Chapter 1

1.1	$(2/3)^{10} = 0.0173$ yd; $6(2/3)^{10} = 0.104$ yd (compared to a total of 5 yd)				
1.3	$5/9$	1.4	$9/11$	1.5	$7/12$
1.6	$11/18$	1.7	$5/27$	1.8	$25/36$
1.9	$6/7$	1.10	$15/26$	1.11	$19/28$
1.13	\$1646.99	1.15	Blank area $= 1$		
1.16	At $x = 1$: $1/(1+r)$; at $x = 0$: $r/(1+r)$; maximum escape at $x = 0$ is $1/2$.				

2.1	1	2.2	$1/2$	2.3	0
2.4	∞	2.5	0	2.6	∞
2.7	e^2	2.8	0	2.9	1

4.1 $a_n = 1/2^n \to 0$; $S_n = 1 - 1/2^n \to 1$; $R_n = 1/2^n \to 0$

4.2 $a_n = 1/5^{n-1} \to 0$; $S_n = (5/4)(1 - 1/5^n) \to 5/4$; $R_n = 1/(4 \cdot 5^{n-1}) \to 0$

4.3 $a_n = (-1/2)^{n-1} \to 0$; $S_n = (2/3)[1 - (-1/2)^n] \to 2/3$; $R_n = (2/3)(-1/2)^n \to 0$

4.4 $a_n = 1/3^n \to 0$; $S_n = (1/2)(1 - 1/3^n) \to 1/2$; $R_n = 1/(2 \cdot 3^n) \to 0$

4.5 $a_n = (3/4)^{n-1} \to 0$; $S_n = 4[1 - (3/4)^n] \to 4$; $R_n = 4(3/4)^n \to 0$

4.6 $a_n = \dfrac{1}{n(n+1)} \to 0$; $S_n = 1 - \dfrac{1}{n+1} \to 1$; $R_n = \dfrac{1}{n+1} \to 0$

4.7 $a_n = (-1)^{n+1}\left(\dfrac{1}{n} + \dfrac{1}{n+1}\right) \to 0$; $S_n = 1 + \dfrac{(-1)^{n+1}}{n+1} \to 1$; $R_n = \dfrac{(-1)^n}{n+1} \to 0$

5.1	D	5.2	Test further	5.3	Test further
5.4	D	5.5	D	5.6	Test further
5.7	Test further	5.8	Test further		
5.9	D	5.10	D		

6.5	(a) D	6.5	(b) D

Note: In the following answers, $I = \int^{\infty} a_n \, dn$; $\rho =$ test ratio.

6.7	D, $I = \infty$	6.8	D, $I = \infty$	6.9	C, $I = 0$
6.10	C, $I = \pi/6$	6.11	C, $I = 0$	6.12	C, $I = 0$
6.13	D, $I = \infty$	6.14	D, $I = \infty$	6.18	D, $\rho = 2$
6.19	C, $\rho = 3/4$	6.20	C, $\rho = 0$	6.21	D, $\rho = 5/4$
6.22	C, $\rho = 0$	6.23	D, $\rho = \infty$	6.24	D, $\rho = 9/8$
6.25	C, $\rho = 0$	6.26	C, $\rho = (e/3)^3$	6.27	D, $\rho = 100$
6.28	C, $\rho = 4/27$	6.29	D, $\rho = 2$	6.31	D, cf. $\sum n^{-1}$
6.32	D, cf. $\sum n^{-1}$	6.33	C, cf. $\sum 2^{-n}$	6.34	C, cf. $\sum n^{-2}$
6.35	C, cf. $\sum n^{-2}$	6.36	D, cf. $\sum n^{-1/2}$		

