a DC power supply capable of delivering 12V, 2A. Connect the cell plates and the potentiometer / reversing switch box to a 300 VDC power supply and a DVM. Check to see that 0-300V can be to the cell plates by varying the potentiomter. When turning on the lamp supply, do not exceed the power / voltage rating of the lamp (usually 24W/12V).

3. Focus the Microscope: insert the probe through the hole in the top of the (set the reversing switch to neutral to avoid shorting the power supply). The eyepiece of the microscope should be slid all the way forward in its tube and the graticule should be sharp focus. Adjust the focus knob of the microscope to bring the Into focus.

4. Adjust Light Source: proper of the light is essential. Line up the lamp and the lens along the axis of the perspex rod. Focus the image of the lamp filament on the end of the rod. Remove the cover of the cell. Adjust the positions of the lamp and the lens to obtain a beam of light (as intense as possible) across the cell. Replace the lid of the cell and with the room lights off, spray some oil droplets. While observing the oil droplets through the microscope, adjust the lamp and the lens for visibility of the droplets. Cover the lamp and the lens foil, in order to reduce stray light in the room, and try to observe the graticule. Some further adjustments of the light source may be necessary to make the graticule You wish to tape a small slip of white paper to the inner of the Millikan cell, directly opposite the perspex rod. This will reflect some light back into the cell, which may improve visibility of the droplets and the graticule. Observing the droplets and the graticule will be easier If your eyes are adjusted to darkness. Try to keep stray light in the room a minimum.

5. Adjust Graticule Inclination: Compare the path of gone falling droplets with the vertical of the graticule. If the graticule is tilted, loosen the retaining ring on the eyepiece tube and rotate the eyepiece tube slightly to make the graticule lines vertical. Note that the eyepiece tube screws into the microscope tube, so rotating the eyepiece tube change the focal length of the microscope and, therefore, the graticule calibration Do not rotate the eyepiece tube by more than a fraction of a turn or you cause a significant change in the graticule calibration. When you have finished, re-tighten the eyepiece retaining ring.

6. Locate charged droplets: Tilt the atomizer slightly and bring the nozzel near the hole in the plastic lid. Squeeze the bulb firmly several times. You should

observe a large number of droplets. Because the microscope gives an inverted image, they appear to be falling up. With the potential set to its maximum value, throw the reversing switch every few seconds and look for droplet that reverse their direction Of notion. Some charged droplets move very quickly, and may be driven out of the field of view if the potential is left in one polarity for too long. The charged droplets are usually the smaller ones; the ones that are the most difficult to see. Be patient charged droplets are often not located until most of the other droplets have drifted of the field of view.

7. Measure the Balance Potential: When a charged droplet is located, reduce the potential until it comes to rest. (You Can vary the potential to move the droplet into a convenient location for viewing). Observe the droplet for at least 1 minute to make sure that you have found the exact value of V required Co balance it. Record V_b

   Alternate Procedure Set the potential difference between the plates for some value V_b and spray a Large number of droplets into the cell. Wait for 5 or 10 minutes any droplets remaining in the field of view after this period of time are charge droplets which balance at the value of V_b set. (Sometimes, very small droplets which fall very slowly may also remain in the field of view). This method easier than actively looking for charged droplets, and may give V_b more accurately on how long you wait for the field of vier to clear. If you start with enough droplets, you will almost always find one charged droplet remaining In the field of view.

8. Measure the Terminal velocity (3 times): Move the droplet into the lower part of the field of view (by increasing the potential), then turn the reversing switch to neutral. Measure the time required for the droplet to fall through two or three graticule spaces. Turn the potential back on and bring the droplet back to the tower part of the field of view. Repeat the measurement. Measure the free fall velocity three times for each droplet. Note that the droplet should be in sharp focus during the velocity measurement, otherwise paralax will cause the graticule calibration to be incorrect.

9. Repeat these measurements for at least 5 or 6 different droplets. Droplets with large values of V_b (which are the larger droplet.) tend to give the best results. This is probably due to the fact that deviations from Stokes Law become significant for very small droplets.

3.7 Trouble Shooting

1. No Droplets are Observed: The small hole in the lid of the cell may be plugged with oil. Consult your instructor for the cleaning procedure. The microscope may not be focused near the center of the cell (use the wire probe to check this) or the light source my be very poorly adjusted.

2. Few Droplets are Observed: The light source is probably not adjusted properly. Move the lamp and / or the lens while observing the droplets through the microscope.

3. No Charged Droplets: If many droplets are visible, but none seen to be charged, check to make sure that a potential difference is being applied to the plates of the cell. Use the DVM to check the potential difference across the plates directly. Make sure that the spring clip is making good contact with the lid of the cell. Remember that the charged droplets that can be balanced are the smaller ones; the droplets that are the most difficult to see. Improper adjustment of the light source may make the charged droplets difficult to detect.

3.8 Calculations

1. Look up the viscosity of air, n, for the temperature at which the experiment was performed. Pay attention to the units of n. (The poise is a cgs unit). This value of n will be referred to as the uncorrected viscosity n_0.

2. There ere deviations from stokes' law for very small objects (dimensions on the order of the mean free path of the molecules of the medium). The viscosity n appearing in equation 3.2 should be the corrected viscosity; .

$$n = \frac{n_0}{1 + \frac{b}{PR}} \tag{3.3}$$

where

n_0	uncorrected viscosity
b	$6.17 \times 10^4 cm^2$
P	barometric pressure (typically 690 cm, at our altitude)
R	Radius of the droplet

Substitute for n in equation 3.2 using equation 3.3 and solve for R. (You will have a quadratic equation in R. The correct root will be obvious). Use this equation, with the average terminal velocity for each droplet, to calculate the radius of each droplet.

3. Calculate the charge on each droplet using equation 3.1. Assume that e is roughly 1.6×10^{-19}C and determine the number of charges on each droplet. This must, of course, be an integer. It is quite likely that one or more of your droplets will have a single unit of charge. Divide the charge on each droplet by the number of charges on each droplet to determine a value of e from each droplet. Present the results of these calculations in a table giving R, q, number of charges and value of e for each droplet.

4. Average the values of e from each droplet to obtain your reported value for e. Determining the uncertainty in this value of e is complicated by the fact that two types of uncertainty are present:

   (a) Random errors - in determining V_b, and the free fall time, for example.

   (b) Unknown systematic errors - The uncertainties in the plate separation, d, and the graticule calibration, for example.

The Standard deviation of the mean value of e, σ_e , represents the uncertainty in e due to the random errors. To find the uncertainty in e due to the unknown systematic errors, treat them as random errors and use propagation of errors. The contributions to Δe due to unknown systematic errors will not be affected by averaging since these errors always act in the same direction. Assume that the uncertainty in $\bar{e}$ is the sum of the uncertainties due to random errors (found from the standard deviation of the mean) and the unknown systematic errors (found using propagation of errors). This procedure can not be justified in a rigorous way, but it should give a reasonable estimate of the overall uncertainty.

3.9 Questions

1. Do the values of e obtained from droplets having a larger radius seem to differ significantly from the values of e obtained from the smaller droplets? Do the larger droplets tend to give values of e that are closer to the accepted value?

2. For the largest droplet and the smallest droplet; calculate the charge-on the droplet using the uncorrected value of n in equation 3.2. What is the percentage change In q due to the correction to Stokes law in each of these cases?

3.10 Calibrating the Graticule

One way to calibrate the graticule is to remove the microscope f rom the base of the Millikan apparatus and clamp it securely to a vertical rod. Clamp a micrometer in front of the microscope, so that the travel of the micrometer shaft can be observed. The microscope should be focused on the forward edge of the shaft. The shaft should move parallel to the graticule lines and should remain in the plane of focus as fc moves across the graticule pattern. Measure the distance traveled by the shaft when it moves from one edge of the graticule pattern to the other. Repeat this measurement several times, focusing on different parts of the edge of the shaft. When you replace the microscope, make sure that it is pointed at the region below the hole in the center of the lid. Note that several small washers are used to shim the microscope in order to achieve the proper inclination. If a micrometer only, not attached to an arm and base shaft, is available, then the calibration can be carried out without removing the microscope from the Millikan apparatus. One could also construct a device which would allow a micrometer to drive a fine wire. Such an arrangement would make it easier to insure that the wire is moving parallel to the graticule lines, however, care must be taken to insure that the wire is being driven along the direction of motion of the micrometer shaft.

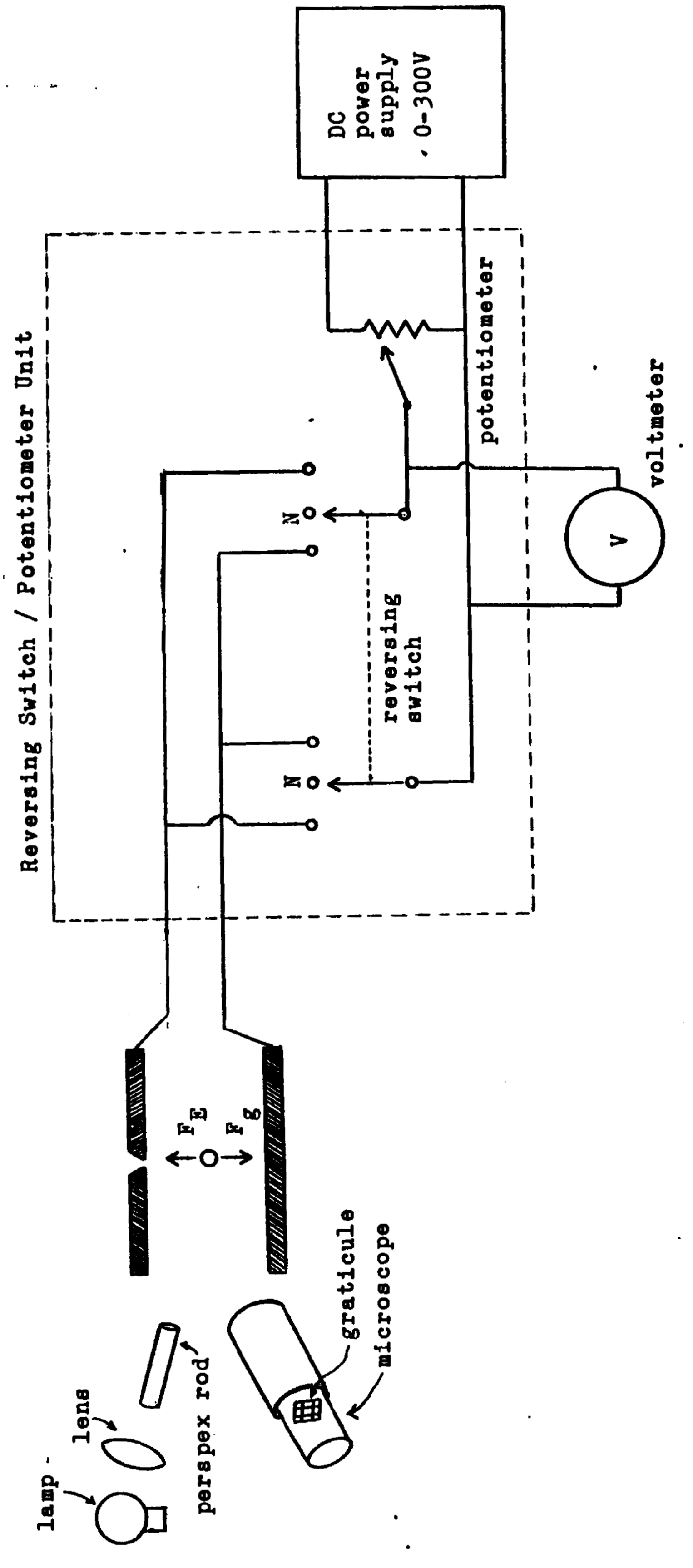
DC
power
supply
0-300V
potentiometer
voltmeter
V
Reversing Switch / Potentiometer Unit
reversing
switch
N
N
F_E
F_g
graticule
microscope
perspex rod
lens
lamp

FIGURE 3.2

EXPERIMENT 4

Measurement of the specific charge of the electron e/m

4.1 Objectives

1. To measure the charge to mass ratio of the electron.

4.2 References

4.3 Preliminary Exercises

1. Draw the magnetic field lines for the Helmoltz coil arrangement (see figures 4.1 and 4.2) looking from the end of the coils. Show that should have only an axial component on the plane half way between the coils (i.e. that $\vec{B} = B\hat{k}$ on the xy plane of figure 4.2).
2. Calculate the magnetic field in this experiment we will assume that the field is approximately uniform over the plan. half-way between the two coils. Suggest a simple, experimental method of testing this assumption. (Hint: the experiment can be done with the e/m apparatus itself. A magnetic field meter is not required).

3. In this experiment, e/m will be determined by observing the motion of an electron beam in a magnetic field. How could e/m be determined by observing the motion of an electron beam in crossed electric and magnetic fields?

4.4 Theory

In this experiment, the charge/mass ratio of the electron will be determined by measuring the trajectory of a beam of electrons (cathode rays) as it is deflected by a magnetic field. This experiment was first performed by J.J. Thomson almost 90 years ago. At that time, a value for e/m had also been deduced by Zeemen from his studies of the splitting of spectral lines of atoms placed in a strong magnetic field (the Zeeman effect). In order to determine e/m from his data, Zeeman assumed a simple model of the atom in which electrons move In circular orbits. The fact that Thomson's direct measurement of e/m for cathode rays was in agreement with Zeeman's measurement of e/m for electrons in atom was of great significance. This result demonstrated that the particles identified with cathode rays were also constituents of the atom. By using different electrodes in his apparatus, Thomson also demonstrated that the same value of e/m is obtained for cathode ray emitted by many different metals.

A diagram of the apparatus used in this experiment is shown in figure 4.1. Electrons emitted by a hot cathode are accelerated by a potential difference V applied between the cathode and the anode. The electron beam emerges from a small hole in the anode. Two external sets of coils produce a relatively uniform magnetic field perpendicular to the direction of the electron beam. The electron beam is bent along e circular path by the magnetic field. The charge to mass ratio elm for the electron can be determined by measuring the diameter of the electron's circular trajectory, the accelerating potential V and the magnet current. . To calculate e/m from the electron's trajectory, we must consider the motion of a charged particle in electric and magnetic fields. (The MKSA system of units will be used here). An electron emitted from the cathode experiences a total force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

where q = charge of the electron
$\vec{v}$ = velocity of the electron
$\vec{E}$ = electric field

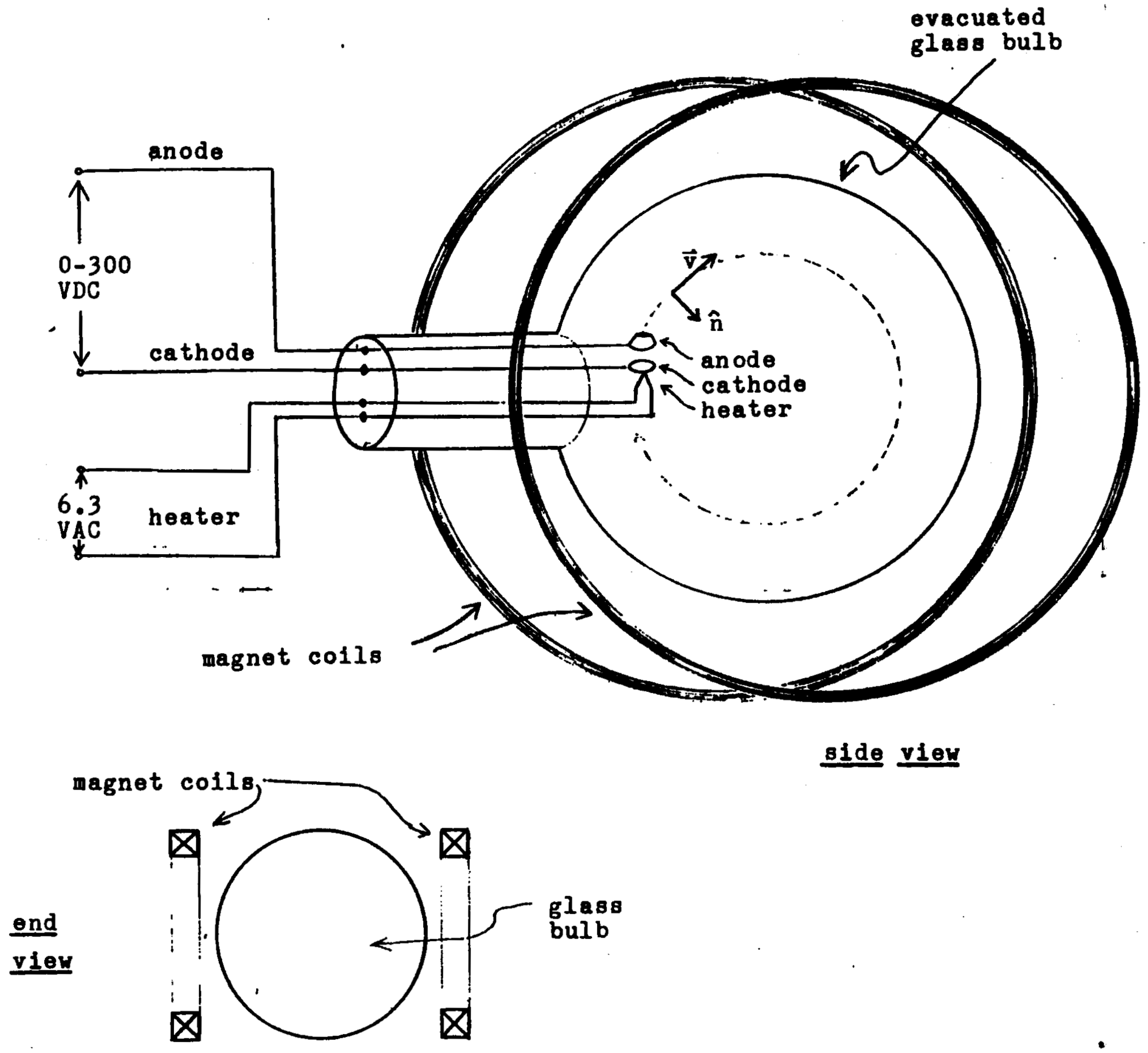

FIGURE 4.1

$\vec{B}$ = magnetic field

The electric field , due to the accelerating potential, exists only in the small region between the anode and the cathode. An electron leaving the cathode and accelerated to the anode will gain a kinetic energy of:

$$\begin{aligned}\Delta KE &= \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2\\ &= \int_C^A \vec{F}\cdot d\vec{r}\\ &= q\int_C^A \vec{E}\cdot d\vec{r}\\ &= -qV\\ &= eV\end{aligned}$$

The change in kinetic energy is given by the integral of over the path from the cathode to the anode. The magnetic field does no work on a moving charge end therefore does not affect the kinetic energy of the electrons. Assuming that the initial kinetic energy of the electrons is small ($v_i \ll v$), we have:

$$eV = \frac{1}{2}mv^2 \tag{4.1}$$

This is the kinetic energy of an electron when it leaves the accelerating region. Electrons emerging from the small noise in the anode experience only a magnetic field, which does not alter the magnitude of their velocity. The magnetic field is applied perpendicular to $\vec{v}$, so we can write:

$$\vec{F} = evB\hat{n}$$

where $\hat{n}$ is a unit vector directed inward, as shown in figure 4.1. (We assume that $\vec{B}$ is directed into the page. This will be true if the magnet coils are connected to the power supply in the proper polarity). As the trajectory of the electrons is bent inward, the magnetic force remains constant In magnitude and perpendicular to the velocity.) The electron will therefore be bent into a circular orbit. For a circular trajectory, we've have:

$$evB = m\frac{v^2}{R} \tag{4.2}$$

where R is the radius of the circular path. We can now eliminate v from equations 4.1 and 4.2 to solve for e/m as a function of v, B and R. From equation 4.1, we have

$$\frac{e}{m} = \frac{v^2}{2V} \tag{4.3}$$

solving equation 4.2 for v we obtain

$$v = (\frac{e}{m})BR$$

and inserting this result into equation 4.3 we obtain

$$\frac{e}{m} = \frac{2V}{B^2R^2} \tag{4.4}$$

The magnetic field B must be calculated from the current applied to the coils and the coil geometry. The Helmlioltz pair consists of two circular loops that are separated by a distance equal to their radius. The magnetic field at some location $\vec{r}$ can be calculated

