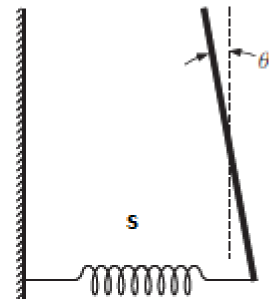
Birzeit University
Department of Physics
Physics of vibrations and waves, Phys236
Spring 2019
Homework 1: Due date Feb. 25th 2019

1. The figure below shows a thin uniform rod of mass M and length 2L that is pivoted without friction about an axis through its mid point. A horizontal light spring of spring constant s has one end attached to the lower end of the rod, and the other end to a fixed wall. The spring is at its equilibrium length when the angle θ with respect to the vertical is zero. Show that for oscillations of small amplitude, the rod will undergo SHM with a period of $2\pi\sqrt{\frac{M}{3s}}$.



2. A particle oscillates with amplitude A in a one-dimensional potential U(x) that is symmetric about the origin, and that U(0) = 0.
   (a) Show that the velocity v of the particle at position x is given by
   $$v = \sqrt{2[U(A) - U(x)]/m}.$$
   (b) Show that the period of oscillation T is given by
   $$T = \sqrt{\frac{8m}{U(A)}} \int \frac{dx}{\sqrt{1 - U(x)/U(A)}}$$
   (c) If the potential U(x) is given by $U(x) = \alpha x^n$ where α is a constant and n = 2, 4, 6, . . . , obtain the dependence of the period T on the amplitude A for different values of n = 2, 4, ...

3. For a body in stable equilibrium, the net force acting on it is zero. If a body is displaced a distance x away from the position of stable equilibrium the force F will be of opposite sign to x. The force can be written as:
   $$F = \sum_{i=0} = C_i x^i$$
   where C_i is a negative constant. For small oscillations about the origin, F = ma becomes
   $$C_1 x = md^2x/dt^2$$
   Comparing this with the standard equation for simple harmonic motion,
   $$C_1 = -(dF/dx)_0 = -[d(-dU/dx)/dx]_0 = (d^2U/dx^2)_0.$$
   Suppose the potential energy in a diatomic molecule can be expressed as the sum of an attractive term b/r^3 and a shorter-range repulsive term a/r^5, that is, $U = (a/r^5) - (b/r^3)$.
   (a) The equilibrium position r_o.
   (b) The spring constant s between the two atoms
   (c) The vibrational frequency of this diatomic molecule if each atom has a mass m.