7.1	C	7.2	D	7.3	C	7.4	C
7.5	C	7.6	D	7.7	C	7.8	C

9.1	D, cf. $\sum n^{-1}$	9.2	D, $a_n \not\to 0$
9.3	C, I $= 0$	9.4	D, I $= \infty$, or cf. $\sum n^{-1}$
9.5	C, cf. $\sum n^{-2}$	9.6	C, $\rho = 1/4$
9.7	D, $\rho = 4/3$	9.8	C, $\rho = 1/5$
9.9	D, $\rho = e$	9.10	D, $a_n \not\to 0$
9.11	D, I $= \infty$, or cf. $\sum n^{-1}$	9.12	C, cf. $\sum n^{-2}$
9.13	C, I $= 0$, or cf. $\sum n^{-2}$	9.14	C, alt. ser.
9.15	D, $\rho = \infty$, $a_n \not\to 0$	9.16	C, cf. $\sum n^{-2}$
9.17	C, $\rho = 1/27$	9.18	C, alt. ser.
9.19	C	9.20	C
9.21	C, $\rho = 1/2$		

9.22 (a) C (b) D (c) $k > e$

10.1	$\lvert x\rvert < 1$	10.2	$\lvert x\rvert < 3/2$	10.3	$\lvert x\rvert \le 1$
10.4	$\lvert x\rvert \le \sqrt{2}$	10.5	All x	10.6	All x
10.7	$-1 \le x < 1$	10.8	$-1 < x \le 1$	10.9	$\lvert x\rvert < 1$
10.10	$\lvert x\rvert \le 1$	10.11	$-5 \le x < 5$	10.12	$\lvert x\rvert < 1/2$
10.13	$-1 < x \le 1$	10.14	$\lvert x\rvert < 3$	10.15	$-1 < x < 5$
10.16	$-1 < x < 3$	10.17	$-2 < x \le 0$	10.18	$-3/4 \le x \le -1/4$
10.19	$\lvert x\rvert < 3$	10.20	All x	10.21	$0 \le x \le 1$
10.22	No x	10.23	$x > 2$ or $x < -4$	10.24	$\lvert x\rvert < \sqrt{5}/2$
10.25	$n\pi - \pi/6 < x < n\pi + \pi/6$				

13.4 $\binom{-1/2}{0} = 1$; $\binom{-1/2}{n} = \dfrac{(-1)^n(2n-1)!!}{(2n)!!}$

Answers to part (b), Problems 5 to 19:

13.5 $-\sum_1^\infty \dfrac{x^{n+2}}{n}$

13.6 $\sum_0^\infty \binom{1/2}{n} x^{n+1}$ (see Example 2)

13.7 $\sum_0^\infty \dfrac{(-1)^n x^{2n}}{(2n+1)!}$

13.8 $\sum_0^\infty \binom{-1/2}{n} (-x^2)^n$ (see Problem 13.4)

13.9 $1 + 2\sum_1^\infty x^n$

13.10 $\sum_0^\infty \dfrac{(-1)^n x^{4n+2}}{(2n+1)!}$

13.11 $\sum_0^\infty \dfrac{(-1)^n x^n}{(2n+1)!}$

13.12 $\sum_0^\infty \dfrac{(-1)^n x^{4n+1}}{(2n)!(4n+1)}$

13.13 $\sum_0^\infty \dfrac{(-1)^n x^{2n+1}}{n!(2n+1)}$

13.14 $\sum_0^\infty \dfrac{x^{2n+1}}{2n+1}$

13.15 $\sum_0^\infty \binom{-1/2}{n} (-1)^n \dfrac{x^{2n+1}}{2n+1}$

13.16 $\sum_0^\infty \dfrac{x^{2n}}{(2n)!}$

13.17 $2\sum_{\text{odd}\,n}^\infty \dfrac{x^n}{n}$

13.18 $\sum_0^\infty \dfrac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$