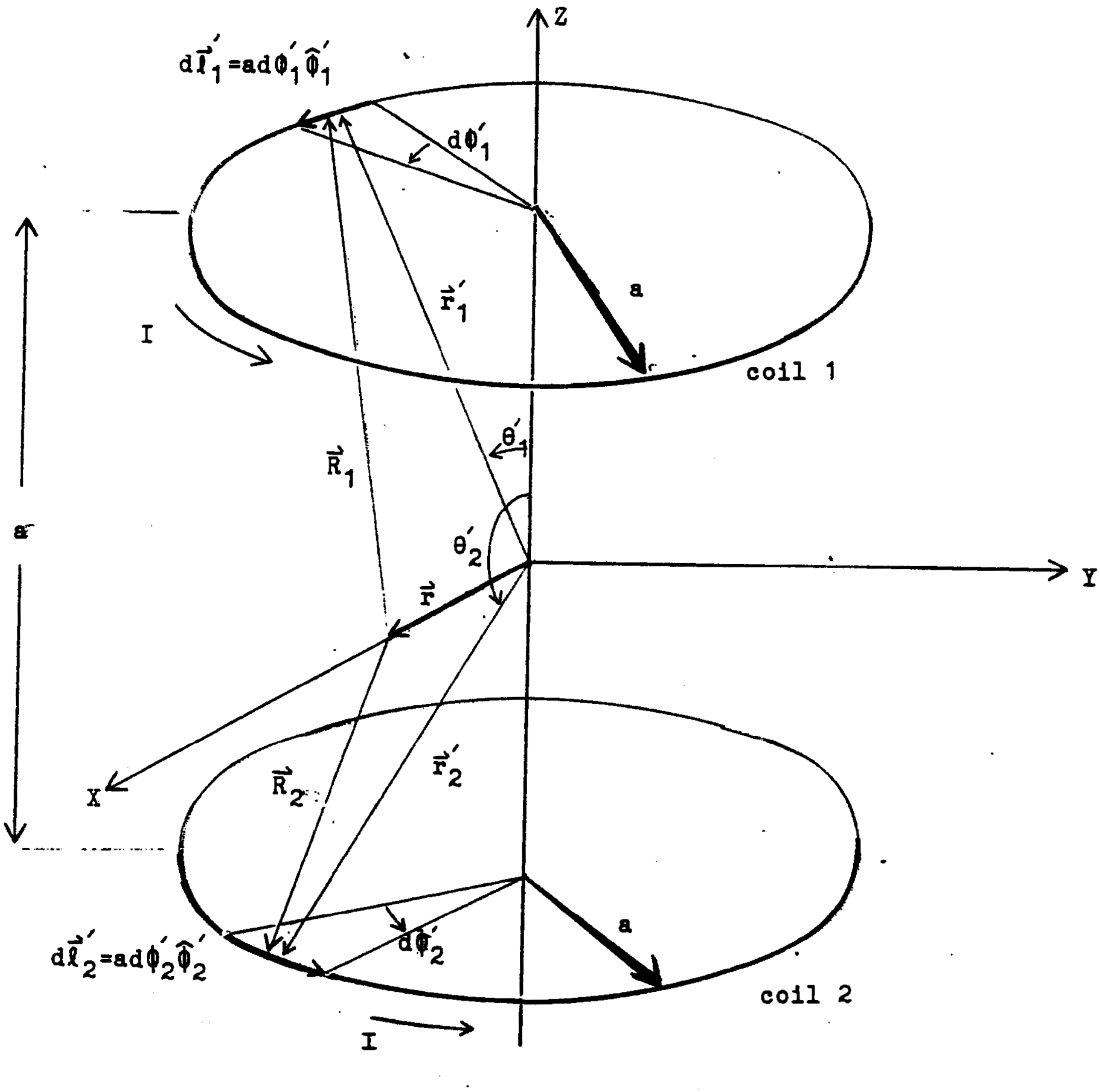

FIGURE 4.2

using the Biot-Savart law. We wish to find $\vec{B}$ on the plane half-way between the two coils, since the circular path of the electron beam will lie approximately in the plane. We choose $\vec{r}$ to lie in the xy plane, on the x axis. (We can choose $\vec{r}$ to be anywhere in the xy plane because of symmetry about the z axis). Applying the BiotSavart law to each coil, we have:

$$\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{Id\vec{l_1} \times \vec{R}_1}{R_1^3} + \frac{\mu_0}{4\pi} \oint \frac{Id\vec{l_2} \times \vec{R}_2}{R_2^3}$$

where:

$$\begin{aligned}
\vec{R}_1 &= \vec{r_1'} - \vec{r} \\
\vec{R}_2 &= \vec{r_2'} - \vec{r} \\
a &= r_1' sin\theta_1' \\
a &= r_2' sin(\pi - \theta_2') = r_2' sin\theta_2' \\
d\vec{l_1} &= ad\phi_1'\hat{\phi}_1' = r_1' sin\theta_1' d\vec{\phi}_1'\hat{\phi}_1' \\
d\vec{l_2} &= ad\phi_2'\hat{\phi}_2' = r_2' sin\theta_2' d\vec{\phi}_2'\hat{\phi}_2' \\
\mu_0 &= 4\pi \times l0^{-7} Newton/(Ampere)^2
\end{aligned}$$

Calculating the cross products in the above integrals, we find that the x and y components of the two integrals cancel while the z components add to give

$$\begin{aligned}
\vec{B} &= \frac{\mu_0 I a^2 \hat{k}}{2\pi} \int_0^{2\pi} \frac{d\phi_1'}{[r^2 + r_1'^2 - 2\vec{r_1} \cdot \vec{r_1'}]^{3/2}} \\
&= \frac{\mu_0 I a^2 \hat{k}}{2\pi} \int_0^{2\pi} \frac{d\phi_1'}{[r^2 + r_1'^2 - 2racos\phi_1']^{3/2}}
\end{aligned}$$

At the center of the coils (r = 0), we obtain

$$\vec{B} = \frac{\mu_0 I a^2 \hat{k}}{r_1'^3} \tag{4.5}$$

The coil separation is equal to the coil radius a, so we can replace $r_1'^3$ by:

$$\begin{aligned}
r_1'^3 &= (r_1'^2)^{3/2} \\
&= (a^2 + (\frac{a}{2})^2)^{3/2} \\
&= a^3(\frac{5}{4})^{3/2}
\end{aligned}$$

The magnitude of B at the center of the coils is then given by:

$$B = (\frac{5}{4})^{3/2}\frac{\mu_0 I}{a}$$

If each colt consists of N turns of wire, each carrying current I, then the current in the above equation must be replaced by NI

$$B = (\frac{5}{4})^{3/2}\frac{\mu_0 NI}{a} \tag{4.6}$$

4.5 Apparatus

A diagram of the apparatus is shown in figure 4.1. The electron beam is generated and observed in a large evacuated glass bulb. A small amount of hydrogen gas in the bulb makes the beam visible and helps to keep the electrons focused in a narrow beam (gas focusing). An electron beam will normally tend to spread out due to the electro static repulsion between the electrons. Positive Ions formed by collisions between the electrons and hydrogen atoms tend to reduce this effect. The pressure in the glass bulb must be very low, otherwise the electrons will be stopped in a short distance by collisions with gas molecules. The electron beam is generated in the usual way. A cathode is heated by a filament, causing it to emit electrons. The electrons are accelerated by a potential difference applied between the anode and the cathode. Electrons emerging from the hole in the anode form the electron beam. (Of course, some electrons will strike the anode, giving a small current between the anode and cathode). You may notice that the beam passes between a pair of plates as it emerges from the anode. These plates can be used to deflect the beam by applying a second electric field, but they arc not used in this experiment.

Two coils located outside the glass bulb produce the required magnetic field. Each of these coils has 130 turns. The magnetic field is varied by adjusting the current through the coils. This adjustment can be made by including a variable resistor in series with the coils. Most power supplies, however, may be operated in current regulating mode and may be connected directly to the coils. The magnet current may then be adjusted at the power supply.

The radius of the electron beam trajectory is measured by viewing the circular beam through a tube, as shown in figure 4.3. The magnet current and accelerating potential are adjusted to make the trajectory appear at the outer rim of the tube. It is then possible to calculate R from geometry.

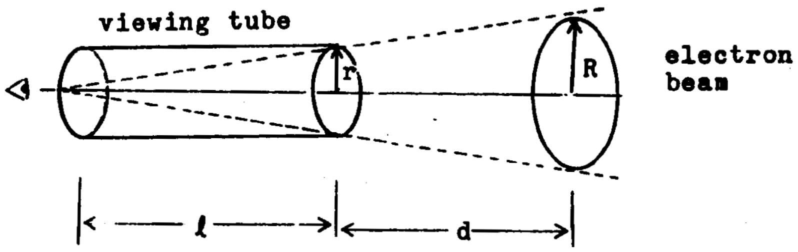

Figure 4.3: Measurement of R

In the experiment, the position of the viewing tube will be fixed. The accelerating potential V will be measured as a function of magnet current for a fixed radius of the trajectory. This will be done for two different viewing tubes (two different values of R).

4.6 Procedure

1. Set up the apparatus: Connect the magnet current, accelerating potential and filament power supplies to the apparatus. Use a 0-300 VDC supply for the accelerating potential and a power supply capable of delivering up to 5A for the magnet coils. The filament operate. from 6.3 VAC. Monitor I and V using digital multimeters. (Note that some multimeters have LED displays which can be read in the dark). Turn off the room lights to observe the electron beam. Adjust V and I to obtain a circular trajectory. If you try to make the curvature of the beam too large, it may strike the electrode structure and you will loose the beam. If this happens, you may need to reduce V and I to zero to recover it. Clamp the viewing tube securely in place. in tube should be located such that the beam will appear along the rim of the tube. Record thr quantities as defined in figure 4.3, and the diameter of the magnet coils.

2. Measure V v.s I for fixed R: Set the magnet current for some value t and adjust V to give a trajectory which just fills the viewing tube. (With this procedure, the uncertainty in I is small while the uncertainty in V includes the effects of uncertainty in viewing the trajectory). Record at least six different values for V and I which give a trajectory which just fills the viewing tube. Make sure that

your eye is in the same position to view the trajectory for each measurement and that the tube does not move during the experiment.

3. Repeat the experiment using different trajectory radius. Repeat the measurements using a different viewing tube.

4.7 Calculations

1. We will assume that the magnetic field is approximately constant near the center of the coils and that equation 4.6 can be used to find the value of B on the trajectory of the beam. Use equation 4.6 to eliminate B from equation 4.4. Rearrange the resulting expression to obtain V as a function of I^2.
2. For each set of measurements, plot V as a function of I^2. The result should be a straight line. Use the equation derived in 1 to calculate e/m from the slopes of the lines. The method of least squares should be used to find the slopes and their uncertainty.
3. Determine the uncertainty in each of the values of e/m. You should consider the uncertainty in the slope as well as the uncertainties in any other quantities that may be significant. If the two values of elm are in reasonable agreement then you may average them to obtain your reported value of elm. If they are not in reasonable agreement then you must decide which value is more reliable and explain why a discrepancy exists.
4. Compare your measured value of elm with the accepted value.

4.8 Questions

1. Compare the values of e/m obtained using the two different viewing tubes. Is one of them closer to the accepted value of e/m than the other by a significant amount? If so, explain why. Can you draw any conclusions regarding the validity of any assumptions or approximations that we have made in this experiment?

EXPERIMENT 5

Radioactivity

5.1 Objective

1. To familiarize the student with the Geiger-Muller counter as a device for detecting radioactivity.
2. To study counting statistics.
3. To estimate the range of particles emitted by some radioactive nuclei.

5.2 References

5.3 Preliminary Problem

1. Refer to the "Handbook of Physics and Chemistry" or one of the above references to find the types of particles emitted by the radioactive nuclide $^{152}_{63}Eu$. Find also the disintegration energy of each particle.
2. The nuclide $^{137}_{55}Cs$ produces γ-rays at 0.66 MeV after decaying into $^{137}_{56}Ba$ by β^- emission. Calculate the maximum energy that these particles lose when they interact with matter through compton scattering.

3. Describe an experiment where you employ a Geiger-Muller (G-M) tube to find out whether a source is an emitter of both β and γ particles or is a pure γ-emitter.
4. If the average counting rate in a G-M tube experiment is 100 counts/min., how long should you wait for each reading in order to obtain an accuracy better than 1%?

5.4 Theory

5.4.1 Introduction

The radiations emitted spontaneously by unstable nuclei are of three types, called alpha (α), beta (β) and gamma (γ). Another type of decay, in which a heavy nucleus undergoes spontaneous fission (SF) emitting a large fragment of nuclear matter, i.e. a nucleus, accompanied by the emission of neutrons and γrays, does not concern us here.) Some radioactive nuclei emit either α, β or γ and some emit two and some all three types.

In this experiment we shalt study the law that governs nuclear decay and some of the properties of the particles emitted in the process. The mechanism that underlies the decay process does not concern us here as it requires e much more involved treatment which goes beyond the level of this course.

1. α-particle: Alpha particles is a helium nucleus (He). Each particle contains two protons and two neutrons and has a double positive charge and about four times the mass of a hydrogen atom. The speed of α-particle in air is less than 0.1c; they penetrate materials only slightly and may be stopped by a few centimeters of air or a thin sheet of paper.
2. β-particle:Naturally produced beta particles are high speed electrons (β^-) or positron (β^+). They are more penetrating than α-particle. Since the mass of β-particle is very small, its ability to ionize atoms is smaller than that of an α-particle. However, since a β-particle penetrates almost a thousand times further into a medium than does an α-particle with the same amount of energy, there is very little, difference in the total ionization produced by either particle.

3. γ-ray: Gamma rays are a form of electromagnetic radiation (photons) with relatively short wavelength. They are the most penetrating of the three types of nuclear radiations. Since they have no electric charge, gamma rays do not directly ionize the molecules of a gas. However, ionization of gas occurs after absorbing γ-particle due to secondary a effects such as (Compton scattering, photoelectric ejection or pair production. (Epla1n each of these effects)

5.4.2 The Decay Law

In a radioactive sample, each atom may decay by the emission of an α, a β or a γ particle. It is found that the number of decays during a short time interval dt is proportional only to the number of nuclei in the sample N(t) and to the time interval. Therefore, the change in the number of nuclei in the sample Is given by:

$$dN = -\lambda N(t)dt \tag{5.1}$$

where λ is a constant called the decay constant. The negative sign in eq. 5.1 indicates that the sample is losing nuclei. To find the number of nuclei at any given time t, we integrate eq. 5.1 and obtain

$$N(t) = N_0 e^{-\lambda t} \tag{5.2}$$

where N_0 is the initial number of radioactive nuclei at t = 0. Differentiating eq. 5.2 we obtain the decay rate

$$-\frac{dN}{dt} = (\lambda N_0)e^{-\lambda N(t)} \tag{5.3}$$

The quantity $-\frac{dN}{dt}$ can be measured using a radiation detector as the counting rate (counts/second).

By plotting the logarithm of the counting rate, on a graph paper as a function of time, the decay constant λ can be determined from the slope. The half-life of a radioactive nucleus is defined as the time that elapses before half of the radioactive atoms in a sample has disintegrated. From eq. 5.3 it follows that the half-life $t_{1/2}$ is given by:

$$t_{1/2} = \frac{0.693}{\lambda}$$

The mean life, or average lite, is defined as the total time of survival of all nuclei divided by their total number

$$\tau = \frac{1}{N_0}\int_0^\infty t(-dN) = \int_0^\infty \lambda t dt = \frac{1}{\lambda} \qquad (5.4)$$

The intensity of decay is expressed by the "activity" of the source which is equal to the number of disintegration per unit time, that is, to the counting rate. The unit of activity is the Curie (Ci) which represents a counting rate of 3.7×10^{10} disintegrations/sec. Eq. 5.3 yields:

$$-\frac{dN}{dt} = -(\frac{dN}{dt})_{t=0}e^{-\lambda t}$$

and therefore the activity A satisfies:

$$A = A_0 e^{-\lambda t} \qquad (5.5)$$

For a source of a long life time (compared to the time of the experiment) the activity is almost constant. For example, if a counting rate of 200 counts/s is detected by a radiation detector with an efficiency of 10% for the type of particles detected, then the activity of the source is:

$$A = \frac{200 \times \frac{100}{10}}{3.7 \times 10^{10}} = 0.05 \mu Ci$$

Nevertheless, the activity varies if long periods are considered.

5.4.3 Statistics of Counting

Consider a radioactive source with an almost constant activity which is placed in front of a radiation detector, The number of counts over a fixed short period of time ($\Delta t \ll \tau$) is measured without altering the position of the source or anything in the experimental setup. If such an experiment is performed repeatedly, then one might expect to obtain the same reading each time, within the precision of the measuring instruments. In reality, however, a different reading is very likely to be obtained each time. This is not the result of faulty equipment or limited accuracy in the experimental method, but an indication of the basic fact that radioactive decay is a random process. Therefore the number of counts registered by a radiation detector is subject to statistical fluctuations.

Let the average count measured in a fixed time interval Δt ($\Delta t \ll \tau$) be $\bar{n}$. The probability of obtaining a count of value n, that is the probability that n nuclei will disintegrate (assuming l00% efficiency of the detector), is given by Poisson's distribution which can be written as:

$$P_n = \frac{\bar{n}^n}{n!} e^{\bar{n}} \tag{5.6}$$

$P_n dn$ gives the probability of obtaining a count between n and n + dn when the average is $\bar{n}$. The above distribution is normalized to unity, that is

$$\int_0^\infty P_n dn = 1 \tag{5.7}$$

It can be shown that when n is very large eq. 5.7 reduces to the gaussian or distribution. The normal distribution can be written as:

$$P_n = A e^{\frac{(n-\bar{n})^2}{2\sigma^2}} \tag{5.8}$$

where A is a constant whose value depend on the normalization. If the normalization of eq. 5.7 is used then $A = (2\pi\sigma^2)^{1/2}$. σ is the standard deviation defined by:

$$\sigma = \frac{\sum_{n=0}^{\infty}(n - \bar{n}P_n}{\sum_{n=0}^{\infty} P_n}$$

and it is easy to verify that

$$\sigma = \sqrt{\bar{n}} \tag{5.9}$$

A plot of the normal distribution is shown in figure 5.1.

5.4.4 Interaction of radioactive particles with matter

5.4.4.1 Charged Particles

When a charged particle traverses matter, it loses energy due to coulomb interaction with atoms. Particles' energy is transferred into kinetic energy of free electrons after the atoms are ionized. Although the energy loss in a single interaction is small compared to the initial energy of the particle, the particle might lose all of its energy through many such interactions. In that case the particle is stopped by the matter. The distance that

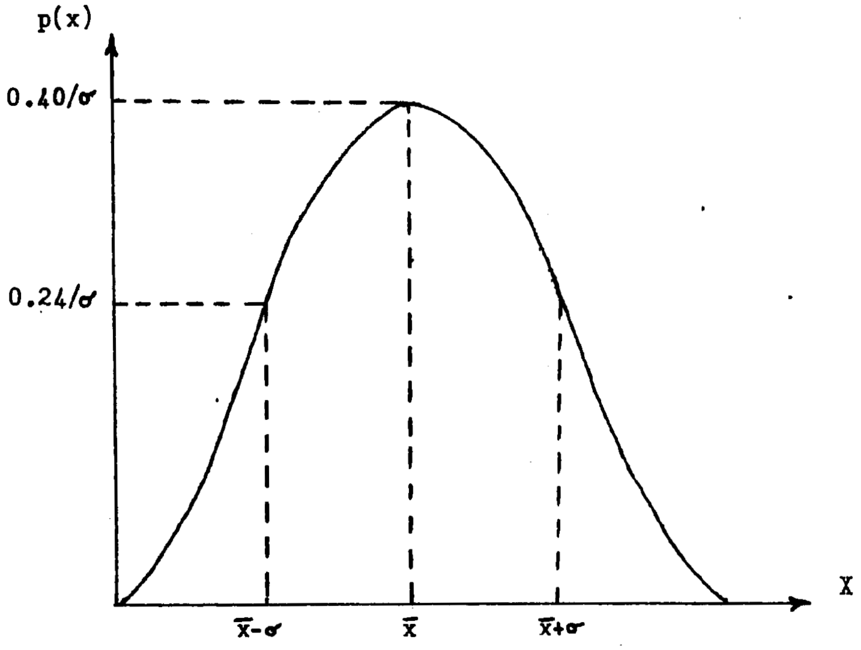

FIGURE 5.1: Normal distribution

the particle travels through matter before it is stopped is called the "range". Clearly, the range of a particle which interacts with a certain material depends on the initial energy of the particle and on the intrinsic properties of both particle and material. An expression for energy loss of an e particle as it passes through matter can be derived as follows: Let the particle travel at an initial speed V. As the particle passes an atom, it imparts an impulse δp to the bounded electron, thereby ionizing the atom. The force which acts on the electron during the interaction is the Coulomb force $\frac{kze^2}{r^2}$ as shown in fig. 5.2 It is easy to show that the energy transferred to the electron during the

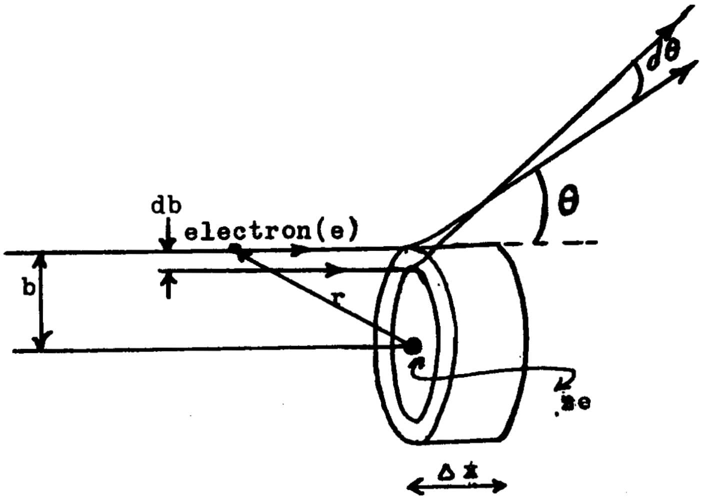

FIGURE 5.2

collision is given by

$$\Delta E = \frac{(\Delta P)^2}{2m} = \frac{2k^2 z e^4}{mV^2 b} \tag{5.10}$$

where m t the electron mass and b is the impact parameter. As the particle moves a distance dx it interacts with M electrons confined in the cylindrical shell of thickness db. For a material of density ρ, atomic weight A and atomic number Z_A this number is given by:

$$M = Z_A(N_A/A)\rho 2\pi b db dX$$

where N_A is Avogadro's number. The particle loses to each electron an amount of energy given by eq. 5.9. There for the energy loss after a distance dx is:

$$-dE = \int_{b_{min}}^{b_{max}} M(b)\Delta E(b) db$$

by direct integration we obtain:

$$-E\frac{dE}{dx} = (\frac{2\pi e^4 N_A}{m})(\frac{Z_A \rho}{A})(z^2 m_z)L = c \tag{5.11}$$

where m_z is the mass of the particle and E(x) is its energy. The constant L ($L = \frac{b_{max}}{b_{min}}$) is discussed in the references.

