Birzeit University

Physics Department

Physics 236-

 **WAVES AND OSCILLATIONS Homework Assignment 1**

1. In steady state of forced vibration show that:

 (a) at low frequencies ω, the phase ϕof the driven system is zero and the amplitude A is independent of ω

 (b) at high frequencies, ϕπ and A depends on ω .

2. Show that for steady state forced vibration with an external force F=F0cosωt:

 (a) the power P is given by

 where b=r the damping constant.

3. A damped harmonic oscillator, driven by a force F0 cosωt, vibrates with an amplitude A(ω) given by :

A(ω)=/ [(ωo/ω − ω/ωo)2 + 1/Q2]1/2

where a is the amplitude as ω → 0, ωo is the natural frequency of oscillation and Q is the quality factor. Show that the amplitude A(ω) is a maximum for a frequency  ωmax = ωo(1 − 1/2Q2)1/2 and that at ωmax the amplitude is equal to  aQ /. (1 − 1/4Q2)1/2  (Hint: Let ωo/ω = u, divide the denominator and numerator by u and investigate the resulting expression inside the square root.)

1. For a value of Q = 10 in Problem 3, find:

 (a) the percentage difference between the natural frequency of oscillation ωo and the frequency ωmax at which the maximum amplitude of oscillation would occur and

(b) the percentage difference between the amplitudes at these two frequencies.

1. A forced oscillator has a natural frequency ωo of 100 rad s−1, a Q-value of 25 and an average input power Pmax at resonance of 50 W. Plot the power resonance curve of the oscillator over the frequency range 92 to 108 rad s−1.
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6. A series *LCR AC-*circuit has C=8.0×10−6 F, L=2.0×10−2 H and R = 75 \_ and is driven by a voltage V (t) = 15 cos ωt V. Determine :

(a) the resonance frequency (Hz) of the circuit and ,

(b) the amplitude of the current at this frequency.

7. A simple pendulum consists of a mass m attached to a light string of length l. When at rest it lies in a vertical line at x = 0. The pendulum is driven by moving its point of suspension harmonically in the *horizontal* direction as ξ = a cos ωt about its rest position (x = 0). There is a damping force Fd = −bv due to friction as the mass moves through the air with velocity v.

(a) Show that the horizontal displacement x of the mass, with respect to its equilibrium position (x = 0), is the real part of the complex quantity z where

and ωo2 = g/l. (b) Assuming a solution of the form z = Aei(ωt−δ), find the phase angle δ between the driving force and the displacement of the mass.

8. The equation of motion of a forced harmonic oscillator with damping is given by

 +kx=F0cosωt.

 

 Assuming a solution x = A(ω) cos(ωt − δ):

* (a)  Give expressions for (i) the instantaneous kinetic energy K, (ii) the instantaneous potential energy U and (iii) the instantaneous total energy E of the oscillator.
* (b)  For what value of ω is the total energy constant with respect to time? What is the  total energy of the oscillator at this frequency?
* (c)  Obtain an expression for the ratio of the average kinetic energy K to the average  total energy E of the oscillator in terms of the dimensionless quantity ωo/ω. Sketch this expression over an appropriate range of ω. For what value of ω are the average values of the kinetic and potential energies equal?
* (d)  Find the variation of the average total energy of the oscillator varies with angular frequency ω.

9.  Two simple pendulums, each of length 0.300 m and mass 0.950 kg, are coupled by attaching a light, horizontal spring of spring constant k = 1.50 N m−1 to the masses. (a) Determine the frequencies of the two normal modes. (b) One of the pendulums is held at a small distance away from its equilibrium position while the other pendulum is held at its equilibrium position. The two pendulums are then released simultaneously. Show that after a time of approximately 12 s the amplitude of oscillation of the first pendulum will become equal to zero momentarily. (Assume g = 9.81 m s−2.)

10.  Two simple pendulums, each of length 0.50 m and mass 5.0 kg, are coupled by attaching a light, horizontal spring of spring constant k = 20 N m−1 to the masses. (a) One of the masses is held at a horizontal displacement xa = +5.0 mm while the other mass is held at a horizontal displacement xb = +5.0 mm. The two masses are then released from rest simultaneously. Using the expressions  xa =1/2 (C1 cosω1t +C2 cosω2t) and xb =1/2 (C1 cosω1t −C2 cosω2t)

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where ω1 and ω2 are the normal frequencies, find the values of C1 and C2. Plot xa and xb as a function of time t over the time interval t = 0 to 10 s. (b) Repeat part (a) for initial conditions: (i) xa = +5.0 mm, xb = −5.0 mm, (ii) xa = +10 mm, xb = 0 mm and (iii) xa = +10 mm, xb = +5.0 mm. (Assume g = 9.81 m s−2.)

11.Consider the example of two identical masses connected by three identical springs on a frictionless horizontal circuit . Combine the equations of motion of the two masses to obtain a pair of equations of the form

+ ω 12 q 1 = 0 a n d + ω 2 2 q 2 = 0

and hence obtain the normal coordinates q1 and q2 and the respective normal frequencies ω1 and ω2.

12.Two identical pendulums of the same mass m are connected by a light spring. The displacements of the two masses are given, respectively, by

xa = A cos(ω2 − ω1)t/2 cos (ω2 + ω1)t/2 , xb = A sin (ω2 + ω1)t/2 sin(ω2 − ω1)t/2

Assume that the spring is sufficiently weak that its potential energy can be neglected and that the energy of each pendulum can be considered to be constant over a cycle of its oscillation.

(a) Find the energies of the two masses , in this case. And show that the total energy of the system remains constant.

(b) Sketch Ea and Eb over several cycles on the same graph. What is the frequency at which there is total exchange of energy between the two masses?

13-Two identical masses of mass m are suspended from a rigid support by two strings of length l and oscillate in the vertical plane as a coupled two simple pendulums. The oscillations are of sufficiently small amplitude that any changes in the tensions of the two string from their values when the system is in static equilibrium can be neglected. In addition the small-angle approximation sin θ=θ can be made.

(a) Find the equations of motions of the upper and lower masses.

  

(b) Assuming solutions of the form x = A cos ωt , find the normal frequencies of the system.