13.19 $\sum_0^\infty \binom{-1/2}{n} \dfrac{x^{2n+1}}{2n+1}$

13.20 $x + x^2 + x^3/3 - x^5/30 - x^6/90 \cdots$

13.21 $x^2 + 2x^4/3 + 17x^6/45 \cdots$

13.22 $1 + 2x + 5x^2/2 + 8x^3/3 + 65x^4/24 \cdots$

13.23 $1 - x + x^3 - x^4 + x^6 \cdots$

13.24 $1 + x^2/2! + 5x^4/4! + 61x^6/6! \cdots$
13.25 $1 - x + x^2/3 - x^4/45 \cdots$
13.26 $1 + x^2/4 + 7x^4/96 + 139x^6/5760 \cdots$
13.27 $1 + x + x^2/2 - x^4/8 - x^5/15 \cdots$
13.28 $x - x^2/2 + x^3/6 - x^5/12 \cdots$
13.29 $1 + x/2 - 3x^2/8 + 17x^3/48 \cdots$
13.30 $1 - x + x^2/2 - x^3/2 + 3x^4/8 - 3x^5/8 \cdots$
13.31 $1 - x^2/2 - x^3/2 - x^4/4 - x^5/24 \cdots$
13.32 $x + x^2/2 - x^3/6 - x^4/12 \cdots$
13.33 $1 + x^3/6 + x^4/6 + 19x^5/120 + 19x^6/120 \cdots$
13.34 $x - x^2 + x^3 - 13x^4/12 + 5x^5/4 \cdots$
13.35 $1 + x^2/3! + 7x^4/(3 \cdot 5!) + 31x^6/(3 \cdot 7!) \cdots$
13.36 $u^2/2 + u^4/12 + u^6/20 \cdots$
13.37 $-(x^2/2 + x^4/12 + x^6/45 \cdots)$
13.38 $e(1 - x^2/2 + x^4/6 \cdots)$
13.39 $1 - (x - \pi/2)^2/2! + (x - \pi/2)^4/4! \cdots$
13.40 $1 - (x - 1) + (x - 1)^2 - (x - 1)^3 \cdots$
13.41 $e^3[1 + (x - 3) + (x - 3)^2/2! + (x - 3)^3/3! \cdots]$
13.42 $-1 + (x - \pi)^2/2! - (x - \pi)^4/4! \cdots$
13.43 $-[(x - \pi/2) + (x - \pi/2)^3/3 + 2(x - \pi/2)^5/15 \cdots]$
13.44 $5 + (x - 25)/10 - (x - 25)^2/10^3 + (x - 25)^3/(5 \cdot 10^4) \cdots$

14.6 Error $< (1/2)(0.1)^2 \div (1 - 0.1) < 0.0056$
14.7 Error $< (3/8)(1/4)^2 \div (1 - \frac{1}{4}) = 1/32$
14.8 For $x < 0$, error $< (1/64)(1/2)^4 < 0.001$
For $x > 0$, error $< 0.001 \div (1 - \frac{1}{2}) = 0.002$
14.9 Term $n + 1$ is $a_{n+1} = \frac{1}{(n+1)(n+2)}$, so $R_n = (n + 2)a_{n+1}$.
14.10 $S_4 = 0.3052$, error < 0.0021 (cf. $S = 1 - \ln 2 = 0.307$)