The range R of the particle is defined by:

$$R = \int_0^R dx = \int_{E_0}^0 \frac{dE}{dE/dx}$$

Therefore

$$R = \int_{E_0}^0 -\frac{E}{c} dE = \frac{1}{c} E_0^2 \tag{5.12}$$

where c is a constant.

The range R can be measured experimentally. As a result, the energy of the incident particle can be calculated from eq. 5.12 for a given material.

It should be noted however that in deriving eq. 5.11 several simplifications were made. We shall only mention the fact that the deflection of the charged particle during interaction was neglected. This assumption which holds for heavy particles cannot possibly be true for e particles. Moreover, relativistic corrections were also ignored. Owing to the difficulties in obtaining an exact expression for the range, empirical formulas were fitted. An expression for the range of particles in aluminum is given by Feather's formula:

$$\begin{aligned} R(gr/cm^2) &= 0.543E_0 - 0.160 \\ E_0 > 0.8MeV & \end{aligned} \tag{5.13}$$

For α particles in air, the following empirical expression was found

$$\begin{aligned} R(cm) &= 0.31E_0^{3/2} \\ 4 < E < 7MeV & \end{aligned} \tag{5.14}$$

5.4.4.2 Absorption of γ-rays

Absorption of γ-ray by matter is characterized by the exponential attenuation which is known to occur for ordinary electromagnetic waves. Let the intensity of the incident beam I be represented by:

$$I = Nh\nu$$

where N is the number of particles in the beam and hν is the energy of each particle. If such a beam passes through a slab of thickness ΔX, the intensity loss ΔI(= ΔNhν) is given by:

$$\Delta I = -\mu I \Delta x$$

or

$$(\Delta N)(h\nu) = -\mu N h\nu \Delta x$$

hence

$$\Delta N = -\mu N \Delta x$$

where μ is the lateral absorption coefficient. Therefore the number of transmitted particles through the slab of width x is given by

$$N = N_0 e^{-\mu x} \tag{5.15}$$

where N_0 is the initial number of particles in the beam. If the density of the radiated material is $\rho(gr/cm^3)$, then the exponent in eq. 5.15 can be written as

$$\mu x = \mu_m(\rho x) = \mu_m \sigma$$

where μ_m is called the mass absorption coefficient and σ is the density thickness (gr/cm^2). Equation 5.15 can then be written as

$$N = N_0 e^{-\mu_m \sigma} \tag{5.16}$$

5.4.5 The Geiger-Huller (G-H) Counter

An inexpensive and yet useful device for counting radioactive particles is the G-M counter. The operation of this detector is based on the ionization of gas caused by the passage of particles emitted by the radioactive source as explained in the previous sections. A Schematic diagram of the counter is shown in figure5.3. When a particle enters the tube through the window it generates electron-ton pairs. The number of initial ionizations depends on the energy of the passing particle. In the absence of external, voltage, these pairs will eventually recombine and no output signal will be obtained. However, when voltage is applied, electrons will move towards the wire electode and ions will be attracted by the metallic wall. As a result an output signal is built up across the resistance R. This signal is fed to the input of an amplifier and then

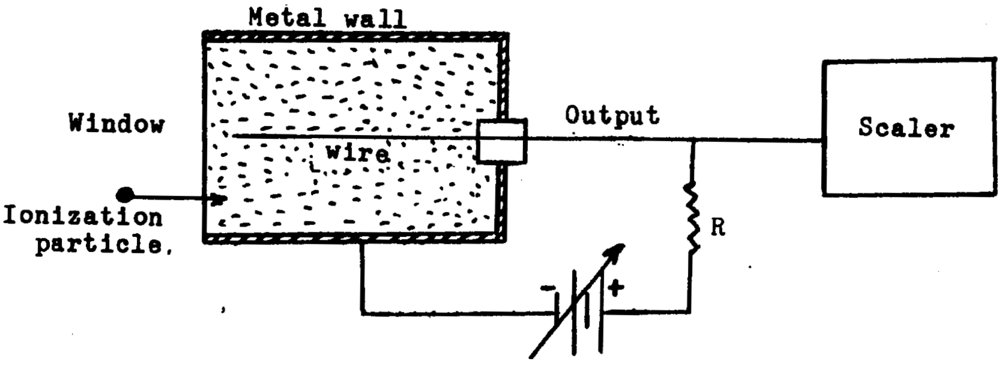

FIGURE 5.3

to a pulse counter. It was found that in a voltage range $V_1 < V < V_2$ the output pulse height does not depend on the applied voltage but only on the initial ionization, that is on the energy of the entering particle. The device which operates in this region is called the ionization chamber. The advantage of the ionization chamber is the ability to respond to a high count rate and to measure the energy of the incoming particle. Its main disadvantage is the low level output signals produced due to the small number of collected ion.

When the voltage is further increased (see figure 5.4), the initially ejected electrons attain sufficient energy as they move towards the wire so that they produce new electron-ion pairs as a result of collision with the gas molecules. Therefore, the output signal which results from the passage of a particle depends on both the initial ionization (energy of the radioactive particle) and the value of the applied voltage. It was found that in a certain voltage range $V_2 < V < V_3$ the output pulse height is proportional to the initial ionization (but not necessarily to the applied voltage). The device which operates in this region is called the proportional counter. The advantage of the proportional counter is its ability to detect weak -rays especially when strong background is present.

For voltages $V > V_3$ a cascade ionization by the secondary electrons occurs. Each particle which enters the tube causes a complete gas breakdown. As a result all incoming particles produce similar output pulses independent of their initial energy. If the discriminator of the scaler is adjusted between two appropriate levels then all emitted radioactive particles are counted. The region between the voltage interval $V_3 < V < V_4$ is called the Geiger-Muller region. In this range the height of the output pulse is independent on both the initial ionization and the applied voltage. It should be noted, however, that gas breakdown, as a result of the passage of a particle, lasts for a finite time τ_d. During this period the counter is "dead" and the passage of any particle through the counter will not be detected. It is therefore necessary to make the counter "recover" as soon as possible after each event or in other words to make the "dead time" τ_d as small as possible. If the actual number of particles per second that enter the window is denoted by R_0, then the detected count rate R is given by:

$$R = R_0(1 - R\tau_d) \tag{5.17}$$

For example if the rate is 100 counts/s and $\tau_d = 500\mu s$ then:
$R/R_0 = 95\%$.
but if R = 1000 count/s then $R/R_0 = 50\%$ which means that only 50% of the entering

particles are detected. This indicates that the G-M counter is not suitable for strong radiopacity. A simple method for measuring the dead time is to use two different sources. Let the count rates recorded when each source is placed alone infront of the window by $R_1^{'}$ and $R_2^{'}$ and when the two sources are placed together $R_{12}^{'}$ From equation 5.17 we obtain:

$$\begin{aligned}
R_1^{'} &= R_1(1 - R_1^{'}\tau_d) \\
R_2^{'} &= R_2(1 - R_2^{'}\tau_d) \\
R_{12}^{'} &= R_{12}(1 - R_{12}^{'}\tau_d)
\end{aligned}$$

where R_1, R_2 and R_{12} are the corresponding incident rates. By solving the above equations and assuming that

$$\frac{R_1^{'} + R_2^{'} - R_{12}^{'}}{R_{12}^{'}} \ll 1$$

the dead time is found to be given by:

$$\tau_d = \frac{R_1^{'} + R_2^{'} - R_{12}^{'}}{2R_1^{'}R_2^{'}} \qquad (5.18)$$

Finally, when the applied volt.age is increased above V_4, spurious gas breakdowns start to occur. In addition these breakdowns become continuous. The counter should not therefore be used in this region.

5.5 Experimental Procedure

1. Determination of the G-M Region: In this part you will determine the operating voltage of the G-M tube. This is chosen to be in the middle of the region where the number of counts per second becomes almost independent on the applied voltage. The 'flat' range is called the 'plateau'. The slope of the plateau is an indication of the state of the tube: A good tube has a flat plateau.

   (a) Connect the G-M tube to the scaler. Set the voltage at its lowest value and switch on the power. Set the scaler on 'COUNT' mode. In this mode the scaler will continue to add up all events until it is reset.

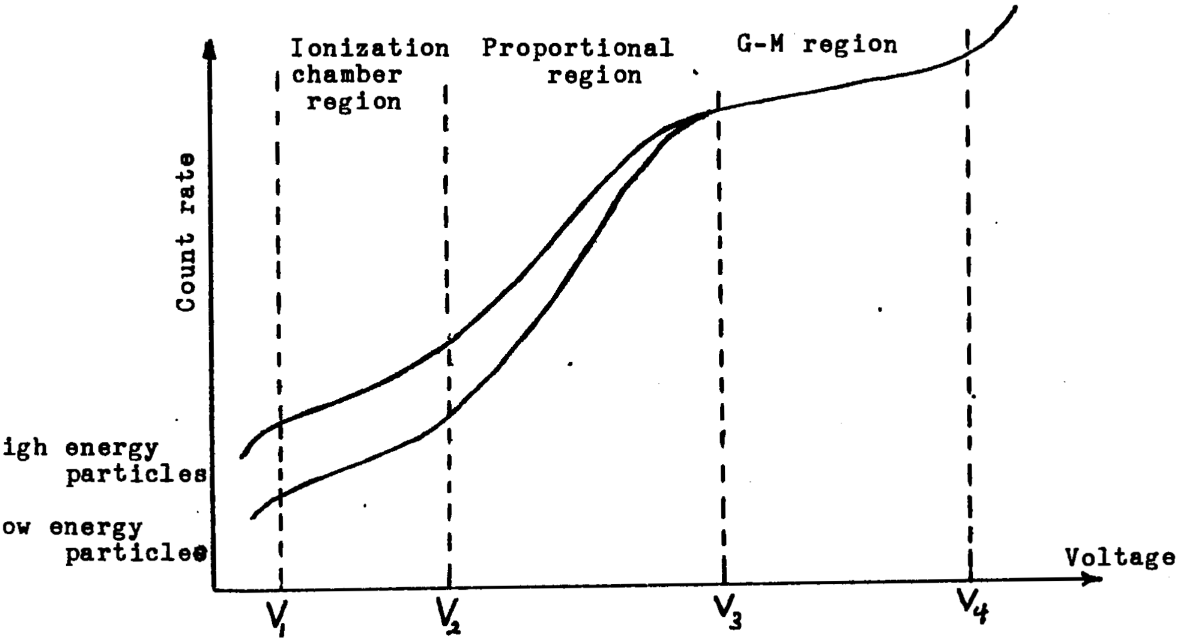

FIGURE 5.4: Operation regions of radiations detector

(b) bring the source infront of the tube so that it faces its window. Increase the voltage slowly until counts are displayed by the scaler. DO NOT TOUCH THE SOURCE.

(c) Switch over to 'COUNT RATE' mode. In this case the counter displays a count every 2.5 seconds. Increase the voltage until the count rate becomes almost constant (within the statistical fluctuations). Advance the source towards the tubes' window until a count rate of about 80 counts/s is obtained. Now keep the position of both the tube and the source fixed.

(d) Switch back to COUNT mode. Now you will, be able to obtain about 10000 counts every 2 minutes which means a precision of 1%. If less counts are obtained then you must either increase the measuring time or advance the source further towards the tube.

(e) Measure the number of counts as a function of voltage for fixed time interval (which you have chosen in (1d)).

(f) Plot the count rate as a function of voltage (use error bars.) Determine the operation point and the slope of the plateau

2. Absorption of Radiation:

(a) Set the voltage at the middle of the plateau region.

   (b) Place sheets of aluminum infront of the source and measure the transmitted intensity as a function of the density thickness. Make the time of measurement long enough to obtain a reasonable number of counts in each measurement. Remember that the error in a measured number of counts N i $\sigma = \sqrt{N}$

   (c) Remove the radioactive source and measure the count rate due to background.

3. Statistics of Counting:

   (a) Switch the scaler to COUNT RATE.

   (b) Advance the source towards tile tube until a count rate of around 80 counts/s is obtained. Record about a 100 successive displayed readings. Notice that if a reading is displayed for more than 2.5 s on the scaler it means that the reading was repeated.

4. Measurement of the 'Dead Time':

   (a) Place two different sources infront of the counter. Measure the count rate R'_{12}. Make sure that the position of one of the sources is sufficiently well determined so that the source can be put back into its previous position after being removed if necessary.

   (b) Remove (or cover with thick lead) one of the sources and measure the count rate R'_1,. Now put back (or uncover) the removed (or covered) source and remove (or cover) the other source. Measure the count rate R'_2

5.6 Results and Conclusions

1. Plot the logarithm of the count rate as a function of the density of the aluminum sheets (use error bars in your plotting). Estimate the range of the β-particles. Use equation 5.12 to estimate the energy of β-particles. Compare your results with those quoted in handbooks.

2.) From the graph you plotted in 1 calculate the mass absorption coefficient of aluminum (for this particular energy of γ-particle.)

3. Divide the data you obtained in part 3 into intervals of about 5 counts/s. Plot a histogram which represents the probability of obtaining a count in each interval. This probability is proportional to the number of times that the count was obtained.

4. To fit a gaussian curve (see figure 5.1) to your histogram, it ta possible to normalize the histogram to unity and use equation 5.8 with $A = \sqrt{2\pi\sigma^2}$. It is easier, however, to choose an arbitrary normalization to suit your histogram. First calculate $\bar{n}$ and σ from your data and substitute their values in eq. 5.8. To calculate A we integrate eq.5.8 which gives

$$\int_{-\infty}^{\infty} P_n dn \quad = \quad A \int_{-\infty}^{\infty} e^{-\frac{(n-\bar{n})^2}{2\sigma^2}} dn = A(2\pi\sigma^2)$$

   The left-hand side of the above equation is the area under the curve:

$$\int_{-\infty}^{\infty} P_n dn \quad = \quad \sum_i f(n_i)\Delta n_i$$

   where Δn_i is the width of the i^{th} interval arid $f(n_i)$ is the number of times that a count in this interval was obtained. If all intervals have the same width then:

$$\begin{aligned} A\sqrt{2\pi\sigma^2} &= F\Delta n \\ A &= \frac{F\Delta n}{\sqrt{2\pi\sigma^2}} \end{aligned}$$

   where F is the total number of runs, i.e. times when readings were taken, and an is the width of each interval. Now fit a gaussian curve to your histogram by substituting A in equation 5.8. Take the values of the n's to be the centers of your intervals.

5. Calculate the dead time of the G-M tube.

5.7 Questions

1. The efficiency of a Gieger counter is high if α and β particles are being detected but is low for γ particles. Why?

2. The activity of a ${}^{214}_{82}Pb$ source is $l\mu ci$ and its halflife is 26.8 min. Determine the weight of the source.

EXPERIMENT 6

Velocity of Light

6.1 Objectives

1. To measure the velocity of light in air.
2. To determine the r.fractive index of water.

6.2 References

6.3 Preliminary Problems

1. Define the following terms; index of refraction, group velocity, phase velocity, modulation of waves, mixing of two signals.
2. The output of a voltage source is a sinusoidal signet at 50MHz. Since the amplifier of an ordinary oscilloscope used by students cuts off such a high frequency; frequency down conversion is required in order to display the signal. Plot a block diagram which shows the basic elements of a frequency converter, which converts the frequency of this signal to 50KHz.
3. In this experiment, the source which emits the light beam should be positioned at the focal point of the lens. Why?

6.4 Theory

In this experiment, the speed of light is d.termin.d by measuring the phase change of a light pulse modulated at 50 MHz, after the pulse travels e distance 2d. We assume that the modulation envelope propagates with the same velocity as the electromagnetic oscillations that make up the light pulse. The principle of this experiment is illustrated in figure 6.1. A 50 MHz oscillator is used to drive a light emitting diode (LED). This frequency was chosen to make the scale of distances in the experiment reasonable (d ≈ lm). The light beam travels a distance d then is reflected back by the reflecting square mirrors arrangement, so that the total distance traveled by the tight pulse is 2d. The reflected light pulse is then converted to a 50 MHz electrical signal by striking a photodiode. This signal is then compared with the original signal (from the oscillator) and the phase shift between them $\Delta\phi$ is measured.

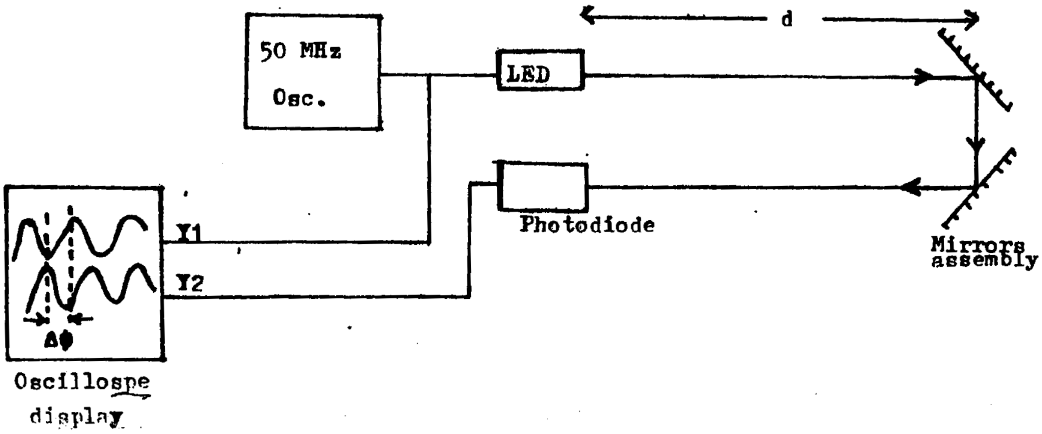

FIGURE 6.1

Let the modulation envelope of the light pulse emitted by the LED be written as:

$$E \;=\; E_0 cos(\omega_M t - kz) \tag{6.1}$$

where ω_M is the modulation angular frequency, k is the wave number and z is the distance that the wave travels. When the signal travels a distance z it suffers a phase shift given by:

$$\Delta\phi \;= 7k_0 z \tag{6.2}$$

where $k_0 = \frac{\omega_M}{C_0}$ and C_0 is the speed of Light in vacuum. Therefore:

$$\Delta\phi = \frac{2\pi f_M z}{C_0} \tag{6.3}$$

where f_M is the modulation frequency.

When the mirrors assembly is at a distance d from the LED, light travels a distance z = 2d before it strikes the photodiode as shown in figure 6.1. Therefore:

$$\Delta\phi = \frac{4\pi f_M d}{C_0} \tag{6.4}$$

The phase shift $\Delta\phi$ can be measured by displaying the two signals (the signal from the driving oscillator and the received signal which travels a distance 2d), on a dual beam oscilloscope for a given distance d. Hence the speed of tight C_0 can be calculated from eq. 6.4.

In the above discussion it was assumed that light was propagating in vacuum. When light travels through material of index of refraction n, its speed is reduced according to the relation:

$$C = \frac{C_0}{n} \tag{6.5}$$

In this experiment the index of refraction of a liquid can be determined within three significant figures. To measure n a tube of length L filled with the liquid is placed in the path of the beam as shown in figure 6.2 From eq. 6.2 the new phase shift after the tube is inserted become:

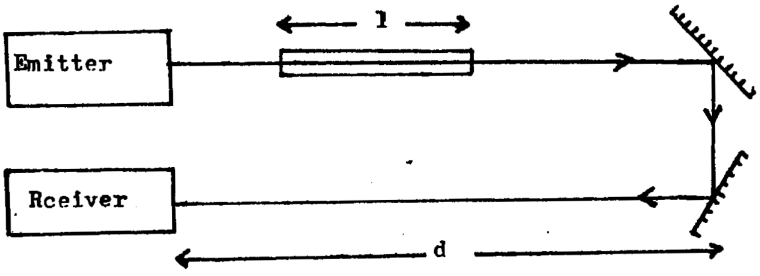

FIGURE 6.2: Calculation of the index of refraction of a liquid

$$\Delta\phi' = k + oz'$$

where now $z^{'}$ is the optical path that the beam travels (from the emitter to the detector). Therefore from figure 6.2 we obtain:

$$z^{'} = nL + (d - L) + d = 2d + L(n - 1)$$

Fran equations 6.3 and 6.4 we obtain:

$$\Delta\phi^{'}(d) = \Delta\phi(d) + \frac{2\pi f_M}{C_0}[L(n-1)] \tag{6.6}$$

Suppose now we change the distance d to $d^{'}$ (by moving the mirrors assembly) until the phase difference assumes the same value before the tube was inserted. Then,

$$\Delta\phi^{'}(d^{'}) = \Delta\phi(d) \tag{6.7}$$

Also, eq. 6.6 then yields

$$\Delta\phi^{'}(d^{'}) = \Delta\phi(d^{'}) + \frac{2\pi f_M}{C_0}[L(n-1)] \tag{6.8}$$

Therefore

$$\Delta\phi^{'}(d) - \Delta\phi(d^{'}) = \frac{2\pi f_M}{C_0}[L(n-1)]$$

Using eq. 6.4 we obtain:

$$2(d - d^{'}) = L(n-1)$$

Or

$$(n-1) = 2\frac{(d-d^{'})}{L} \tag{6.9}$$

Notice that $d - d^{'}$ is the distance by which the mirrors assembly must be shifted in order ro obtain the same phase shift before the insertion of the tube. By measuring $d - d^{'}$ and from the known value of L, the index of refraction can be determined.

6.5 Apparatus

A Schematic diagram of the experimental set up is shown in figure 6.3. Light emitted by the LED L; made into a parallel beam by means of the lens L_1. 1f the two plane mirrors M_1 and M_2 are oriented at 90^o relative to each other then the reflected beam travels in an exactly opposite direction to the incident beam. It is then focused on the surface of the photodiode by the lens L_2. Oscilloscope display of the envelops of both the emitted and the received light pulses is made by using a frequency converter circuit whose block diagram is shown in figure 6.4. The driving oscillator which modulates the emitted light supplies a 50MHz signal. to a mixer. This signal beats at the mixer with a 50.05MHz signal which is supplied by a stabilized oscillator. The beat signal whose frequency is equal to the difference between the the two mixed signals (which is in this case 50KHz) is fed to the input of an oscilloscope. A 1:1000 frequency reduction is thus obtained. The amplitude of the displayed signal is proportional to the amplitude of the light pulse envelop. Similarly the received 50MHz voltage signal which is generated from the reflected light after striking the photodiode, is converted into a 50KHz signal by using a second mixer. Its amplitude is proportional to the envelop of the received (reflected) light pulse.