15.1 $-x^4/24 - x^5/30 \cdots \simeq -3.376 \times 10^{-16}$
15.2 $x^8/3 - 14x^{12}/45 \cdots \simeq 1.433 \times 10^{-16}$
15.3 $x^5/15 - 2x^7/45 \cdots \simeq 6.667 \times 10^{-17}$
15.4 $x^3/3 + 5x^4/6 \cdots \simeq 1.430 \times 10^{-11}$
15.5 0 15.6 12 15.7 10!
15.8 1/2 15.9 −1/6 15.10 −1
15.11 4 15.12 1/3 15.13 −1
15.14 $t - t^3/3$, error $< 10^{-6}$ 15.15 $\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}$, error $< \frac{1}{7}10^{-7}$
15.16 $e^2 - 1$ 15.17 $\cos\frac{\pi}{2} = 0$
15.18 $\ln 2$ 15.19 $\sqrt{2}$
15.20 (a) 1/8 (b) $5e$ (c) 9/4
15.21 (a) 0.397117 (b) 0.937548 (c) 1.291286
15.22 (a) $\pi^4/90$ (b) 1.202057 (c) 2.612375
15.23 (a) 1/2 (b) 1/6 (c) 1/3 (d) −1/2
15.24 (a) $-\pi$ (b) 0 (c) −1
(d) 0 (e) 0 (f) 0
15.27 (a) $1 - \frac{v}{c} = 1.3 \times 10^{-5}$, or $v = 0.999987c$
(b) $1 - \frac{v}{c} = 5.2 \times 10^{-7}$
(c) $1 - \frac{v}{c} = 2.1 \times 10^{-10}$
(d) $1 - \frac{v}{c} = 1.3 \times 10^{-11}$
15.28 $mc^2 + \frac{1}{2}mv^2$
15.29 (a) $F/W = \theta + \theta^3/3 \cdots$
(b) $F/W = x/l + x^3/(2l^3) + 3x^5/(8l^5) \cdots$

15.30 (a) $T = F(5/x + x/40 - x^3/16000 \cdots)$
(b) $T = \frac{1}{2}(F/\theta)(1 + \theta^2/6 + 7\theta^4/360 \cdots)$
15.31 (a) finite (b) infinite

16.1 (c)

overhang:	2	3	10	100
books needed:	32	228	2.7×10^8	4×10^{86}

16.4 C, $\rho = 0$ 16.5 D, $a_n \not\to 0$ 16.6 C, cf. $\sum n^{-3/2}$
16.7 D, $I = \infty$ 16.8 D, cf. $\sum n^{-1}$ 16.9 $-1 \le x < 1$
16.10 $|x| < 4$ 16.11 $|x| \le 1$ 16.12 $|x| < 5$
16.13 $-5 < x \le 1$
16.14 $1 - x^2/2 + x^3/2 - 5x^4/12 \cdots$
16.15 $-x^2/6 - x^4/180 - x^6/2835 \cdots$
16.16 $1 - x/2 + 3x^2/8 - 11x^3/48 + 19x^4/128 \cdots$
16.17 $1 + x^2/2 + x^4/4 + 7x^6/48 \cdots$
16.18 $x - x^3/3 + x^5/5 - x^7/7 \cdots$
16.19 $-(x-\pi) + (x-\pi)^3/3! - (x-\pi)^5/5! \cdots$
16.20 $2 + (x-8)/12 - (x-8)^2/(2^5 \cdot 3^2) + 5(x-8)^3/(2^8 \cdot 3^4) \cdots$
16.21 $e[1 + (x-1) + (x-1)^2/2! + (x-1)^3/3! \cdots]$
16.22 arc tan $1 = \pi/4$ 16.23 $1 - (\sin\pi)/\pi = 1$
16.24 $e^{\ln 3} - 1 = 2$ 16.25 -2
16.26 $-1/3$ 16.27 $2/3$
16.28 1 16.29 6!
16.30 (b) For $N = 130$, $10.5821 < \zeta(1.1) < 10.5868$
16.31 (a) 10^{430} terms. For $N = 200$, $100.5755 < \zeta(1.01) < 100.5803$
16.31 (b) 2.66×10^{86} terms. For $N = 15$, $1.6905 < S < 1.6952$
16.31 (c) $e^{e^{200}} = 10^{3.1382 \times 10^{86}}$ terms. For $N = 40$, $38.4048 < S < 38.4088$

Chapter 2

	x	y	r	θ
4.1	1	1	$\sqrt{2}$	$\pi/4$
4.2	-1	1	$\sqrt{2}$	$3\pi/4$
4.3	1	$-\sqrt{3}$	2	$-\pi/3$
4.4	$-\sqrt{3}$	1	2	$5\pi/6$
4.5	0	2	2	$\pi/2$
4.6	0	-4	4	$-\pi/2$
4.7	-1	0	1	π
4.8	3	0	3	0
4.9	-2	2	$2\sqrt{2}$	$3\pi/4$
4.10	2	-2	$2\sqrt{2}$	$-\pi/4$
4.11	$\sqrt{3}$	1	2	$\pi/6$
4.12	-2	$-2\sqrt{3}$	4	$-2\pi/3$
4.13	0	-1	1	$3\pi/2$
4.14	$\sqrt{2}$	$\sqrt{2}$	2	$\pi/4$
4.15	-1	0	1	$-\pi$ or π
4.16	5	0	5	0
4.17	1	-1	$\sqrt{2}$	$-\pi/4$
4.18	0	3	3	$\pi/2$
4.19	4.69	1.71	5	$20^\circ = 0.35$
4.20	-2.39	-6.58	7	$-110^\circ = -1.92$
5.1	$1/2$	$-1/2$	$1/\sqrt{2}$	$-\pi/4$
5.2	$-1/2$	$-1/2$	$1/\sqrt{2}$	$-3\pi/4$ or $5\pi/4$
5.3	1	0	1	0
5.4	0	2	2	$\pi/2$
5.5	2	$2\sqrt{3}$	4	$\pi/3$
5.6	-1	0	1	π
5.7	$7/5$	$-1/5$	$\sqrt{2}$	$-8.13^\circ = -0.14$
5.8	1.6	-2.7	3.14	$-59.3^\circ = -1.04$
5.9	-10.4	22.7	25	$2 = 114.6^\circ$
5.10	$-25/17$	$19/17$	$\sqrt{58/17}$	$142.8^\circ = 2.49$
5.11	17	-12	20.8	$-35.2^\circ = -0.615$
5.12	2.65	1.41	3	$28^\circ = 0.49$
5.13	1.55	4.76	5	$2\pi/5$
5.14	1.27	-2.5	2.8	$-1.1 = -63^\circ$
5.15	$21/29$	$-20/29$	1	$-43.6^\circ = -0.76$
5.16	1.53	-1.29	2	$-40^\circ = -0.698$
5.17	-7.35	-10.9	13.1	$-124^\circ = -2.16$
5.18	-0.94	-0.36	1	201° or -159°, 3.51 or -2.77

5.19 $(2+3i)/13$; $(x-yi)/(x^2+y^2)$
5.20 $(-5+12i)/169$; $(x^2-y^2-2ixy)/(x^2+y^2)^2$
5.21 $(1+i)/6$; $(x+1-iy)/[(x+1)^2+y^2]$
5.22 $(1+2i)/10$; $[x-i(y-1)]/[x^2+(y-1)^2]$
5.23 $(-6-3i)/5$; $(1-x^2-y^2+2yi)/[(1-x)^2+y^2]$
5.24 $(-5-12i)/13$; $(x^2-y^2+2ixy)/(x^2+y^2)$
5.26 1 5.27 $\sqrt{13/2}$ 5.28 1
5.29 $5\sqrt{5}$ 5.30 3/2 5.31 1
5.32 169 5.33 5 5.34 1
5.35 $x=-4$, $y=3$ 5.36 $x=-1/2$, $y=3$
5.37 $x=y=0$ 5.38 $x=-7$, $y=2$
5.39 $x=y=$ any real number 5.40 $x=0$, $y=3$
5.41 $x=1$, $y=-1$ 5.42 $x=-1/7$, $y=-10/7$
5.43 $(x,y)=(0,0)$, or $(1,1)$, or $(-1,1)$
5.44 $x=0$, $y=-2$
5.45 $x=0$, any real y; or $y=0$, any real x
5.46 $y=-x$
5.47 $(x,y)=(-1,0)$, $(1/2,\pm\sqrt{3}/2)$
5.48 $x=36/13$, $y=2/13$
5.49 $x=1/2$, $y=0$
5.50 $x=0$, $y\geq 0$
5.51 Circle, center at origin, radius $=2$
5.52 y axis
5.53 Circle, center at $(1,0)$, $r=1$
5.54 Disk, center at $(1,0)$, $r=1$
5.55 Line $y=5/2$
5.56 Positive y axis
5.57 Hyperbola, $x^2-y^2=4$
5.58 Half plane, $x>2$
5.59 Circle, center at $(0,-3)$, $r=4$
5.60 Circle, center at $(1,-1)$, $r=2$
5.61 Half plane, $y<0$
5.62 Ellipse, foci at $(1,0)$ and $(-1,0)$, semi-major axis $=4$
5.63 The coordinate axes
5.64 Straight lines, $y=\pm x$
5.67 $v=(4t^2+1)^{-1}$, $a=4(4t^2+1)^{-3/2}$
5.68 Motion around circle $r=1$, with $v=2$, $a=4$