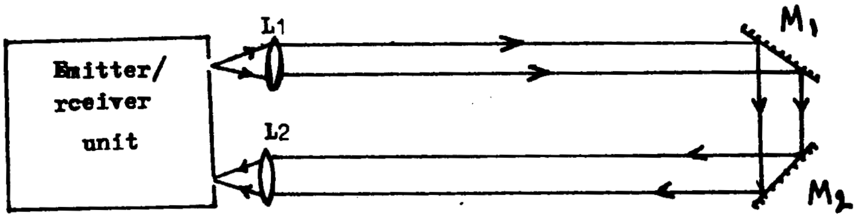

FIGURE 6.3: Experimental setup

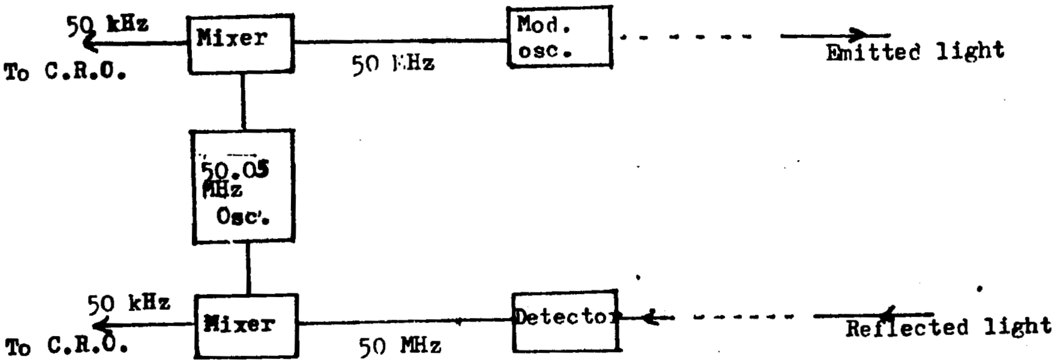

FIGURE 6.4: frequency converter circuit

it should be noticed that the phase difference between the two 50kHz signals might not in fact be equal to the phase difference between the emitted and the received Light pulses. This is because the electronic circuit itself might introduce a phase shift. Let the phase of the emitted Light when It leaves the LED be ϕ_e and the phase of the light wave as it hits the detector be ϕ_r. One would of course like the phase difference of the two 50Hz signals to be $\Delta\phi = \phi_e - \phi_r$. In reality however, the measured phase difference is $\Delta\phi = \phi_e - \phi_r + \Delta\psi$ where $\Delta\psi$ is a phase shift created by the electronic circuit. Fortunately, the presence of this instrumental. phase shift does not influence our final result. The student will notice throughout this experiment that what he really is measuring is not the absolute value of the phase difference $\Delta\phi$, but the change in $\Delta\psi$ as a result of changing the optical path difference (see equation 6.8). As long as the instrumental phase shift $\Delta\phi$, remains constant as the optical path varies we need not worry about it. In fact the apparatus design provides us with the facility of varying the value of $\Delta\phi$. This permits the setting of a convenient initial value of $\Delta\phi$, which is easily measured (0 or π). This can be done by adjusting the knob marked ”phase” which is mounted on the case which houses the emitter/receiver unit.

6.6 Procedure

1. Alignment:

   The most difficult pert in this experiment is the optical. alignment: whence alignment is properly done, the measurements become straightforward.

   First, the distance between the LED and the lens L_1 should be adjusted so that a collimated beam is obtained. This can be done by longitudinal movement of the lens L_1, which is mounted on a magnetic base, along the sheet-steel bench. To ensure that, an aligning card which has a circular aperture at its middle is used.

   (a) Switch on the mains of the transmitter/receiver unit. You should observe red light coming from the emitting diode. Connect the xoutput to one channel of a dual beam oscilloscope. A sinusoidal signal at a frequency equal to $f_M/1000$ (why?) which comes from the oscillator which modulates the emitted light is observed. Measure the frequency of this signal and find f_M.

   (b) Clamp the alignment card to the mirror M_1 so that the circular aperture uncovers the mirrors center. Adjust the lateral and vertical position of the lens L_1 so that the light spot hits the mirror M_1 at its center. Move the mirrors square about 70cm back from the light source. Now move the holder of the lens L_1 backwards or forwards until the light spot covers the whole apertures of the card. The emitted beam is now made approximately parallel.

   (c) Connect the y-output of the emitter/receiver unit to the second channel of the oscilloscope. This output gives the signal due to the received light. When the optical system is not aligned no signal will be detected. Focus the light beam which is reflected by the mirror M_2 onto the surface of the photodiode. The received sinusoidal signal should now appear. Adjust the tilting screws on the back of each mirror for maximum amplitude of the received signal.

   (d) Move the mirrors assembly forward and backward. If the amplitude of the received signal varies significantly it means that the light beam is not well collimated (why?). Move the lens L_1 slightly forward or backward. Make fine adjustment of the lenses position, and the tilting screws of the mirrors. When alignment is optimum, the amplitude of the received signal should be at least 1/5 of the amplitude of the modulating signal.

2. Measurement of the Velocity of Light:

   (a) Adjust the gains of the oscilloscope channels so that the emitted and received signals have equal amplitudes. With the mirrors assembly near the far end of the optical bench set the "Phase" knob to obtain a phase difference $\Delta\phi_1 = 0$ or π. (It is convenient to Switch the scope to "External" and use Lissajous figures). Record the position d_1 the mirrors assembly.

   (b) Move the mirrors assembly towards the transmitter/receiver unit until a phase difference $\Delta\phi_2 = \pi$ or 0 is obtained. Record the position d_2 of the mirrors assembly.

   (c) Repeat this measurement 6 times each time starting from a slightly different position.

3. Index of Refraction of Water:

   (a) Fill the tobular cell with water. Move back the mirror assembly and use the metallic supports to fit the tube between L_2 and M_2. Make fine adjustment of the tilting screws for maximum received signal (switch back the scope to "Internal" to observe the received amplitude).

(b) Adjust the phase difference to give $\Delta\phi = 0$ or π

(c) Remove the water tube carefully. A change in the phase difference will now be observed. Move the mirrors assembly backwards until the original phase difference (0 or π) is restored. Record the displacement of the mirrors assembly (dd').

(d) Repeat the measurement 6 times, each time starting from a new position.

(e) Measure the length L of the water column.

6.7 Calculations

1. Using equation 6.4, calculate the velocity of light in vacuum.
2. Using equation 6.9 calculate the index of refraction of water.

6.8 Questions

1. Why can not we use light modulated at 50 KHz (instead of 50MHz) and hence avoid mixing?
2. Does the fact that the experiment was performed in air rather than vacuum influence your results? How long should the length of the optical bench be in order that the error caused by the presence of air becomes significant? (The index of refraction of air is 1.0003).
3. When the emitted beam is not made parallel, the amplitude of the received signal changes as the mirrors assembly is moved. Why?
4. When no light is reflected to the photo-diode and the gain of the oscilloscope is made maximum, a weak signal at 50KHz can be observed. Where does this signal come from?

EXPERIMENT 7

Michelson Interferometer

7.1 Objectives

1. To measure the refractive index of glass.
2. To measure the splitting of the yellow doublet of mercury or sodium.

7.2 References

7.3 Assessment

1. Explain the following concepts: .
    - Coherence
    - Coherence length
    - Optical path length.
2. Why can we obtain interference fringes in a Michelson interferometer using a source that is not spatially coherent, while we can not obtain interference fringes by illuminating a double slit with a spatially incoherent source?

3. Show that the slope of the line plotted in part 4 of this experiment is equal to $\lambda/2c$, where c is the calibration constant for the mirror movement.

4. Why is it possible to obtain fringes from a white light source only when the optical path length between the two beams is very close to zero

7.4 Theory

7.4.1 Basic Operation of the Interferometer

A diagram showing the essential features of the interferometer is given in figure 7.1.

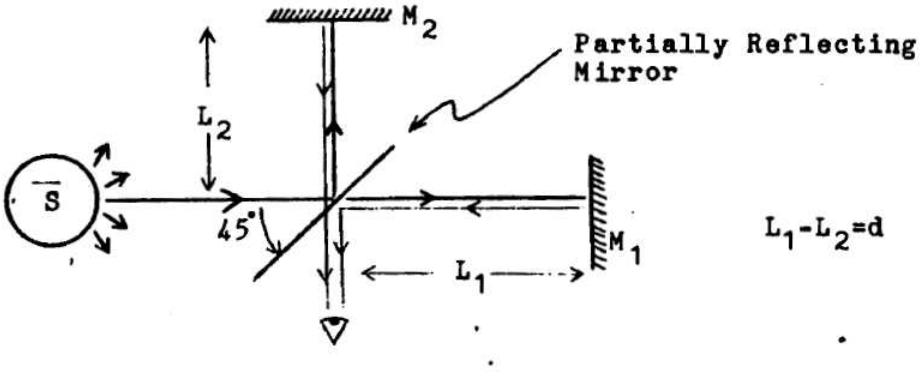

Figure 1

FIGURE 7.1: Schematic diagram of Michelson Interferometer

Light from a source s is divided into two beams by a partially ref1ectig mirror. (A partially reflecting mirror is a piece of glass with a thin coating of aluminum on one side. Part of the incident light is transmitted and part is reflected). Each of these beams is reflected back by a fully silvered mirror (M_1 and M_2). Again, part of the returning light is transmitted and part is reflected by the partially reflecting mirror. The two beams of light reaching the eye of the observer have travelled different paths. The two rays are in phase when they are first separated at the partially reflecting mirror. If they travel different distances, a phase difference between the two waves will be introduced and interference will occur when the two rays are recombined by moving one of the mirrors forward or backward, the path difference $2d = 2(L_1 - L_2)$ and therefore the phase difference $\Delta\phi$ may be varied.

7.4.2 The Fringe pattern

To understand the fringe patterns that may be observed in the interferometer, note that the observer is viewing the reflection of mirror M_1 in the partially reflecting mirror. The image formed by a plane mirror lies behind the mirror Surface on a line perpendicular to the mirror surface and passing through the object. The image and object distances are equal. This will place the image of M_1 just behind M_2 by a distance d (in front of M_1 if $L_1 < L_2$). In effect, we have an air gap of thickness d. The resulting interference patterns are similar to those produced by a thin film. see figure 7.2

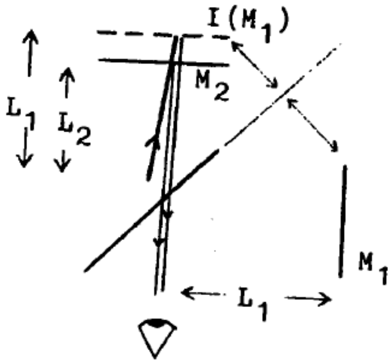

FIGURE 7.2: Thin film interference in Michelson Interferometer

Consider the case in which the mirrors M_1 and M_2 are exactly parallel.

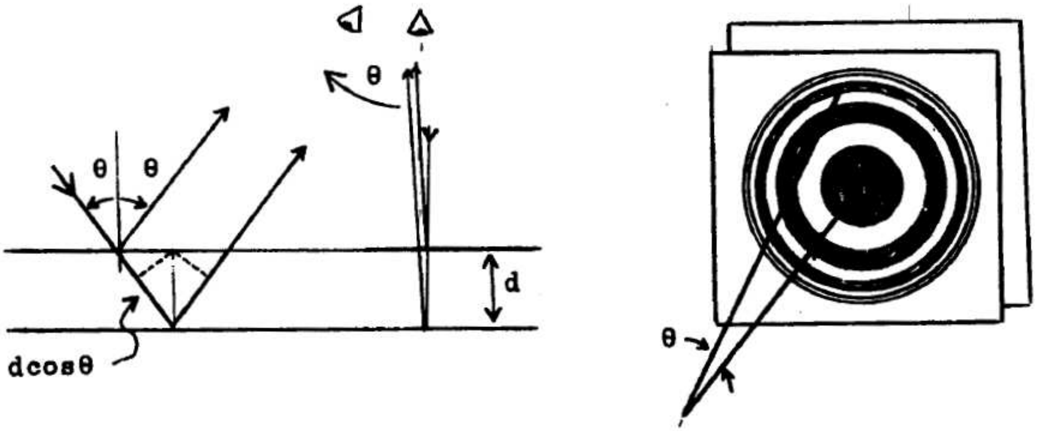

FIGURE 7.3: Formation of Michelson fringes

An observer looking directly at the mirrors would see rays incident and reflected normally to the surface. Looking at an angle θ to the normal, the observer will see rays incident and reflected at angle θ. (We assume that the observer is relatively far away or that a telescope adjusted to focus parallel ray is used). The path difference for the two rays will be $\Delta x = 2d\cos\theta$. For light of wavelength λ, this gives a phase difference $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{4\pi}{\lambda}d\cos\theta$.

The condition for constructive interference is:

$$2d\cos\theta \;=\; m\lambda, \qquad\qquad m = 1, 2, 3, ...$$

We will have the same angle θ on a ring about the normal, so the interference fringes will, be a set of concentric rings. if d is small, the angular spread of the rings will be large, and we will have a few broad rings in the field of view. If d is large, the angular separation of the rings will be small and we will have many fringes in the field of view.

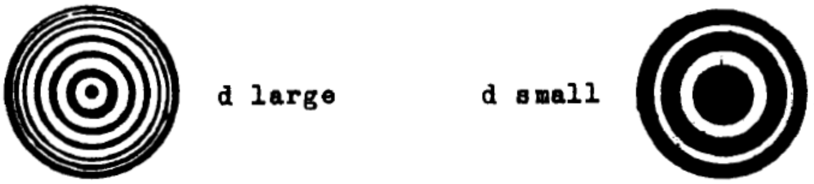

Figure 7.4: Michelson fringes

For $d = \pi/4$, we expect a single, very broad dark fringe. For $d = 0$, the entire field should appear bright. If one mirror is tilted with respect to the other, we will have an air wedge. Curved fringes will be obtained. (looking out from the normal at an angle θ, we must move into the narrower part of the wedge to keep the path difference constant). If the angle of tilt is made sufficiently large, the curvature of the fringes will not be noticeable and they will appear to be straight.

Figure 7.5: Curved fringes

In order to.observe these fringe patterns, light rays must be incident on the mirrors over a range of angles θ. It is there fore necessary to use an extended source, rather than a well collimated beam of light, to illuminate the interferometer.

7.4.3 Coherence

A light source can be characterized by two types of coherence; spatial coherence and temporal coherence. To define spatial coherence, we consider the phase of the light at two observation points that are equal distances from the sources

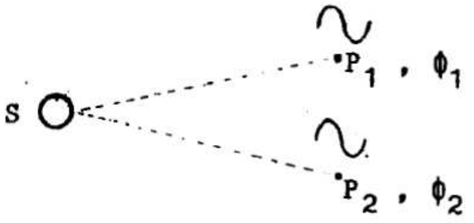

Figure 7.6: Coherence

If $\Delta\phi = \phi_1 - \phi_2$ is zero or a constant, the source is spatially coherent. If $\Delta\phi =$ varies rapidly and randomly, the source is spatially incoherent. A source has temporal coherence if, at a fixed point in space,the phase increases regularly in time as $\phi = \omega t$ If the phase,at a fixed point in space, varies rapidly and randomly, the source is temporally incoherent. A light source that emits light in long wave trains will have temporal coherence, while a light source that emits light in short bursts will be temporally incoherent.

If a source emits wave trains that are typically of length l_c, then the phase would increase regularly for a time$t_c = l_c/c$ (this is the time taken for the wavetrain to be emitted). We call l_c the coherence length of the source and t_c the coherence time. A source which has a long coherence length will, also have a well defined wavelength. A discharge tube which emits a single narrow spectral line will have a long coherence length (it will have temporal coherence). For a typical discharge tube source, $t_c \approx 10^{-9}s$ and $1_c = ct_c = 30\ cm$. An incandescent lamp, giving off white light, has a very short coherence length. Lasers have a very long coherence length (many meters) and the beam is spatially coherent as well. To obtain interference fringes in the double slit experiment we must use a source which is spatially coherent, so that the two slits will act as sources which oscillate in phase. The source used in the double slit experiment need not have temporal coherence, since we only require the phase difference between the two slits to be zero. The source used for the Michelson interferometer need not be spatially coherent. When an incoming light wave strikes a given point on the partially reflecting mirror, it is divided into two parts. The two outgoing waves will be in phase. The phase at a neighboring point on the partially reflecting mirror is not important, so long as the two outgoing waves are in phase. The coherence length of the source is important, however, ti the difference in path length for the two rays, ax, is greater than the coherence length of the source, then we will not obtain interference fringes. When the two rays are recombined, their phase difference will vary rapidly and randomly in time, and interference will not be observed. In order for the phase difference between the two waves reaching the observer to be. constant, the path difference for the two rays must be less than the coherence length of the source. The Michelson interferometer may be used to measure the coherence length of a source.

7.4.4 Applications

We will discuss two applications of the Michelson Interferometer;

1. The measurement of the index of refraction of a material and
2. The measurement of the wavelength separation of two closely spaced spectral lines.

To understand how the Michelson interferometer may be used to measure the index of refraction, we must introduce the idea of optical path length. When light enters a material, having index of refraction n, the wavelength of light in the material will be $\lambda' = \lambda/n$, where λ is the wavelength of the light in the vacuum. (The index of refraction of air is very close to 1 and the presence of air can usually be neglected). If a light wave passes through a length T of this material, we will have $N = T/\lambda'$ waves in the material,) In the vacuum, the same number of waves would span a length $L = N\lambda - nT$. We call $L = nT$ the optical path length of the material.

To measure the index of refraction of a material using the Michelson interferometer we first adjust the position of the movable mirror to give zero path difference between the two rays. (Assume that the movable mirror is M_2 for this discussion). This can be done by adjusting the position of M_2 to give white light fringes. Because the coherence length of a white light source is very short, we will only obtain white light fringes if $\Delta x = 0$. We then place our sample of thickness T in the path to the fixed mirror. This increases the optical path length to M_1 by $(n1)T = \Delta D$. To return to the condition$\Delta\phi = 0$, and recover the white light fringes, we must move M_2 back a distance ΔD so that the optical path lengths for the two rays will again be equal.

By measuring the distance D that M_2 must be translated to recover the white light fringes and the thickness of the sample, T, the index of refraction can be calculated. The spectra of many atoms are found to contain sets of closely spaced spectral lines called multiplets. The existence of multiplets gives important information about atomic structure,, and one would often like to measure the difference in wavelength between the lines in a multiplet. It is difficult to measure the wavelength separation of closely spaced lines directly because a very accurate, highresolution spectrograph would be required. We will see how the Michelson interferometer can be used to measure the difference in wavelength between the two lines of a doublet. If the interferometer is illuminated by light consisting of two wavelengths, λ_1 and λ_2, then two sets of interference fringes will be obtained. Often, however, we are not able to observe two-distinct sets of fringes because the separation of the fringes is less than their width. In this case, the individual fringes due to λ_1 and λ_2 can not be resolved. We can still detect the presence of the doublet by observing changes in the contrast of the fringe pattern. (The contrast is the difference in brightness between the bright and dark fringes). For certain values of the mirror spacing, d, the bright fringes of one pattern will coincide with the bright fringes of the other. This will give maximum contrast. For other values of d the bright fringes of one pattern will fall on the dark fringes of the other and we will have minimum contrast. If the intensities of the two spectral lines are approximately equal, the fringe pattern may seem to disappear altogether at minimum contrast. The spacing of fringes in the two patterns will not be exactly the same (since λ_1 and λ_2 are different), so the frtnge.patterns will not overlap for all angles θ. If λ_1 and λ_2 differ only by a small fraction, however, the fringe patterns will almost line up, at least over the angular width of several fringes.

By measuring the values of d at which minimum (or maximum)contrast is obtained, the difference in wavelength λ_2 and λ_1 can be determined. Consider the central fringes; $\theta = 0$. The phase difference between the two rays of wavelength λ_1 will be

$$\Delta\phi_1 = 2d\frac{2\pi}{\lambda_1}$$

Similarly, for the two rays of wavelength λ_2,

$$\Delta\phi_2 = 2d\frac{2\pi}{\lambda_2}$$

Suppose that $\lambda_2 > \lambda_1$. Starting at $d = 0$, we will have;

$$\Delta\phi_2 - \Delta\phi_1 = 0$$

, bright fringe at $d = 0$.

As d increases, $\Delta\phi_2$ increases more slowly than $\Delta\phi_1$. When $\Delta\phi_2$ fall behind $\Delta\phi_1$ by π , the first minimum in contrast will be obtained

$$\begin{aligned} \Delta\phi_1 &= 2\pi m, \qquad (m\ integer) \quad brightfringe \\ \Delta\phi_2 &= \Delta\phi_1 - \pi, \qquad\qquad\qquad darkfringe \end{aligned}$$

As we continue to increase d, $\Delta\phi_2$ will fall further behind $\Delta\phi_1$ The next minimum in contrast will be obtained when $\Delta\phi_2 - \Delta\phi_1 = 3\pi$. In general, minima in contrast will be obtained when

$$\Delta\phi_2 = \Delta\phi_1 - (n + \frac{1}{2})2\pi, \qquad n = 0, 1, 2, \ldots$$

Using our expressions for $\Delta\phi_1$ and $\Delta\phi_2$;

$$\begin{aligned} \frac{4\pi d}{\lambda_2} &= \frac{4\pi d}{\lambda_2} - (n + \frac{1}{2})2\pi \qquad n = 0, 1, 2, \ldots \\ 2d(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}) &= n + \frac{1}{2} \\ 2d(\frac{\Delta\lambda}{\lambda^2}) &= n + \frac{1}{2} \qquad (7.1) \end{aligned}$$

where $\Delta\lambda = \lambda_2 - \lambda_1$ and $\lambda^2 = \lambda_1\lambda_2 \approx \lambda_1^2 \approx \lambda_2^2$

Let d' be the value of d at which contrast minimum n' is obtained. Let d'' be the value of d at which the next contrast minimum is obtained ($n'' = n' + 1$). Then

$$\begin{aligned} \Delta d &= d'' - d' \\ &= \frac{\lambda^2}{2\Delta\lambda} \\ \Delta\lambda &= \frac{\lambda^2}{2\Delta d} \end{aligned} \tag{7.2}$$

By measuring the values of mirror spacing, d, at which successive minima in contrast are obtained, we can determine $\Delta\lambda$. (It is assumed that approximate values of λ_2 and λ_1 are known from low resolution spectroscopic measurements).

7.4.5 Details of Interferometer Operation

In the actual interferometer, it is convenient to add an additional component, a compensation plate, as shown in figure 8

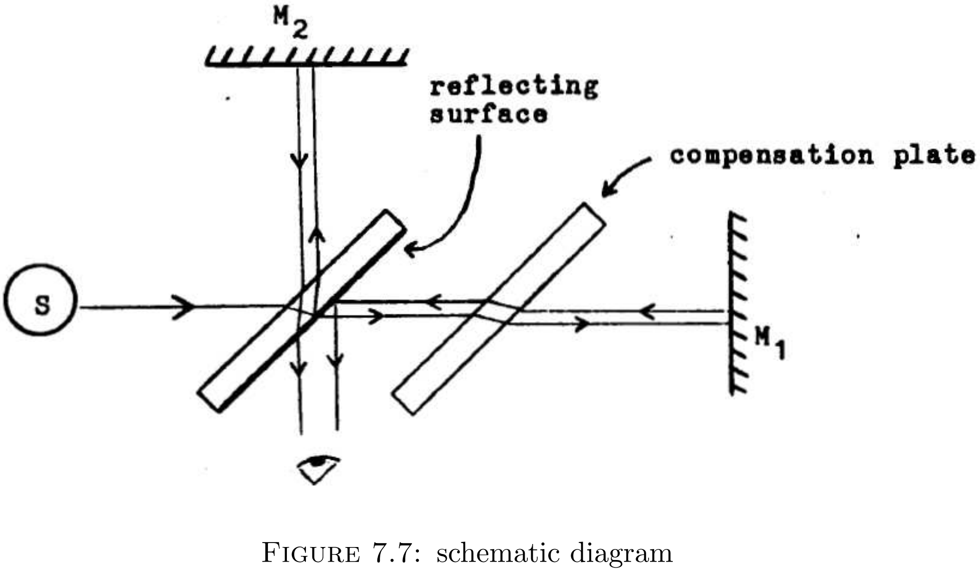

Figure 7.7: schematic diagram

After the incident ray is split at the partially reflecting surface, the ray taking the path to M_2 must pass through the glass plate of the partially reflecting mirror twice before reaching the observer. The ray traveling to M_1 would not pass through this glass plate. The glass plate increases the optical path length to mirror M_2 . Without the compensation plate, we would have to make the physical distance to M_1 greater than the distance to M_2 in order to make the optical path lengths for the t(io rays equal.

The compensation plate has the same thickness as the glass plate of the partially reflecting mirror. With this plate in the path to mirror M_1 , the optical path lengths for the two rays will be equal when the physical distances to M_1 and M_2 are equal. This makes setting up the interferometer somewhat easier if we wish to position the mirrors near zero optical path difference. In our interferometer, the mirror M_2 is the movable mirror. It can be moved forward or backward by rotating a micrometer screw. The interference pattern will be very sensitive to small changes in the position of M_2. Changing the distance to M_2 by $\lambda/4 = 10^{-4}$ mm would cause a change from constructive to destructive interference at the center of the field of view ($\theta = 0$).

to achieve fine control over the mirror position, a lever is used .

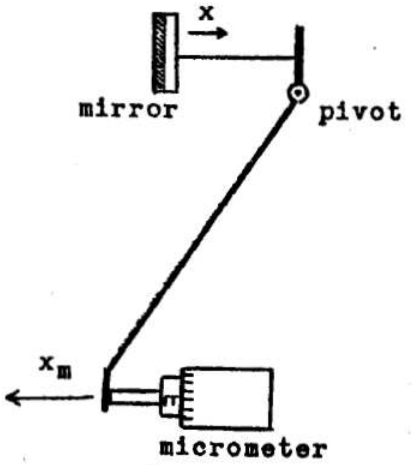

Figure 7.8: Micrometer and Mirror movement

$x = cX_m$.

X - actual mirror displacement

X_m- micrometer displacement

c - calibration constant

The value of the calibration constant c is 0.2 (changing the micrometer by 0.5 mm will move the mirror by $\simeq$ 0.1 mm). The exact value of the calibration constant will be determined in the experiment. This is done by illuminating the interferometer with light of a known wavelength λ and observing the central ($\theta = 0$) fringe in a circular fringe pattern. If the micrometer is displaced by X_m, we will note that the central fringe changes from bright to dark N times. Every time the central fringe changes from bright to dark, we know that the mirror position changed by $\lambda/4$. The total change in the mirror position is therefore $\Delta x = N\lambda/4$. By counting the number of times the central fringe changes from bright to dark for a certain micrometer displacement aXm, we can determine the / calibration constant.

The mirrors M_1 and M_2 are front surfaced mirrors; the aluminum coating is applied to the front surface of the glass for maximum reflectivity and flatness. The mirror surfaces are carefully polished to be flat to within $\lambda/4$ for $\lambda = 5890A^o$ (sodium yellow doublet). The mirror surfaces are easily damaged; never touch or attempt to clean them. Because the mirrors are not perfectly flat, the fringe patterns will become badly distorted near $d = 0$. The tilt of the fixed mirror, M_2, can be adjusted by two screws on the back of the mirror holder.

7.5 Procedure

1. Set up the interferometer: Identify the components of the interferometer. Make the distances to M_1 and M_2 (from the partially reflecting mirror) equal, as measured by a ruler. Turn on the mercury lamp and insert the green filter, so that the interferometer will be illuminated by the green line of mercury $\lambda - 5461A^o$. A white glass plate should be mounted on the interferometer table, in front of the light source, to diffuse the light.

2. Align mirror M_1 to obtain circular fringes: Clip a metal pointer on the diffusing screen. Looking down the axis of the interferometer (toward M_2). You should see three images of the pointer. Two of these images will be fixed, while the third will move when mirror M_1 is tilted. Of the two fixed images, the one on the right hand side should be darker. The faint image on the left is due to a reflection from the unsilvered surface of the partially reflecting mirror. Adjust the tilt controls of mirror M_1 to exactly superimpose the two darker images (the moving image and the fixed image on the right). When the two images are superimposed, the mirrors are approximately parallel and curved fringes should appear. Adjust the tilt controls in a direction that seems to increase the curvature and the contrast of the fringes. Careful adjustment of the tilt controls should yield a circular fringe pattern.

3. Final alignment with telescope: Take the telescope to a window and adjust it to bring distant objects into focus. This insures that the telescope will be focused for parallel incoming rays. Clamp the telescope into position. Make careful adjustments to the tilt controls to maximize the contrast of the fringe pattern, then adjust the position of the telescope to inter this fringe pattern in the field of view. Since the telescope is focused for parallel rays, we can only obtain fringes if the two mirrors are exactly parallel. (If the mirrors are tilted,the angles of reflection will be different). Change the micro meter screw which controls the position of M_2. Note the changes in the fringe pattern.

4. Calibration of the mirror movement: Adjust the mirror position until only a few fringes are in the field of view. As the mirror M_2 moves in the direction which decreases the mirror spacing, d, the bright fringes appear to shrink and disappear into the center of the pattern. Every time a fringe disappears into the center, the mirror position changes by δx, (we go from destructive → constructive → destructive interference at $\theta = 0$). Turn the micrometer screw and count the disappearing fringes. Note the micrometer reading every time 50 fringes have disappeared. (Record the initial micrometer reading as well). Obtain 7 micrometer readings (300 fringes disappear). Hake a rough calculation of c from your data and check to see that it is near the expected value.

5. Measurement of refractive index: Remove he telescope. We will wish to observe straight fringes later in this section,and this fringe pattern can not be obtained using the telescope,since the rays are not parallel at infinity. Turn the micro meter screw in the direction which decreases d. As d decreases, you will observe fewer fringes in the field of view and the remaining fringes will be broader. The fringe pattern will become distorted as d ≈ 0, making it difficult to recognize the exact point at which d $= 0$. If you go past the point at which d $= 0$, the circular fringes will reappear. (They will appear to be growing from the center as d increases). Once you have adjusted the position of M_2 to be near the point of zero optical path difference, remove the green filter and switch to the white tight. Slowly turn the micrometer until fringes appear. (They will be colored rings). Tilt the fixed mirror to obtain straight fringes. Turn the micrometer screw to Center these fringes and note the micrometer reading. (This is d $= 0$, zero optical path length distance). (Turn the micro meter screw back and forth a few times. Note how easy it is to miss these fringes if M_2 is moving too rapidly). Place a thin glass plate in the path to M_1. The surface of the plate should be

parallel to M_1 in both the horizontal and vertical directions. Estimate the new micrometer setting for d = 0. Assume that n ≈1.5 (typical, for glass) and c = 0.2. Turn the micrometer to this setting and search for the white light fringes. Record the micrometer setting at which the fringes reappear ($d^{'} = 0$).Remove the glass plate and check the location of d = 0 again. Insert the glass plate and check the location of $d^{'} = 0$ again. The difference in micrometer readings $D_M = X_M - X^{'}_M$. should be the same, to within the limits of error, both times. Measure the thickness of the glass plate in several locations using a micrometer.

6. Wavelength separation of a doublet: Realign the interferometer for circular fringes using the mercury green line. Both mercury and sodium have a yellow doublet. Consult your instructor for the source to be used. For the mercury doublet replace the green fitter by a yellow or an orange filter. Note the variations in contrast of the fringe pattern as the location of M_2 is varied. Record the micrometer reading for at least six successive minima in contrast. (If there are variations in contrast over the field of view, concentrate on the fringes at the center. The telescope may be helpful here).

7.6 Calculations

1. Plot the micrometer reading X_m vs. number of disappearing fringes from your data from part 4. Determine the slope and its uncertainty using the least squares method. Determine the calibration constant C and its uncertainty.

2. From your data, calculate the refractive index of the glass slide.

3. For your data from part 6, plot the Micrometer reading $X_m VS.n$, the number of contrast minima. (Where we start counting n is arbitrary, since it would just shift the intercept). From the slope of the graph, determine $\frac{\lambda^2}{\Delta\lambda}$. (See equation 7.4.4, and do not forget that you must convert X to actual mirror displacement, X). Using $\lambda^2 = \bar{\lambda}^2$ calculate the wavelength separation $\Delta\lambda$ and its uncertainty. Compare your value with the value from the wavelength tables. (See, for example, handbook of Chemistry and Physics).

7.7 Questions

1. How could you measure the wavelength of a spectral line using the Michelson interferometer? Estimate the precision with which you could measure wavelengths.
2. Why is it necessary to remove the telescope in order to observe straight fringes?
3. Derive an expression for the intensity of the circular fringe pattern obtained in the Michelson interferometer. Take the following approach; consider two plane waves propagating in the direction. Let them have equl intensity and phase difference $\phi = \frac{4\pi}{\lambda} dcos\theta$, where θe iss the angle of observation, as shown in figure 3. Let both waves be polarized in the k direction.

$$\vec{E}_1 = E_0 \hat{x} sin(kx - \omega t)$$
$$\vec{E}_2 = E_0 \hat{x} sin(kx - \omega t + \phi)$$

when these two waves are combined, the resulting time averaged intensity will be given by:

$$< I >= \frac{1}{T} \int_0^T (\vec{E}_1 + \vec{E}_2)^2 dt$$

where the time average is performed over one period, $T = 2\pi/\omega$. Evaluating the above expression and using the appropriate trigonometric identities, you should find:

$$I = 2E_0 cos^2 (\frac{2\pi}{\lambda} cos\theta)$$

EXPERIMENT 8

The Photo-Electric effect

8.1 objectives

1. Verification of Einsteins interpretation of the photo-electric effect.
2. Determination of Plancks constant.

8.2 References

8.3 Preliminary Problems

1. Calculate the Longest wavelength, above which the photo-electric effect would not occur. Take the work function of the radiated metal 2 eV. Would you expect the photoelectric effect to occur at infra-red frequencies?
2. Estimate the stopping potential V_0 of the photo-current emitted by a metal of work function $\phi = 2$ eV after being radiated by a monochromatic light of wavelength $\lambda = 5000 A^o$.
3. A point source emits radiation, uniformly in all directions at a rate of $P_0 = 0.1$ Watt. A metallic surface of small area is placed at a distance R = 2 meters from the source.

(a) Assuming that all energy which falls on the atoms is absorbed by them, show that the power absorption by a single atom of radius r is given approximately by:

$$P = P_0(\frac{2}{2R})^2$$

(b) Let ϕ the work function of one metal, be = 3 eV. Calculate the minimum irradiation time required to make an atom eject an electron.

8.4 Theory

The photo-electric effect was first observed by Hertz in 1887. It was noticed that negative charges were emitted from the surfaces of some metals when exposed to light. Later, these charges were shown to be electrons. Experiments also suggested that the photocurrent was proportional to the intensity of the incident radiation. Although the latter result was consistent with what is nowadays called the classical. electromagnetic theory, further experiments revealed other results which could not be understood by the classical theory. To illustrate the idea consider the experimental setup shown in figure 8.1.

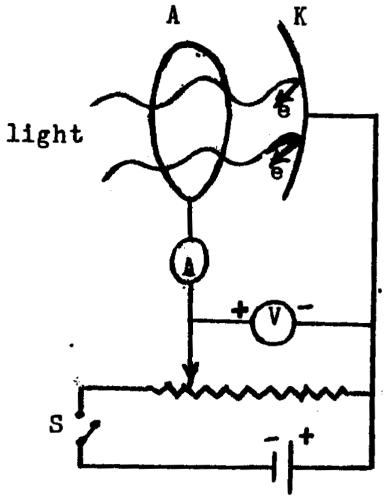

FIGURE 8.1: Schematic experimental arrangement of the photo-electric effect

Light of frequency ν falls on the cathode K causing electrons to be released from its surface. If the switch S is open (no potential is applied to the anode) then some electrons emitted with sufficient kinetic energy, will be able to reach the anode A causing Current to be detected by the ammeter. If now the switch S is closed, negative potential will be applied to the anode (with respect to the cathode). This potential will retard the motion of the electrons thereby decreasing their velocity and thus the measured photocurrent. If the retarding potential is further increased then a critical voltage value will be reached where the fastest electrons are stopped. No current will now be measured by the ammeter. The critical voltage value V8 is called the stopping potential. Since the energy of an electron in a potential difference V_a is eV_a and assuming that the fastest electrons are emitted at a velocity v_m, then at zero current:

$$e|V_s| = \frac{1}{2}mv_m^2 \tag{8.1}$$

where e and m are respectively the charge and the mass of the electron and $V_S < 0$.

The photo-electric effect was investigated, employing the setup described in figure 8.1. The detailed study could be summarized as follows:

1. The stopping potential value V_s depends strongly (In fact linearly) on the frequency of the incident radiation but not on the light intensity
2. The emission of photo-electrons occurs within a very short time interval after the incidence of radiation. The above results were a complete puzzle to physicists at that time. As the intensity of the incident radiation is increased, the energy which is supplied to the atoms per second ta also increased. There fore one would expect electrons to be ejected at a higher kinetic energy. From equation 8.1 it follows that electrons should be stopped at a higher potential. Nevertheless measurement showed that the stopping voltage is independent of intensity. Furthermore since intensity is energy per unit of time per unit of area then the amount of energy which is absorbed by atoms is expected to be proportional to the length of time during which the cathode is irradiated. Straightforward calculations show that the time required for an atom to absorb the minimum energy needed for the release of an electron could typically exceed hours or even days (see problem
3. Careful measurement showed however that electrons were either emitted almost immediately (within leas than 10^{-8}sec.) or never.

An explanation of the photo-electric effect was proposed by Einstein in 1905. Einsteins proposals went beyond a satisfactory explanation of the photo-electric effect phenomenon by offering a new insight into the nature of the electromagnetic radiation. It was suggested that Plancks quantization mechanism of black-body radiation is a general feature of light. He assumed that light is not only emitted but also absorbed by matter in quanta of energy. The energy of each quantum is given by:

$$E = h\nu \tag{8.2}$$

where ν is the light frequency and h is Plancks constant ($h = 6.6 \times 10^{-34} J \cdot s$). Since the kinetic energy of the ejected electron is equal to the energy absorbed by the atom minus the energy spent on releasing the electron from the atom (the binding energy), then:

$$\frac{1}{2}mv^2 = h\nu - \phi_c \tag{8.3}$$

where ϕ_c is the work function of the metal. From equation 8.3 it is clear that the kinetic energy (and thus the stopping potential) of the photo-electrons do not depend on the intensity but only on the frequency of the incident light. Also, it shows that whether an electron will be released from an atom is independent on the time of illumination. The minimum frequency of the least energetic photon capable of releasing electrons is given by:

$$\nu_{min} = \frac{\phi_c}{h} \tag{8.4}$$

8.5 The Experimental Setup

To study the photocurrent as a function of frequency, several spectral lines of the desired wavelengths should be isolated. To avoid the use of filters (why?) a spectral discharge tube which emits discrete spectral lines with well separated wavelengths is used. These spectral Lines can then be easily dispersed by an ordinary prism without much loss of intensity. Since the intensity of such sources is limited, great care should be taken whit. aligning the optical system in order to minimize the losses. The optical system used in our laboratory is described schematically in figure 8.2.

Light from the discharge tube is focused by the lens L_1 (focal length $f_1 = 50$ mm.) on the slit S_1. An image of the slit S_1 is formed by the projecting objective L_2 ($f_2 = 150$ mm) on the slit S_2 which masks the lens L_3 ($f_3 = 200$ mm). When the prism is placed

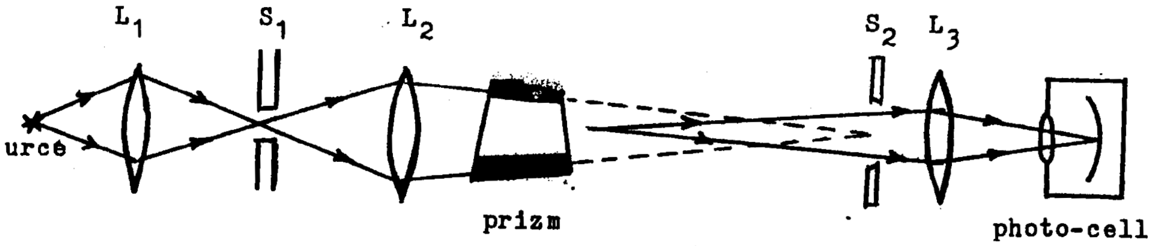

FIGURE 8.2: Optical system arrangement

near the lens L_2, an image of its rectangular cross section is formed by the lens L_3 on the cathode of the photo-cell. The cathode is a potassium fila coated on the inside of a glass cell and the anode is a platinum wire ring mounted inside the glass cell. The photo cell is mounted inside a special casing to provide adequate insulation. Since the potassium evaporates continuously and condenses on the platinum anode it is necessary to heat the anode ring for about 1 minute before every series of measurements. A switch has been installed so that the electric circuit need not to be altered when heating is desired, the electric circuit used for obtaining measurements is described in figure 8.3. When the switch is on position I, the anode wires are shorted and a negative potential

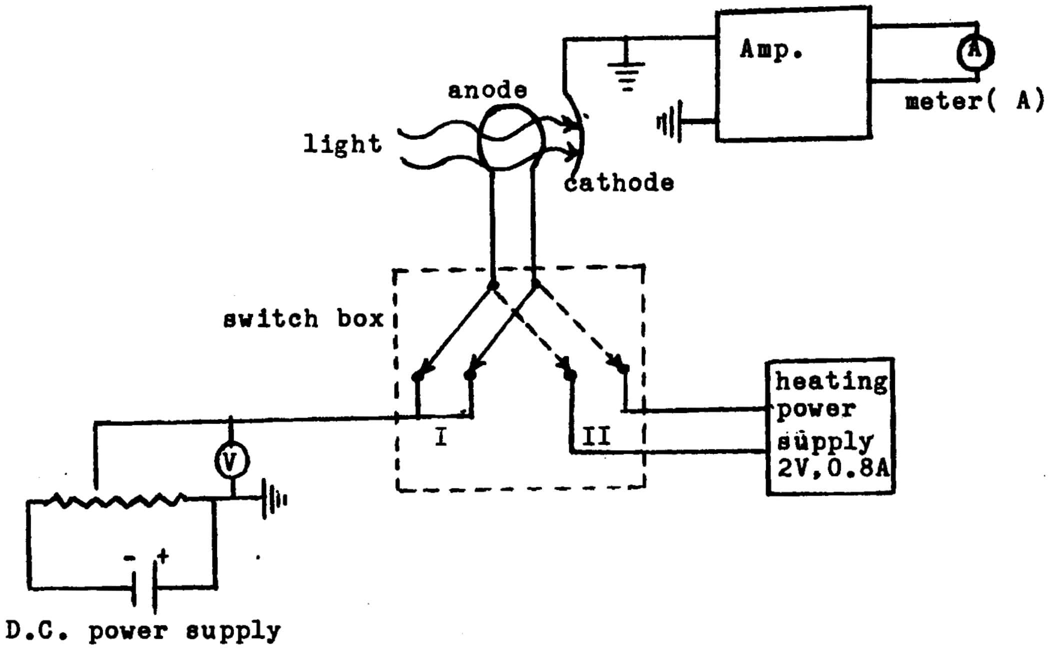

FIGURE 8.3: Electric circuit for measurement of stopping potential

is applied to the anode, through the potentiometer. The photo-current which passes through the cathode (and through the anode) is reed on the meter A. When the switch is on position I, a potential difference is applied by the heater power supply between the two terminals of the ring wire and therefore the anode is heated. When the switch is on position I the value of the anode potential is varied by changing the potentiometer resistance.