6.2 D, $\rho=\sqrt{2}$ 6.3 C, $\rho=1/\sqrt{2}$ 6.4 D, $|a_n|=1\not\to 0$
6.5 D 6.6 C 6.7 D, $\rho=\sqrt{2}$
6.8 D, $|a_n|=1\not\to 0$ 6.9 C 6.10 C, $\rho=\sqrt{2}/2$
6.11 C, $\rho=1/5$ 6.12 C 6.13 C, $\rho=\sqrt{2/5}$

7.1 All z 7.2 $|z|<1$ 7.3 All z
7.4 $|z|<1$ 7.5 $|z|<2$ 7.6 $|z|<1/3$
7.7 All z 7.8 All z 7.9 $|z|<1$
7.10 $|z|<1$ 7.11 $|z|<27$ 7.12 $|z|<4$
7.13 $|z-i|<1$ 7.14 $|z-2i|<1$ 7.15 $|z-(2-i)|<2$
7.16 $|z+(i-3)|<1/\sqrt{2}$

8.3 See Problem 17.30.

9.1	$(1-i)/\sqrt{2}$	9.2	i	9.3	$-9i$
9.4	$-e(1+i\sqrt{3})/2$	9.5	-1	9.6	1
9.7	$3e^2$	9.8	$-\sqrt{3}+i$	9.9	$-2i$
9.10	-2	9.11	$-1\ -i$	9.12	$-2-2i\sqrt{3}$
9.13	$-4+4i$	9.14	64	9.15	$2i-4$
9.16	$-2\sqrt{3}-2i$	9.17	$-(1+i)/4$	9.18	1
9.19	16	9.20	i	9.21	1
9.22	$-i$	9.23	$(\sqrt{3}+i)/4$	9.24	$4i$
9.25	-1	9.26	$(1+i\sqrt{3})/2$	9.29	1
9.30	$e^{\sqrt{3}}$	9.31	5	9.32	$3e^2$
9.33	$2e^3$	9.34	$4/e$	9.35	21
9.36	4	9.37	1	9.38	$1/\sqrt{2}$

10.1 $1,\ (-1\pm i\sqrt{3})/2$ 10.2 $3,\ 3(-1\pm i\sqrt{3})/2$

10.3 $\pm 1,\ \pm i$ 10.4 $\pm 2,\ \pm 2i$

10.5 $\pm 1,\ (\pm 1\pm i\sqrt{3})/2$ 10.6 $\pm 2,\ \pm 1\pm i\sqrt{3}$

10.7 $\pm\sqrt{2},\ \pm i\sqrt{2},\ \pm 1\pm i$ 10.8 $\pm 1,\ \pm i,\ (\pm 1\pm i)/\sqrt{2}$