8.6 Procedure

1. Switch on the power supply of the mercury light source and wait until the emitted light reaches its ultimate intensity. If the optical setup is properly adjusted you will observe coloured strips on the screen which masks the lens L_3. Do not alter the adjustment of the optical system without consulting the instructor.

2. Switch on the measuring amplifier (if it had not been switched on for you). The amplifier should be left on for at least half an hour. before it gain becomes stable.

3. By slight rotation of the prism, focus the blue colour on the masking card. The colour is focused when it is made as narrow as possible. Now rotate the optical bench which carries the photo-cell and move it sideways until the blue strip penetrates L the slit S_2 and falls on the cathode inside the glass cell at its middle. If the optical adjustment is correct, the shape of the light spot which fall, on the cathode will, be similar to the cross section of the prism. Hake sure that the light does not hit the anode ring in order to minimize the reverse electron current from the anode to the cathode.

4. Connect the circuit shown in figure 8.3. Turn on the heater power supply and switch to position "II". After a few seconds you will notice that the anode ring glows. If ir does not consult your instructor. Do not attempt to change the heating voltage or you might fry the anode ring. After 4 minutes of heating turn off the heating voltage and switch to position "I".

5. Turn on the D.C. power supply which supplies the anode potential and make sure that the anode potential can be varied from 0-2V by varying the potentiometer resistance.

6. Make the room completely dark. Place a screen in front of the prism to prevent light from entering the glass cell. Wait until the current reading is constant and adjust the zero setting of the amplifier to make this reading approximately zero. Remove the screen and observe the reading of the photo-current. When the amplifier is at about its maximum gain and the anode potential is zero, you

would read a current of about $500\mu A$. If the current is much greater, reduce the width of the slit (What for?).

7. You will notice that the zero of the amplifier, will drift as the anode voltage is varied. There is no need, however, to readjust the zero. It is sufficient to read its value for each potential and to subtract it from the total current value. Let the value of current when the photocell is masked be I_0 and the current when the cathode is irradiated be I_c. The net current due to the tight source is then

$$I_l = I_c - I_0 \tag{8.5}$$

8. Measure I_c, I_0, as a function of the negative anode potential in steps of 0.1V for the following colours:

colour	wavelength (A^o)	maximum anode voltage
Blue	4358	-1.5 V
Violet	4047	-1.7 V
Green	5461	-1.0 V
Yellow	5770	-1.0 V
Blue-Green	4916	-1.3 V

Do not forget to focus each colour and to heat the anode before starting a new set of measurements. It should be pointed out that the above suggested values for maximum increment of retardation voltages were found to be necessary when an old photo-cell was used. For such a cell the anode voltage should be raised until the reversed current is isolated (see the next section). The student should be able to terminate the measurement as soon as the value of I_c becomes constant within the experimental error. Notice that if an old cell is used then I_c takes negative value also. Notice also that the photo-current decreases significantly when the blue is substituted by green and yellow. It is possible that you might have to open the slit S_1 in order to obtain about $50\mu A$ of photo-current at zero anode potential.

8.7 Calculations

Substituting equation 8.1 in 8.3 we obtain:

$$|V_0| = \frac{h}{e}\nu - \frac{\phi_c}{e} \tag{8.6}$$

However, it can be shown that ϕ_c in equation 8.6 should be replaced by ϕ_a the work function of the anode. Thus:

$$|V_0| = \frac{h}{e}\nu - \phi_a \tag{8.7}$$

where ϕ_a is in electronvolts. Plotting $|V_0|$ against the frequency ν yields both the Planck constant h and the work function ϕ_a. In practice, there is a problem, however, because determining the stopping voltage $|V_0|$ is not straightforward. The difficulty is caused by the presence of the reversed current which arises from electrons emitted from the anode and collected by the cathode. The emission of electrons by the anode when it is irradiated by light is a result of potassium being continuously evaporated by the cathode and deposited on the anode ring. By heating the anode ring, as we have done, we want to evaporate any traces of potassium from the anode ring. Nevertheless, if the cell is old, reversed photo-emission is bound to occur. The actual curve would look as given in figure 8.4b, tether than in figure 8.4a. For voltage values $|V| < |V_0|$, the measured current is the sum of cathode plus anode currents. When the curve becomes a straight line parallel to the voltage axis, the current will be solely due to the reversed current (why?). At $V = V_0$ the total current is equal to the anode current and thus all electrons emitted by the cathode are stopped. Plot I_l, as a function of voltage for the four colours and determine $V_0 \pm \Delta V_0$ as shown in figure 8.4b. Next, plot the stopping potential as a function of frequency and calculate Planck's constant h and the work function ϕ_a. Calculate Δh and $\Delta\phi_a$ from the graph.

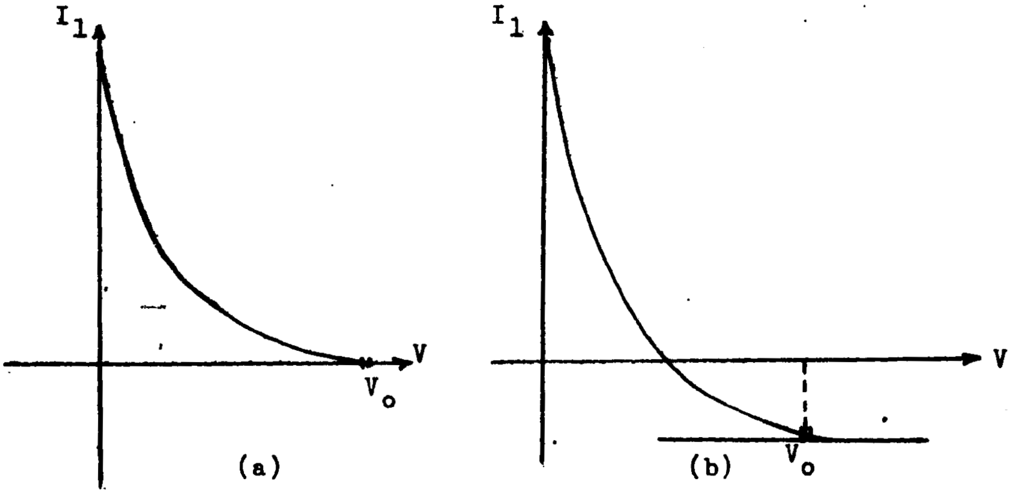

FIGURE 8.4

8.8 Questions

1. What are the advantages and the disadvantages of using a prism rather than coloured filters to isolate spectral lines of the mercury source.
2. The work function of platinum is $\phi_a = 5.3eV$. Does this value agrees with the one you obtained from your data? Explain?
3. When your instructor was taking the data for this experiment, he measured a current of $500\mu A$ for the blue at zero anode potential. Suddenly the door was opened and to his surprise the current dropped to $200\mu A$. Can you offer an explanation?
4. Explain why the anode work function ϕ_a rather than the cathode work function ϕ_e, appears in equation 8.7
5. In the present text it was pointed out that the reversed current can be reduced by reducing the intensity of the incident light. Explain why?

EXPERIMENT 9

Atomic Spectra

9.1 Objectives

1. to measure the wavelengths of the spectral lines of hydrogen and mercury in the visible region using discharge tube sources and a diffraction grating spectroscope.
2. To measure the resolving power of the diffraction grating spectroscope.

9.2 References

9.3 Preliminary Problems

1. Derive the Balmer formula (equation 9.1) from the Bohr model of the atom.
2. Calculate the wavelengths of the first three lines of the Balmer series.
3. Discuss the factors that may influence
   (a) The width of a spectral line.
   (b) The resolving power of a grating spectrometer.

4. A 2mm wide section of a diffraction grating is illuminated by light from the sodium yellow doublet ($\lambda_1 = 5890A^o, \lambda_2 = 5896A^o$). What is the minimum number of lins/mirn required If we are to resolve the yellow doublet in the first order? What would the angular separation of the two lines be?

9.4 Theory

9.4.1 Atomic Spectra

We can describe the electronic structure of atoms by imagining that an atom has a set of energy levels which are partially occupied by the atomic electrons. In the bohr model, these energy levels correspond to the allowed orbits of the electrons about the nucleus. In quantum theory, we discard the idea of electrons traveling In welldefined orbits, but the concept. of energy levels or "allowed states" is still valid. Each energy level can hold up to two electrons, but no more than two, due to the Pauli exclusion principle.

An atom is in its ground state when the electrons fill the lowest energy levels available. This is the lowest energy state of the atom, and an atom will be in this state unless energy is supplied to it by some external source. If one or more electron. are placed in higher energy levels, Leaving one or more vacancies in the lower energy levels, the atom is said to be in an excited state. An atom in the ground state may absorb energy and be promoted to an excited state, in several ways; by absorbing a photon of Light, for example, or by undergoing an inelastic collision with another atom. In a discharge tube, atoms of the gas are excited through inelastic collisions with tree electrons from the discharge. An atom which is placed in an excited state will return to its ground state in a very short time (typically $\approx 10^{-8}$ s). In doing so, it must release its excitation energy, which Is the energy difference between the ground and excited states. This is often done by the emission of a photon. (An atom can also undergo de-excitation through collisions, in which case no photon is emitted).

When an atomic electron makes a transition from a higher energy level to a lover energy level by emitting a photon of light, the frequency of the light will be given by:

$$\nu = \frac{\Delta E_{21}}{h}$$

where

$$\Delta E_{21} = E_2 - E_1$$

and h is Planks' constant.

Here, E_2 and E_1 are the energies of the upper and lower states respectively. An atom has many possible excited states, so many transitions will be possible.

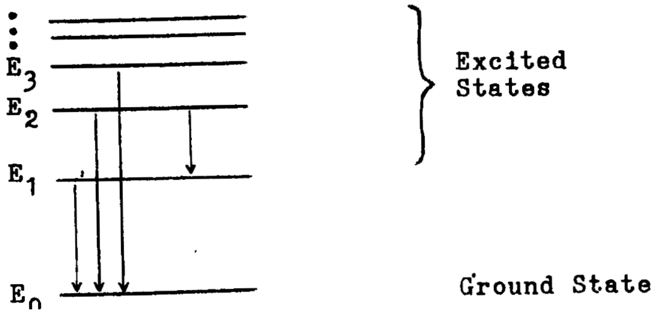

FIGURE 9.1

For the transitions shown in figure 9.1 light would be emitted at frequencies:

$$\nu_{10} = \frac{E_1 - E_0}{h}$$
$$\nu_{20} = \frac{E_2 - E_0}{h}$$
$$\nu_{30} = \frac{E_3 - E_0}{h}$$
$$\nu_{21} = \frac{E_2 - E_1}{h}$$

Transitions between excited states, as well es transitions from an excited state to the ground state, are possible. In general, the transitions between excited states will have a lower probability, and therefore a lower intensity, than transitions to the ground state. Also, these transitions will generally lie outside of the visible region.

A collection of excited atoms, as in a discharge tube, for example, would give off light at a set of specific frequencies. These frequencies correspond to the energy level spacings of the atom. The spectrum of a light source is the distribution 0f light intensity as a function of frequency or wavelength. A spectrum in which light is found only at specific frequencies is called a discrete spectrum. 1f the light intensity is distributed continuously over a range of wavelengths, then we have a continuous spectrum. The visible portion of the electromagnetic spectrum extends f tom wavelengths of approximately 4000 A^o (violet) to approximately 7000 A^o (red). Since each type of atom has its own particular set of energy levels, each atom will have its own, characteristic discrete spectrum. If atoms are supplied with sufficient energy they will be ionized and they will give off a continuous spectrum as well as a discrete spectrum due to the recombination of ions and electrons.

The light given off by an atom at a specific wavelength when the atom makes a transition from one energy state to another is called a spectral line. The width, in wavelength, of a spectral tine La normally very narrow. A spectral line will have a natural line width which is determined by the lifetime of the excited state. According to the uncertainty principle, it a state exists for a time Δt, its energy will be uncertain by an amount $\Delta E = h/\Delta t$. This will cause a corresponding uncertainty in the frequency of the photon emitted when the atom returns to its ground state. Since $\nu = (E_2 - E_1)/h$, the spread in frequency will be $\Delta\nu = \frac{\Delta E_2}{h} - \frac{1}{\Delta t}$. The corresponding uncertainty in wavelength, obtained by differentiating the relation $\nu = c/\lambda$, is $\Delta\lambda = (c/\lambda^2)$. For the green line of mercury ($\Delta t \approx 1^{-8}s$), the corresponding natural line width is $\approx .001A^o$. The observed line width will be much greater than this (typically $\approx 0.1A^o$) for several reasons. The atoms of the source will normally be in random motion, with some atoms moving toward the observer and some moving away. The Doppler shift will cause a corresponding spread in the observed wavelengths. Also, collisions between excited atoms and other atoms will also cause broadening of the spectral line.

9.4.2 The Spectrum of Hydrogen

It is found that the discrete, visible portion of the hydrogen spectrum is described by the Balmer formula;

$$\frac{1}{\lambda_n} = R(\frac{1}{2^2} - \frac{1}{n^2}) \tag{9.1}$$

where

λ_n = wavelength of the spectral line
R = $1.097 \times 10^7 m^{-1}$; the Rydberg constant
n = 3,4,5, ...

The values of λ generated by this formula are in good agreement with the measured wavelengths of the spectral lines of hydrogen. This formula does not, however, describe the spectra of more complicated atoms such as mercury or sodium.

When you observe the spectrum of hydrogen from a discharge tube, you may find that there are broad bands (almost like a continuous spectrum) superimposed over the discrete spectrum of atomic hydrogen. These are due to molecular transitions. (Recall that hydrogen occurs as the diatomic molecule, H_2).

9.4.3 The Spectroscope

Spectroscopes, spectrographs and spectrometers are instruments which separate electromagnetic radiation according to its wave length. These instruments can be employed to study the spectra of atoms. The difference between these three instruments is in the way that the outgoing light is detected;

spectroscope	the spectrum is observed directly
spectrograph	the spectrum is recorded on photographic film
spectrometer	the outgoing light intensity (at a particular wavelength) is measured by a detector such as a photocell or a photomultiplier tube.

In this experiment, a diffraction grating spectroscope will be used to measure the wavelengths of the spectral lines of hydrogen and mercury. A diagram of this instrument is shown in figure 9.2. Light from an extended source, such as a discharge tube, illuminates a narrow slit. (The long narrow alit acts as a point source in 2 dimensions). Light emerging (ro. the slit is focused into a parallel beam by the collimator. The collimator, which may be a single lens or a telescope, is necessary to measure that all

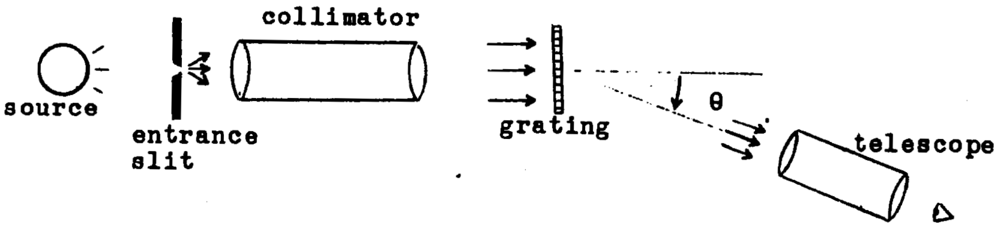

Figure 9.2

of the rays strike the grating at the same angle of incidence. when the beam strikes the grating, it will be diffracted through an angle which depends on the wavelength of the light. When a telescope, focused for parallel rays, is turned to the angle θ the image of the slit will, be observed. (The long narrow slit looks like a line, and hence the term "spectral line"). By measuring the angle θ at which a spectral line appears, its wavelength can be calculated.

The diffraction grating was discussed in experiment 1. We will review only the main points here. When a diffraction grating is illuminated by a monochromatic, parallel beam of light which strikes the grating normal to the plane of the slits, the diffracted intensity will show principle maxima at angles given by;

$$m\lambda = hsin\theta \tag{9.2}$$

where
m = 0,1,2, ...
λ = wavelength of the light
h = grating spacing = $(rulings/mm)^{-1}$

The integer m is called the order of the principle maximum. There will be many secondary maxima between each principle maximum, but these are too weak to be observed. The intensity of the principle maxima will be modulated by an overall diffraction envelope. The angular width of the principle maxima is given by;

$$\delta\theta = \frac{2\lambda}{Nhcos\theta} \tag{9.3}$$

This is the angular separation of the the first intensity minimum on either side of the principle maximum. grating at some angle of incidence ϕ, the grating equation becomes $m\lambda = h(sin\theta + sin\phi)$

If the grating is illuminated by light consisting of several discrete wavelengths we will, obtain a set of principle maxima for each wavelength.

We will obtain a spectrum for each order m. From equation 9.1, we see that for a given order m, the longer wavelengths will be deviated through larger angles. It is possible that some of the orders will overlap. Note that the angular dispersion, the spread in angle between different wavelengths, is larger for higher orders. The angular separation $\Delta\theta$ between the principle maxima of two wavelengths which differ by some

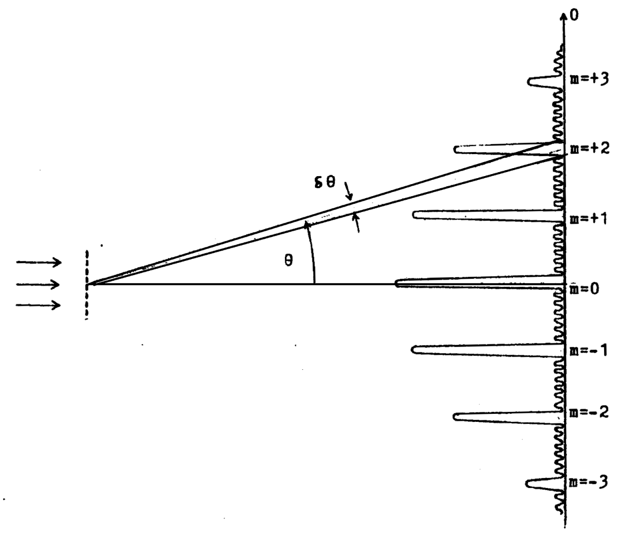

FIGURE 9.3

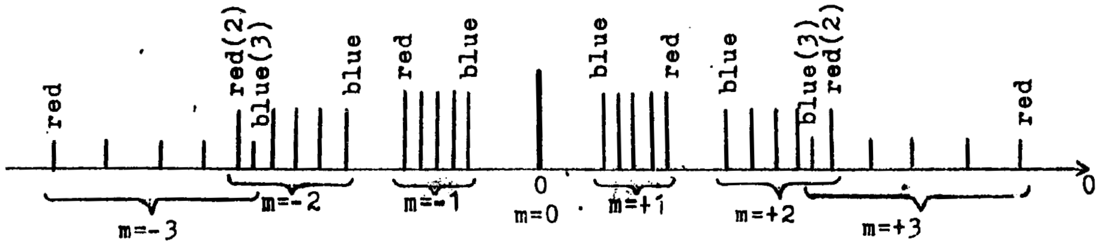

FIGURE 9.4

amount $\Delta\lambda$). can be found by differentiating equation 9.2.

$$\Delta\theta = \frac{m}{hcos\theta}\Delta\lambda \tag{9.4}$$

The resolving power of a spectroscope, spectrograph or spectrometer is the ability of the instrument to resolve two spectral lines which differ in wavelength by a small amount. To define resolving power, suppose that we have two spectral lines at wavelengths λ_1 and λ_2 which can be ”just resolved” by our spectroscope; that is, we can

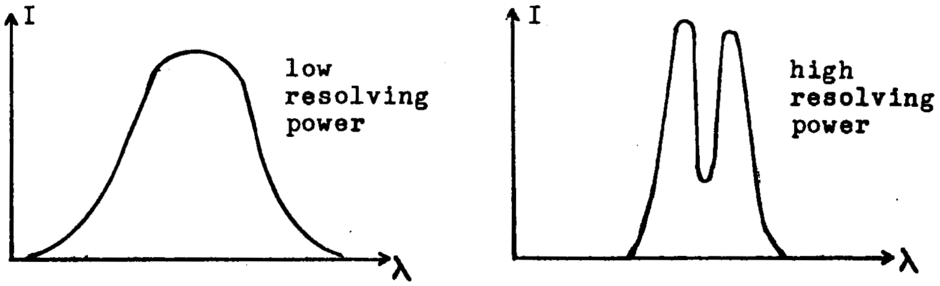

FIGURE 9.5

just barely tell that there are actually two lines present, and not a single line. The resolving power of the spectroscope is defined as;

$$R.P. = \frac{\bar{\lambda}}{\Delta\lambda} \tag{9.5}$$

where

$\bar{\lambda} = \frac{\lambda_1+\lambda_2}{2}$
$\Delta\lambda = \lambda_2 - \lambda_1$

How will we determine when two spectral lines are "just resolved"? If we are looking at the two Lines in a spectroscope, we must use our judgment. We can define the term "just resolved" in a more quantitative way by examining the angular intensity of the lines as shown in figure 9.6. The "lines" that we are observing are principle maxima of the diffraction pattern, with angular width $\delta\theta$. given by equation 9.3. The angular separation of the two principle maxima is given by equation 9.4, with $\Delta\lambda = \lambda_2 - \lambda_1$.

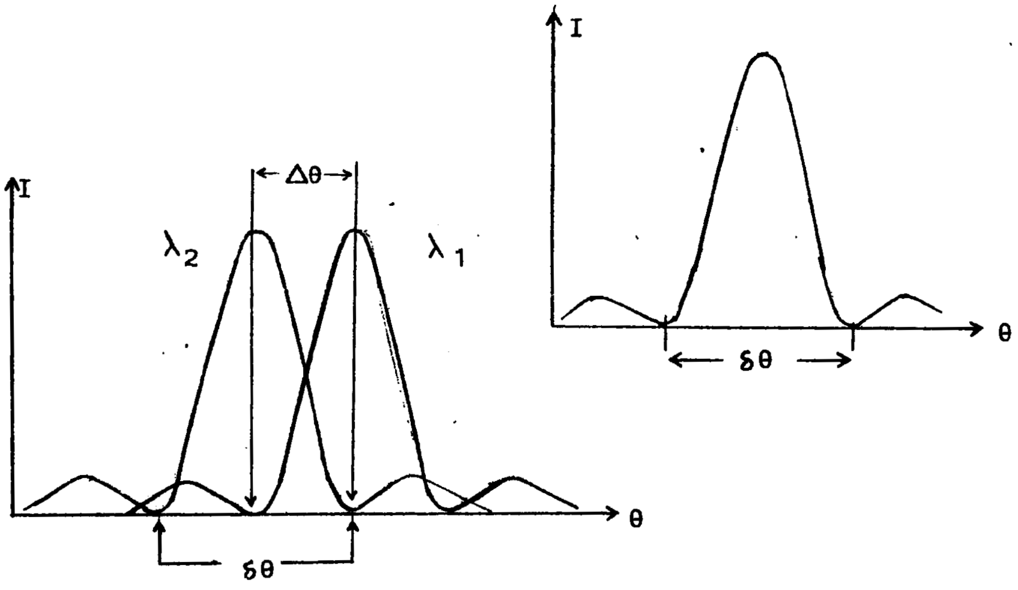

FIGURE 9.6

A common definition of "just resolved" is the Rayleigh criteria which state, that two lines can be considered as just resolved if the peak of one principle maximum falls on the minimum of the other, as shown in figure 9.6. (An equivalent statement of the Rayleigh criterion is that the separation of the peaks should be one half their width). This definition of "just resolved" is somewhat arbitrary, however, it is useful because it gives us a way to compare the resolving power of different instruments. Using equations 9.3 and 9.4, the Rayleigh criterion implies;

$$\Delta\theta(separation) = \frac{1}{2}\delta\theta(width)$$
$$\frac{m}{hcos\theta}\Delta\lambda = \frac{\bar{\lambda}}{Nhcos\theta}$$

$$R.P. = \frac{\bar{\lambda}}{\Delta\lambda} = mN \tag{9.6}$$

The widths of the two line, will differ slightly because the wavelengths of the two lines are different. The difference In wavelength will normally be quite small, however, so the difference in the widths of the principle maxima will be negligible. We therefore use the verge nf the two wavelengths to calculate the width of the principle maxima. Note that the resolving power is proportional to the order m and to the total number of rulings of the grating that are illuminated. To obtain high resolving power, it is important to use a grating having a large number of lines/mm and to use the ent.re area of the grating by making the collimator sufficiently large in diameter. Equation 9.6 represents the maximum resolving power of a diffraction grating instrument. The maximum resolving power is limited by the width of the principle maxima of the diffraction pattern. In the actual, instrument, factors such as the finite width of the entrance slit (which means that not all of the rays striking the grating will be parallel), limited sensitivity of the detector and imperfections in the optical. system will further reduce the resolving power of the instrument.

In some instruments, a prism is used in place of a grating to disperse the incoming Light according to wavelength. The prism has an advantage in that all, of the light intensity is concentrated in one spectrum, while in a diffraction grating the light intensity is spread over several orders. A disadvantage of the prism is that one must know the refractive index of the prism material as a function of wavelength in order to calculate the wavelength of a spectral line from the angle S at which it is observed. (In practice, one would usually use a known spectrum to calibrate the instrument). Ln the case of the diffraction grating, however, it is easy to calculate . from I using equation 9.2. Also, the resolving power available from diffraction grating instruments is generally higher than the resolving power that can be obtained from prism instruments. High quality spectrographs and spectrometers usually use a reflection grating rather than a transmission grating.

9.5 Procedure

NOTE: DO NOT TOUCH THE SURFACE OF THE DIFFRACTION GRATING

1. Focus telescope and collimator - Take the spectroscope to a window. First, bring the telescope cross wires into focus by pushing or pulling the eyepiece. Then, focus the telescope on a distant object such as another building. (Use the focus knob only, do not change the position of the eyepiece). Do not make any change in the telescope focus after you have done this. To focus the collimator, place a discharge tube behind the entrance slit arid view the image of the slit through the telescope. Adjust the collimator focus knob to bring the image of the slit into focus. Since the telescope is focused for objects at infinity, the collimator will now be focused at infinity as well.

2. Mount grating and set plane of grating perpendicular to rays from collimator Place the 80 ruling/mm grating in the holder. (If necessary, bend the retaining clips to hold the grating securely in place). The grating holder should be normal to the line AB of the leveling screws, as shown in figure 9.7. To set the plane of the grating normal to the rays from the collimator, use the following procedure:

   (a) Adjust the position of the telescope to view the slit directly (i.e., the 0th order). Adjust the slit width to obtain a narrow line and center the line on the cross wires. Now, turn the telescope through an angle of exactly $90^o0'$.

   (b) Release the grating table locking screw. Rotate the grating table until you observe the reflected image of the slit. Center the image on the cross wires. this will place the grating at 45^o with respect to the collimator and telescope.

   (c) Turn the grating table through exactly $45^o00'$ and lock it in place. The plane of the grating is now normal to the collimator. Return the telescope to its original position, opposite the collimator, and again center the direct image of the slit on the cross wires: Record the reading of the telescope angle verner. This wilt serve as your 0^o reference point.

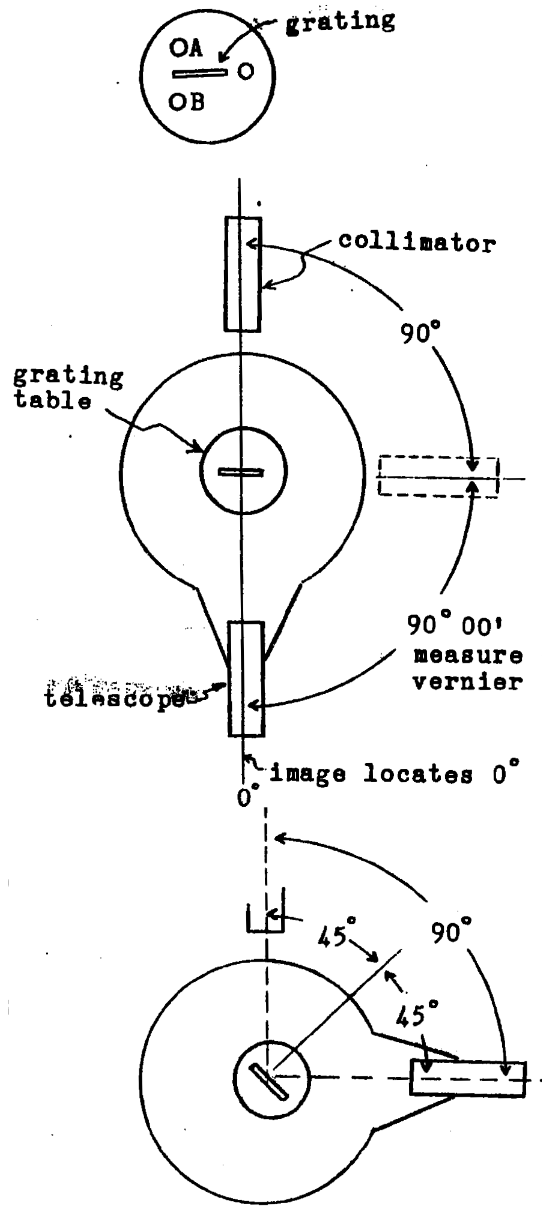

FIGURE 9.7

3. Level the grating table - Observe the diffracted orders on both sides of the zero order and adjust leveling screw C (figure 9.7) until all of the orders appear at the same height within the field of view.

4. Measure the diffraction angles for the spectral lines of hydrogen-Place a hydrogen discharge tube behind the slit, while observing one of the diffracted orders through the telescope, adjust the position of the discharge tube to obtain maximum intensity. Measure the diffraction angles of the observed spectral lines for as many orders as possible. For each wave length in each order, you will have two angles, $\theta+m$ and θ_{-m} measured on either side of the principle maximum. If

necessary, you may increase the slit width to make the less intense lines visible in the higher orders.

5. Measure the diffraction angles for the spectral Lines of mercury - Repeat the above measurements using a mercury discharge tube.

6. Measure the resolving power of the spectroscope - Illuminate the slit using a sodium lamp. Make the entrance slit as narrow as possible. Determine the lowest order, m_0, in which the yellow doublet is just resolved. Also, record the largest order in which the doublet can be observed. Hold a white piece of paper in front of the grating and estimate the width of the grating that is being illuminated.

7. Observe the sodium doublet and measure the diffraction angle. of the green line of mercury using e grating having 572.8 rulings/mm. - Replace the 80 ruling/me grating by one having 14550 ruling/inch. Observe the sodium yellow doublet. You should find that it is clearly resolved. Replace the sodium lamp by the mercury discharge tube. Measure the diffraction angles for the green line of mercury (as in part 4).

9.6 Calculations

1. From your data on the hydrogen and mercury spectra (parts 4 and 5) compute the angle between each line and the zero order (0^o reference point) for all orders. Take the sine of each angle and average the values of $sin\theta_{+m}$ and $sin\theta_{-m}$. Use equation 9.2 to calculate the wavelength corresponding to each angle.

2. Calculate the uncertainty in each wavelength due to the uncertainty in the angle readings. (Neglect the uncertainty in the grating spacing). Present the results of the above calculations in a table.

3. To obtain a value of the wavelength for each spectral line we wish co average the values of λ obtained from the different orders. Since we are averaging numbers which have different uncertainties weighted average should be used. The weighted average is given by:

$$\bar{X} = \frac{\sum w_i X_i}{\sum w_i}$$

where

$$w_i = \frac{1}{(\Delta X_i)^2}$$

ΔX_i = uncertinity in X_i

In the weighted average, points having a smaller uncertainty will have a larger weight w_i, and will Contribute more in determining $\bar{X}$ than points having a larger uncertainty. The uncertainty in the weighted average is

$$\sigma_{\bar{x}} = \sqrt{\sum w_i}$$

Compute the weighted average of the wavelengths for each spectral line and its uncertainty. Compare your measured wavelengths for hydrogen with the values calculated using the Balmer formula. Compare the measured wavelengths for mercury with the values given in wavelength tables (see the CRC handbook of Chemistry and Physics, for example).

4. Look up the wavelengths of the yellow lines of sodium and compute the observed resolving power of the spectroscope with the 80 ruling/mm grating. Estimate the maximum theoretical resolving power, m_0N, and compare with the observed resolving power. If mm is the maximum order in which spectral lines can be detected then the ultimate resolving power of the spectroscope is:

$$R.P.(m_m) = R.P.(m_0)\frac{m_m}{m_0}$$

   Compute the ultimate resolving power.

5. Calculate the wavelength of the green line of mercury using your data taken with the 572.8 ruling/mm gratin3. Compare this result with the wavelength obtained using the 80 ruling/izi. grating and with the accepted value obtained f ro. the wavelength tables.

9.7 Questions

1. What is the minimum doublet separation that can be resolved by the spectroscope that you used with the 80 ruling/mm grating.

2. If the value of the grating spacing used in your calculations is not correct then there will be a systematic error in your measured wavelengths of the hydrogen and mercury spectra. Do you see any evidence that such a systematic error exists?

3. Since the uncertainty in the angle measurement is fixed, wavelengths can be measured more accurately at large diffraction angles than at small diffraction angles. Large diffraction angles can be obtained by using a gracing with a small number of ruling/mm and measuring the diffracted wavelengths in a high order or by using a grating with a sufficiently large number of ruling/ma to place the first order spectrum at a large angle. A similar remark applies to achieving large resolving power. We can make the product mN large by using large m / small N or vice versa. Would there be any advantages to using a grating with a large number of ruling/mm? Any disadvantages?

EXPERIMENT 10

Electron diffraction

10.1 Objectives

To observe the diffraction of electrons and to employ it In probing the structure of crystal.

10.2 References

10.3 Preliminary Problems

1. Calculate the de Brogue wavelength of a microscopic particle of mass m = 1 mg moving with a speed v = 3 m/s. If such a particle passes through a slit of width w = 0.01 mm. Would you expect to observe a diffraction pattern?
2. A beam of electrons is accelerated by a potential difference of 3kv. The beam passes through randomly oriented crystals and thus forms a diffraction ring on a screen whose distance is 10 cm from the crystal holder. If the separation between Bragg's planes in the crystals is d = 1.5 A^o, calculate the radius of the ring.

10.4 Theory

10.4.1 Matter Waves

In the experiment on physical optics (Exp. 1), the wave aspect of electromagnetic radiation was studied. It was observed that diffraction occurs when light passed through a narrow slit which demonstrated the wave nature of light. In the photoelectric effect experiment, however, the wave theory failed to explain the experimental observations. Light had then to be treated a a beam of particles (photons) In order to completely explain the photoelectric effect. A particlelike behaviour had also to be attributed to light in order to explain the compton effect.

This wave-particle dual behavior of light, namely that light behaves like a wave under some conditions and like a particle under others was very puzzling from the point of view of classical physics. In classical mechanics the motion of a particle is described by a well defined position and momentum while a wave is characterized by its amplitude and its wavelength.

In 1924 de Broglie suggested that the failure of the theory to account for the experimental results was due to the traditional distinction between waves and particles which he claimed was incorrect. He argued that all particle, not just photon, exhibit a wave aspect. According to de Broglie's hypothesis, a particle moving with momentum p would have a wavelength λ given by:

$$\lambda = \frac{h}{p} \tag{10.1}$$

where h is Planck's constant. The wave amplitude which was associated with the motion of the particle A(x,t) was chosen so that $|A(x,t)|^2 \Delta x$ gives the probability of finding the particle within an interval Δx centered at x_0. What sort of wave amplitude might we use to describe the motion of a particle? A plane monochromatic wave, for example, is not adequate. This is because such a wave extends from $-\infty$ to ∞ with constant amplitude. This means that the particle can be found at any point in space with equal probability. The position of a particle whose motion is represented by a plane monochromatic wave is thus completely uncertain. If, on the other hand, the particle is represented by a wave packet of finite extension, then the position of the particle would be known within a finite uncertainty Δx as shown in figures 10.1 and

10.2 where the amplitude distribution and the probability distribution $p(x, t_0)$ at a fixed time t_0 are shown

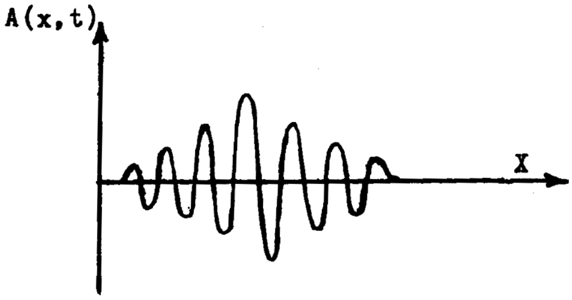

FIGURE 10.1: A wave packet

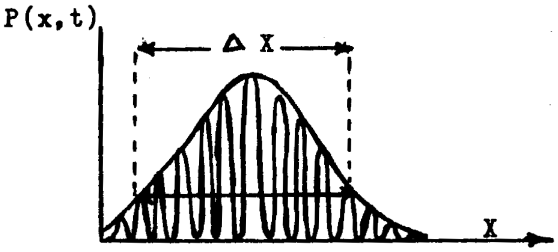

FIGURE 10.2: Probability distribution

The wave represented in figure reffig-1-diff could be regarded mathematically as a superposition of many plane waves of different frequencies ai in the ease of ordinary waves (see the experiment on harmonic analysis). Moreover it is only reasonable to assume that the particle travels at the group velocity v_g of the wave packet. Therefore:

$$v_g = \frac{P}{m} = \frac{d\omega}{dk} \tag{10.2}$$

where ω is the angular frequency, m is the mass of the particle and k is the wave number ($k = \frac{2\pi}{\lambda}$). Assuming that Planck's energy quantization applies for material particles, then

$$E = \frac{P^2}{2m} = \hbar\omega \tag{10.3}$$

where E is the energy of the particle and $\hbar = h/2\pi$. From equations 10.2 and 10.3 we obtain:

$$\frac{dv}{dk} = h/m$$

and assuming that k = 0 for v = 0, we obtain

$$P = \hbar k$$

which is equivalent to de Broglie's formula.

10.4.2 Braggs Diffraction

If material particles behave like waves then why don't they show a wave behaviour in ordinary experiments? Why don't particles form diffraction patterns when they pass through narrow slits in the same manner as light waves do? To answer these questions, we first recall that the wave properties of light (diffraction) were noted only when the dimensions of the apertures through which tight passed were of the same order as the wavelength. Otherwise, interference and diffraction were not observed and light obeyed the rules of ray optics. Microwaves, for example, show diffraction when they pass through apertures of a size of about 1 cm while to observe diffraction of light waves the dimensions of the aperture should be a fraction of a millimeter. To design a diffraction experiment for de Broglie waves we should therefore estimate their wavelength. Consider an electron which is accelerated by a potential difference V. Assuming non-relativistic conditions the velocity of this electron can be found from the equation

$$\begin{aligned} \frac{1}{2}mv^2 &= eV \\ v &= \sqrt{\frac{2eV}{m}} \end{aligned}$$

From equation 10.1, the wavelength is then given by

$$\lambda = \frac{h}{mv} = \sqrt{\frac{h^2}{2emV}}$$

which Can be written

$$\lambda = \frac{\sqrt{\frac{(hc)^2}{2mc^2}}}{\sqrt{E}}$$

where E is the energy of the electron, in electron volts. With $hc = 1.24 \times 10^4 ev - A^o$, $mc^2 = 0.510 \times Mev$ we obtain the convenient from

$$\lambda = \frac{12.25}{\sqrt{E}} A^o \tag{10.4}$$

For E = 1kv, $\lambda = 0.4A^o$.

This value of the wavelength is obviously much smaller than the dimensions of any ordinary slit. For a microscopic particle of a much larger mass the wavelength is even shorter and thus it will not normally show a wave behaviour.

It took three years to verify the de Broglie hypothesis experimentally. It was realized that since the wavelength associated with an accelerated electron beam is of the order of lA^o, a diffraction pattern could be obtained if such a beam is scattered from the atomic planes of a crystal, in the same manner that x-rays where diffracted by crystals. The separation between these planes that are known as Bragg's planes is comparable to the wavelengths given by equation 10.4. Consider a beam of electrons which is incident on a series of atomic planes oriented at an angle $\theta/2$ relative to the beam as shown in figure 10.3.

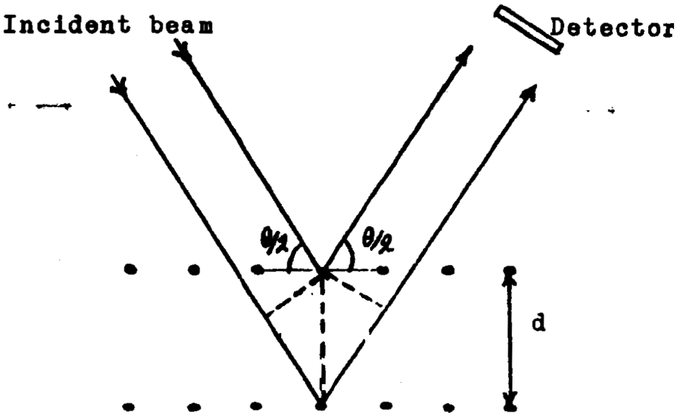

FIGURE 10.3: Bragg diffraction of an electron beam

The path difference between the two rays shown in figure 10.3 is $2dsin(\theta/2)$. Therefore, to obtain constructive Interference at the detector this distance should be an integral multiple of λ that is

$$2dsin\theta/2 = m\lambda \qquad (m = 1, 2, ...)$$

and for small angles Braggs condition becomes

$$(d)(\theta) = m\lambda \tag{10.5}$$

10.5 The experimental set up

The diffraction tube (TEL 555) comprises a gun which emits a converging narrow beam of electrons as shown in figure 10.4.

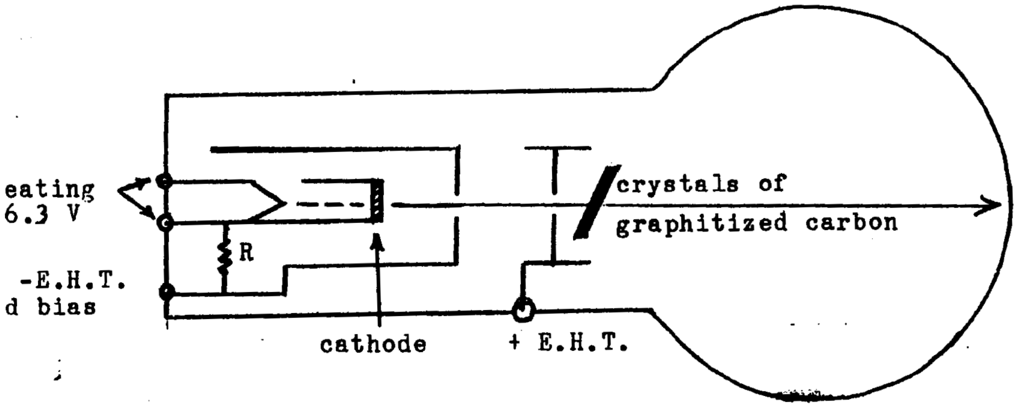

FIGURE 10.4: The electron diffraction tube

The gun assembly contains a cathode which is heated indirectly by a 6.3 V Ac source (4 mm sockets). The emitted electrons are collimated by a narrow aperture. The intensity of the beam can be controlled by applying a negative DC voltage of 50V across the resistor R (2mm socket). The electron beam is then accelerated by a few kilovolts applied to the anode. Diffraction of the beam occurs after it passes through a layer of graphitized carbon supported on a fine mesh grid on the exit of the gun. The diffraction pattern is observed visually when the beam hits the inner surface of the tube which is coated with a luminescent screen. To prevent collisions with air molecules, the tube is evacuated.