10.9 $1,\ 0.309\pm 0.951i,\ -0.809\pm 0.588i$

10.10 $2,\ 0.618\pm 1.902i,\ -1.618\pm 1.176i$

10.11 $-2,\ 1\pm i\sqrt{3}$ 10.12 $-1,\ \left(1\pm i\sqrt{3}\right)/2$

10.13 $\pm 1\pm i$ 10.14 $(\pm 1\pm i)/\sqrt{2}$

10.15 $\pm 2i,\ \pm\sqrt{3}\pm i$ 10.16 $\pm i,\ (\pm\sqrt{3}\pm i)/2$

10.17 $-1,\ 0.809\pm 0.588i,\ -0.309\pm 0.951i$

10.18 $\pm(1+i)/\sqrt{2}$ 10.19 $-i,\ (\pm\sqrt{3}+i)/2$

10.20 $2i,\ \pm\sqrt{3}-i$ 10.21 $\pm(\sqrt{3}+i)$

10.22 $r=\sqrt{2},\ \theta=45^\circ+120^\circ n$: $1+i,\ -1.366+0.366i,\ 0.366-1.366i$

10.23 $r=2,\ \theta=30^\circ+90^\circ n$: $\pm(\sqrt{3}+i),\ \pm\left(1-i\sqrt{3}\right)$

10.24 $r=1,\ \theta=30^\circ+45^\circ n$:
$\pm(\sqrt{3}+i)/2,\ \pm\left(1-i\sqrt{3}\right)/2,\quad \pm(0.259+0966i),\ \pm(0.966-0.259i)$

10.25 $r=\sqrt[10]{2},\ \theta=45^\circ+72^\circ n$: $0.758(1+i),\ -0.487+0.955i,$
$-1.059-0.168i,\ -0.168-1.059i,\ 0.955-0.487i$

10.26 $r=1,\ \theta=18^\circ+72^\circ n: i,\ \pm 0.951+0.309i,\ \pm 0.588-0.809i$

10.28 $\cos 3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$
$\sin 3\theta=3\cos^2\theta\sin\theta-\sin^3\theta$

11.3	$3(1-i)/\sqrt{2}$	11.4	-8	11.5	$1+i$	11.6	$13/5$
11.7	$3i/5$	11.8	$-41/9$	11.9	$4i/3$	11.10	-1

12.20 $\cosh 3z=\cosh^3 z+3\cosh z\sinh^2 z,\quad \sinh 3z=3\cosh^2 z\sinh z+\sinh^3 z$

12.22 $\sinh x=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}\ ,\ \ \cosh x=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$

12.23 $\cos x,\ |\cos x|$

12.24 $\cosh x$

12.25 $\sin x\cosh y-i\cos x\sinh y,\quad \sqrt{\sin^2 x+\sinh^2 y}$

12.26 $\cosh 2\cos 3-i\sinh 2\sin 3=-3.725-0.512i,\ 3.760$

12.27 $\sin 4\cosh 3+i\cos 4\sinh 3=-7.62-6.55i,\ 10.05$

12.28 $\tanh 1=0.762$ 12.29 1

12.30 $-i$ 12.31 $(3+5i\sqrt{3})/8$

12.32 $-4i/3$ 12.33 $i\tanh 1=0.762i$

12.34 $i\sinh(\pi/2)=2.301i$ 12.35 $-\cosh 2=-3.76$

12.36 $i\cosh 1=1.543i$ 12.37 $\cosh\pi$

14.1 $1 + i\pi$
14.2 $-i\pi/2$ or $3\pi i/2$
14.3 $\text{Ln}\,2 + i\pi/6$
14.4 $(1/2)\,\text{Ln}\,2 + 3\pi i/4$
14.5 $\text{Ln}\,2 + 5i\pi/4$
14.6 $-i\pi/4$ or $7\pi i/4$
14.7 $i\pi/2$
14.8 -1, $(1 \pm i\sqrt{3})/2$
14.9 $e^{-\pi}$
14.10 $e^{-\pi^2/4}$
14.11 $\cos(\text{Ln}\,2) + i\sin(\text{Ln}\,2) = 0.769 + 0.639i$
14.12 $-ie^{-\pi/2}$
14.13 $1/e$
14.14 $2e^{-\pi/2}[i\cos(\text{Ln}\,2) - \sin(\text{Ln}\,2)] = 0.3198i - 0.2657$
14.15 $e^{-\pi\sinh 1} = 0.0249$
14.16 $e^{-\pi/3} = 0.351$
14.17 $\sqrt{2}\,e^{-3\pi/4}e^{i(\text{Ln}\sqrt{2}+3\pi/4)} = -0.121 + 0.057i$
14.18 -1
14.19 $-5/4$
14.20 1
14.21 -1
14.22 $-1/2$
14.23 $e^{\pi/2} = 4.81$