Let an electron beam with an angle of incidence $\theta/2$ be scattered by a single series of parllel Brags planes as shown in figure 10.5. It is easy to verify that the beam would be scattered at an angle θ with respect to the direction of incidence. The beam, would thus hit the screen at a distance $r = L.tan\theta$ from the axis. For small angles

$$r = L\theta \tag{10.6}$$

If the diffraction crystals consist of many Bragg planes, randomly oriented, but all with the same separation d, then the outcome would be a ring of diameter D = 2r. If other Bragg planes of different d's are present then additional rings corresponding to different θ's (which satisfy Braggs condition) would be present. Substituting equation 10.6 into equation 10.5 we obtain

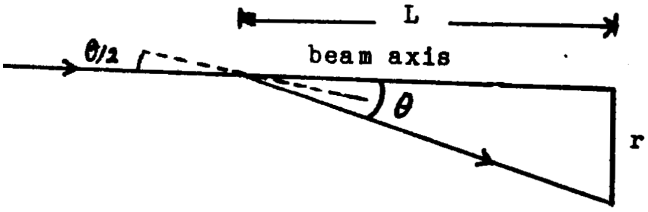

FIGURE 10.5: Calculation of the ring radius

$$\lambda = \frac{dD}{2L} \tag{10.7}$$

where the value m = 1 in equation 10.5 was chosen. By measuring the diameter D as a function of the accelerating voltage and calculating λ from equation 10.4, the separation of Bragg's planes can be calculated.

10.6 Measurement and Calculations

1. take sure that the Electric High Tension is switched off. Connect the circuit as shown in figure 10.3 and show it to your instructor.
2. Switch on the mains of the power supply. This will operate the heating voltage, but not the E.H.T. circuit. Adjust the E.HT. to about 3kv and then switch it on. Observe the two rings on the screen.
3. You will be supplied with a permanent magnet. Adjust its position and its orientation on the table so that the center of the rings, lies in the middle of the screen. If a biasing voltage source is supplied, adjust its value for best focusing.
4. Vary the E.H.T. and observe the corresponding change in the: diameter of the rings. Decide on the working range.
5. Measure the diameters of the inner and outer rings as functions of the accelerating voltage. Estimate the error in each measurement.

6. Plot the diameters of both rings as functions of voltage. From the slope calculate the spacings of the two sets of diffracting planes. The distance between the crystal holder and the screen is L = 13.5 cm.

10.7 Problems and Questions

1. Derive the de Broglie relation using the relativistic expressions for energy and momentum.
2. The structure of carbon atoms is hexagonal. Is this consistent with your results?
3. Does the permanent magnet you used affect the diameter of the ring?

EXPERIMENT 11

The Frank-Hertz Experiment

11.1 Objectives

Verification of energy quantization of atoms.

11.2 References

11.3 Preliminary Problems

1. Estimate the population ratio of energy Levels at room temperature for typical optical and microwave transitions.
2. In a transition between two energy levels of a certain atomic system, radiation is emitted at a frequency of 1000 GHz. Through what potential difference should an electron beam be accelerated in order to excite transitions between these two levels in inelastic collisions with the atoms?
3. Why are two grids used in the Frank-Hertz tube? Why not use a single grid and apply a higher potential to it if necessary?
4. How would the Frank-Hertz curve shown in figure 11.4 change if the terminals of the battery shown in figure 11.5 are inverted.

11.4 Theory

The study of the photoelectric effect led to the conclusion that electromagnetic radiation energy is quantized. Moreover, it was already known from spectroscopic experiments, that radiation is emitted by atoms at discrete frequencies. A satisfactory interpretation of these results was offered by Bohr's model, which assumed that atoms may only exist in discrete energy states. To understand the idea, consider a pair of energy levels as shown in figure 11.1. In a system of $(N_1 + N_2)$ atoms, let N_1 atoms have an energy equal to E_1 and N_2 atoms have an energy equal to E_2. In this case we say that N_1 atoms occupy the energy level E_1 or that the population of E_1 is N_1. Similarly, N_2 atoms occupy the energy level E_2 or the population of E_2 is N_2. The energy levels E_1 and E_2 are measured with respect to some reference energy $E_0 = 0$. At thermal equilibrium the population ratio is given by Boltzmann's distribution

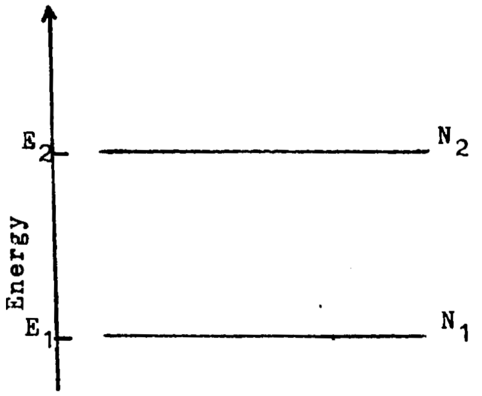

FIGURE 11.1: Two-level quantum system

$$\frac{N_2}{N_1} = e^{-[E_2-E_1]/kT]} \tag{11.1}$$

where T is the absolute temperature of the atomic system and k is Boltzmnnn's constant ($k = 1.38 \times lO^{-23} J/K$). For $E_2 > E_1$ it follows that $N_2 < N_1$, or in other-words, the higher the energy level, the smaller the population (at thermal equilibrium). Transition, from E_2 to E_1 and from $E_1$to E_2 could of course occur. An atom which makes a transitions from to E_2 absorbs an amount of energy equal to $\Delta E = E_2 - E_1$. An atom which makes a transition from E_2 to E_1 radiates an equal amount of energy. We write

$$\Delta E = E_2 - E_1 = h\nu \tag{11.2}$$

where h is Planck's constant and ν is the frequency of the radiated wave. It should be noted that according to Bohr's model, transfer of energy to atomic systems by any mechanism, electromagnetic radiation or otherwise, could only occur in discrete quantities. In the FrankHertz experiment, transfer of energy to atomic mercury vapour occurs via inelastic collision with energetic electrons.

11.4.1 The Frank-Hertz tube

An atomic vapour that constitutes a two-level quantum system as in figure 11.1 is contained in the space between the two grids g_1 and g_2.

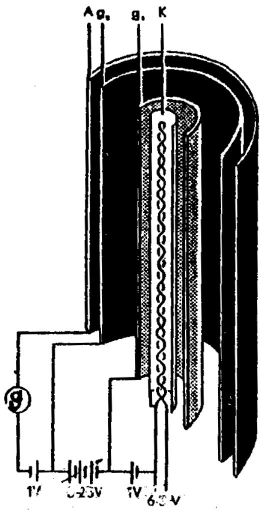

FIGURE 11.2: Diagram of Frank-Hertz tube

The tube contains coaxial electrodes as shown in figure 11.2, K: cathode, g_1: space charge grid, g_2: accelerating grid, A: collector anode. The tube is evacuated to prevent collisions with air molecules. A drop of metallic mercury is placed in the space between the two grids. When the tube is inserted in an oven and heated to a temperature of about 180^{o}C the mercury vapour causes the pressure in the tube to reach a value of 10 torr.

Emission of electrons from the cathode is obtained by heating it, by applying a 6.3V to a filament which is attached to it. Initial acceleration of the emitted electrons is made by applying a potential of about 1V on the grid g_1. This helps the electrons to penetrate through the potential barrier which is established by the space charge near the cathode. The potential configuration which is used to excite the atomic vapour is shown in figure 11.3. As it will be shown later this configuration is suitable when excitation of a single transition between a pair of energy Levels is desired. If excitation of several transitions is needed, a different potential configuration should be used.

The electrons penetrating through the grid g_1 are accelerated by a potential difference (with respect to the cathode which is set at zero potential). A retarding potential of about -1.5V (with respect to g_2) is applied to the Anode. Only those electrons that reach with kinetic energy greater than 1.5 eV will be collected by the anode. Obviously

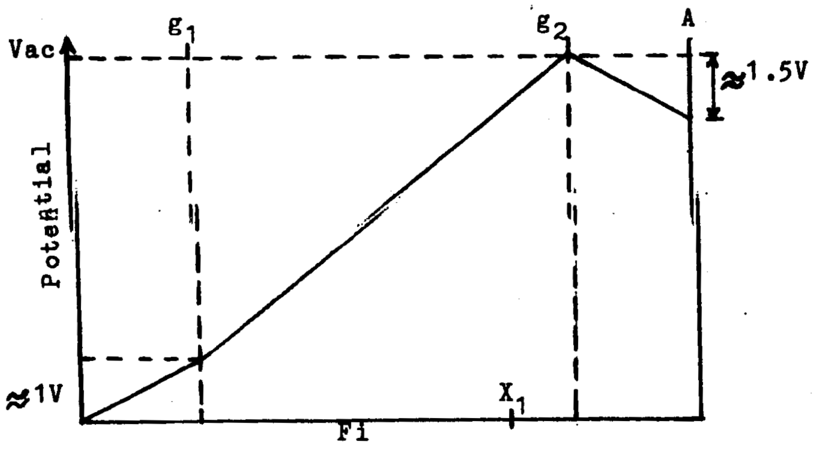

FIGURE 11.3: The applied potential in the Frank-Hertz tube

the retarding, potential barrier will only be effective in stopping those electrons which lost most of their energy via collision, with the mercury atoms in the interaction region between the two grids. The arrival of electrons to the anode is indicated by the reading of the current meter C.

Let us follow the motion of electrons traversing the interaction region between the two grids and neglect for simplicity their velocity distribution. For $1eV < eV_{ac} < \Delta E$ no electrons will have an energy high enough to excite the mercury atoms. Electrons will be scattered elastically and arrive at the grid g_2 with kinetic energy higher than the 1.5 eV anode retarding potential. They will be collected by the anode causing a deflection on the current meter. Further increase of the accelerating potential causes the current to increase monotonously. If V_{ac} is further increased so that $eV_{ac} = \Delta E$ near the grid g_2, electrons will attain, as they reach g_2, enough energy to excite the mercury acorns via inelastic collisions. They will therefore loose their energy and become unable to penetrate through the negative anode potential. A drop in the anode current is thus observed. Now let the accelerating potential be increased so that its value at g_2 reaches a value above ΔE but not beyond $2\Delta E$, $(\Delta E < eV_{ac} < (2\Delta E)$. In this case electrons will gain kinetic energy equal to the energy level separation ΔE at an earlier distance, say x_1. Electrons that do not collide at will be further accelerated and arrive at the anode at a higher energy than before. Moreover, electrons that do collide at x_1 and thus loose their energy at x_1 will of course be accelerated in the space betweenx_1 and g_2 and therefore contribute to the anode current. Monotonous rise in the anode current will again be observed in this energy Interval. As the voltage V_{ac} is further increased the point moves back towards g_1 until a value $eV_{ac} = 2\Delta E$ Is reached. x_1 is now halfway between the two grids. Electrons that collide with mercury atoms at this position and hence loose their energy there, will again attain an amount of energy equal to ΔE as they arrive at g_2. They will loose their energy by exciting the same transition near g_2 and cause the anode current to drop. In general the anode current in a Frank-Hertz tube rises continuously in the energy intervals $m\Delta E < eV_{ac} < (m+1)\Delta E$ (m 1,2,3, ...). Dips (minima) in the anode current are obtained for $eV_{ac} = \Delta E$. A typical plot of the current behaviour is shown in figure 11.4.

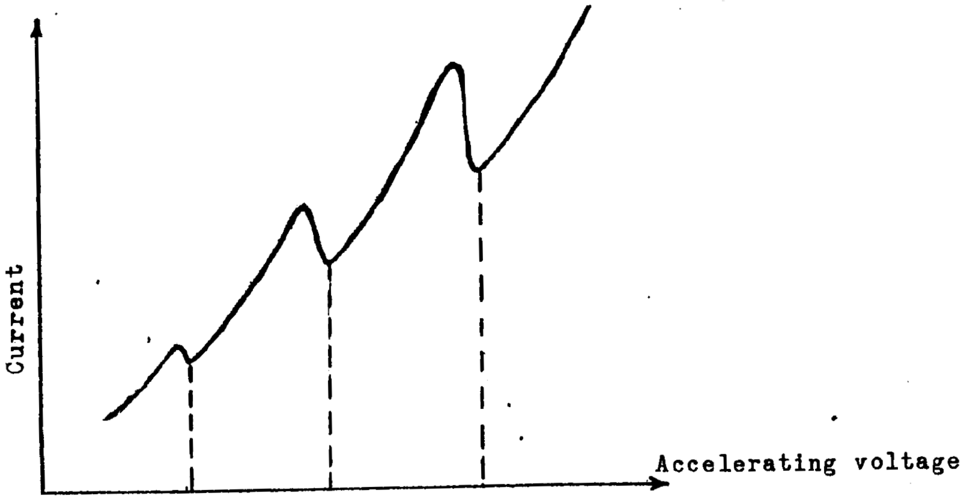

FIGURE 11.4: Current behavior in Frank-Hertz tube

11.5 Experimental Hints

1. Heating: Heating of the Frank-Hertz tube to the correct temperature is very critical. If the tube is not heated to the appropriate temperature there is no way that peaks (and dips) will, be obtained. If the tube is overheated the current will be too small (why?) and consequently the minima, especially the low voltage ones, would be less pronounced. On the other hand if the tube is too cold, the mean free path of electrons increases. At high voltages therefore, electrons may acquire an energy high enough to ionize the mercury atoms before they have the chance to make an excitation. As a result, the high voltage minima will be washed out. Heating the tube is accomplished by applying the mains voltage to the oven via a variac. It has been found that the optimal operating heating voltages of the tube are between 80 and 90 volts.

2. The experimental setup: The circuit diagram for measuring the tube current as a function of the applied accelerating potential is shown in figure 5. power supply The accelerating voltage is supplied to the grid g_2 across the $5Ok\Omega$ potentiometer.

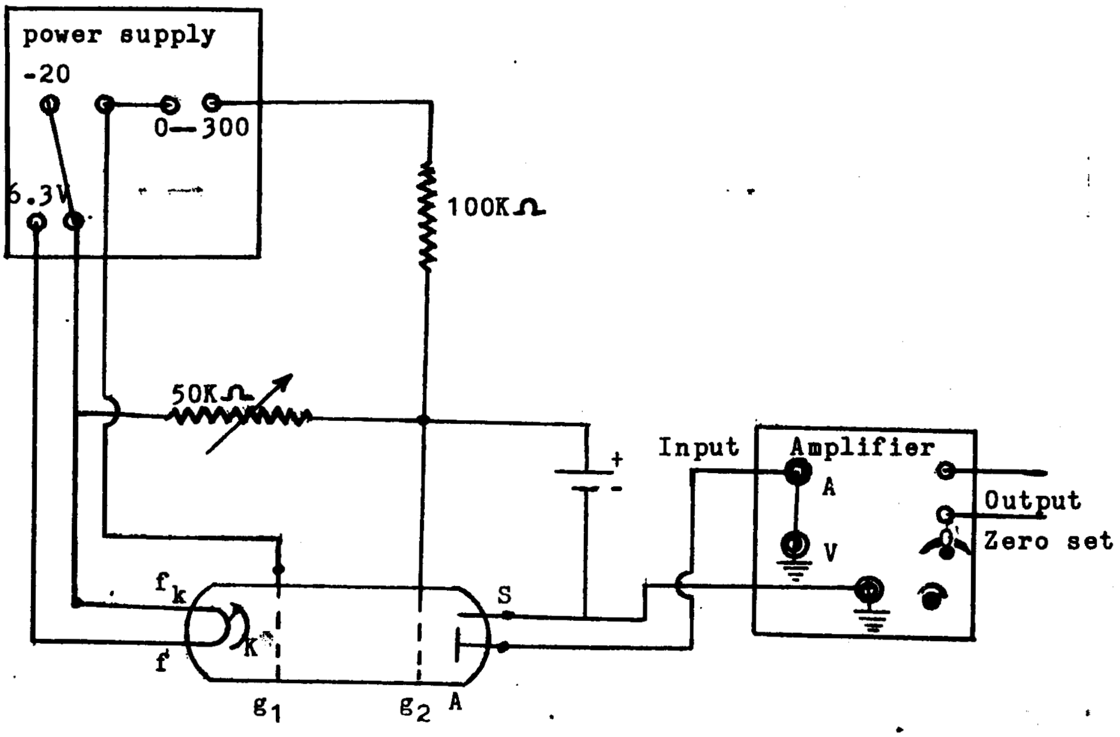

FIGURE 11.5: Current behavior in Frank-Hertz tube

The $lOOk\Omega$ resistance serves as a potential divider so that the full 300 V of the power supply will not fall on the grid. It also prevents damaging the tube if the current gets too high as a result of ionization. It was found mote convenient to fix the supply voltage and vary the accelerating potential by changing the resistance of the potentiometer. The positive potential of the grid g_1 is supplied by applying a negative voltage to the cathode. The screening electrode S is a short pin fused into the glass press of the tube between the supply wires of the anode and the grid g_2. Its purpose is to prevent leakage current to flow between these electrodes. Finally, since the anode current is very low ($\approx 10^{-9}A$), a current amplifier is employed. In this case an ammeter is connected to the output. Alternatively a $20O\Omega$ resistance could be connected to the output and a DVM or an XY plotter could be used.

3. Warnings:

   (a) The Frank-Hertz tube can withstand temperatures only up to $200C^o$. Never let the oven voltage exceed 120V.

   (b) The Frank-Hertz tube contains metallic mercury. Some mercury droplets may, as a result of movement, settle between the electrodes causing short circuit there. The tube, therefore, must not be investigated at low temperatures or before it is thoroughly heated.

(c) Do not apply more than 30V to grid g_2 and more than 6V to grid g_1, otherwise ionization by collision may occur.

11.6 Procedure

1. If the oven heating voltage is adjusted to its operating value ($\approx$ 85V) then an hour is needed before the tube attains its operating temperature. Usually the oven will be switched on for you an hour before the laboratory session begins. If only a short period is available then the following method can be applied: The cold oven is operated for exactly 6 minutes at 220V then the voltage is turned down immediately to 85V. The latter procedure should only be applied in the presence of the instructor.

2. Insert the tube into the oven after wrapping it with aluminium foil. The foil should be earthed. Make sure that the power supply is switched off and connect the circuit es shown in figure 11.5. Show the circuit to your instructor. Connect a voltmeter to measure the voltage of the grid g_1 (with respect to the cathode), a DVM to measure accurately the accelerating potential and the long scale ammeter to measure the anode current.

3. Increase the voltage of the 0-20V supply very slowly up to a value of 1.5V. Adjust the resistance of the potentiometer Adjust the resistance of the potentiometer to about a quarter of its maximum value and increase the voltage of the 0-300 supply until the DVM reads about 10V then reduce the resistance immediately so that the DVM reads zero.

4. Switch the amplifier to the 10^{-9} current scale and increases its sensitivity to about a quarter of its maximum value. Now increase the accelerating potential slowly and observe the current reading. Make further adjustments to the sensitivity and the zero setting of the amplifier as necessary. If the current increases continuously but no "dips" are observed, check to make sure that the terminals of the 1.5V battery are tightly connected.

5. If the current rises very slowly with no dips, the tube is too hot. Reduce the oven voltage by 3V. If the current rises smoothly but then drops suddenly at a voltage

below 20V, the tube is too cold. Reduce the accelerating voltage immediately and increase the voltage oven by 3 volts. At least 3 ”dips” should be obtained.

6. Determine roughly the voltage at the dips and then plot the current of the tube as a function of the accelerating potential.

7. Apply the output voltage built up across the internal resistance of the ammeter($\approx 200\Omega$) to the Y-input and the accelerating potential across the potentiometer to the x-input of an x-y recorder. Plot the Frank-Hertz curve on the graph paper of the recorder by varying the resistance if the potentiometer, for several grid voltage (up to 4V). Record the voltage calibration of the two axes.

11.7 Results and Conclusion

1. Determine the location of the voltage peaks (with the estimated error) Notice that the values you obtained for the peak do not correspond accurately to the excitation energy ΔE of mercury. This is because these voltages should be corrected against the contact potential between the anode and cathode, and against the initial thermal energy of the electrons. However, since this correction remains constant shift of the actual values. The excitation energy should then be take as the difference between the successive peaks.

2. Calculate the separation between the energy levels ΔE. Knowing that mercury atoms were found from spectroscopic measurements to emit radiation at a wavelength of $2537A^o$ when deexcited, evaluate the quality of the method you used.

11.8 Questions

1. When the FrankHertz tube is overheated the low voltage peaks become weak but if the tube is too cold, the high voltage peaks cannot be observed. Why?

2. Why is metallic mercury used in the Frank-Hertz tube? Why cant we simply fill the tube with gas and thus avoid the tedious process of heating? .

3. Your results show that the excitation potential corresponding to each peak is higher than the potential corresponding to the excitation energy of the two levels in question. Why?

4. Your results show that the peaks are not sharp but have a finite width. What causes the broadening?

5. What changes on the circuit diagram shown in figure 11.5 should be made in order to display the Frank-Hertz curve on an oscilloscope ?