15.1 $\pi/2 + 2n\pi \pm i\,\text{Ln}\left(2 + \sqrt{3}\right) = \pi/2 + 2n\pi \pm 1.317i$
15.2 $\pi/2 + n\pi + (i\,\text{Ln}\,3)/2$
15.3 $i(\pm\pi/3 + 2n\pi)$
15.4 $i(2n\pi + \pi/6)$, $i(2n\pi + 5\pi/6)$
15.5 $\pm\left[\pi/2 + 2n\pi - i\,\text{Ln}\left(3 + \sqrt{8}\right)\right] = \pm[\pi/2 + 2n\pi - 1.76i]$
15.6 $i(n\pi - \pi/4)$
15.7 $\pi/2 + n\pi - i\,\text{Ln}(\sqrt{2} - 1) = \pi/2 + n\pi + 0.881i$
15.8 $\pi/2 + 2n\pi \pm i\,\text{Ln}\,3$
15.9 $i(\pi/3 + n\pi)$
15.10 $2n\pi \pm i\,\text{Ln}\,2$
15.11 $i(2n\pi + \pi/4)$, $i(2n\pi + 3\pi/4)$
15.12 $i(2n\pi \pm \pi/6)$
15.13 $i(\pi + 2n\pi)$
15.14 $2n\pi + i\,\text{Ln}\,2$, $(2n+1)\pi - i\,\text{Ln}\,2$
15.15 $n\pi + 3\pi/8 + i\,\text{Ln}\,2)/4$
15.16 $(\text{Ln}\,2)/4 + i(n\pi + 5\pi/8)$

16.2 Motion around circle $|z| = 5$; $v = 5\omega$, $a = 5\omega^2$.
16.3 Motion around circle $|z| = \sqrt{2}$; $v = \sqrt{2}$, $a = \sqrt{2}$.
16.4 $v = \sqrt{13}$, $a = 0$
16.5 $v = |z_1 - z_2|$, $a = 0$
16.6 (a) Series: $3 - 2i$
Parallel: $5 + i$
(b) Series: $2(1 + i\sqrt{3})$
Parallel: $i\sqrt{3}$
16.7 (a) Series: $1 + 2i$
Parallel: $3(3 - i)/5$
(b) Series: $5 + 5i$
Parallel: $1.6 + 1.2i$
16.8 $\left[R - i(\omega CR^2 + \omega^3 L^2 C - \omega L)\right] / \left[(\omega CR)^2 + (\omega^2 LC - 1)^2\right]$; this simplifies to $\dfrac{L}{RC}$ if $\omega^2 = \dfrac{1}{LC}\left(1 - \dfrac{R^2 C}{L}\right)$, that is, at resonance.
16.9 (a) $\omega = \dfrac{R}{2L} \pm \sqrt{\dfrac{R^2}{4L^2} + \dfrac{1}{LC}}$
(b) $\omega = 1/\sqrt{LC}$
16.10 (a) $\omega = -\dfrac{1}{2RC} \pm \sqrt{\dfrac{1}{4R^2C^2} + \dfrac{1}{LC}}$
(b) $\omega = 1/\sqrt{LC}$
16.12 $(1 + r^4 - 2r^2\cos\theta)^{-1}$

17.1 -1

17.2 $(\sqrt{3}+i)/2$

17.3 $r=\sqrt{2}$, $\theta=45^\circ+72^\circ n: 1+i$, $-0.642+1.260i$, $-1.397-0.221i$, $-0.221-1.397i$, $1.260-0.642i$

17.4 $i\cosh 1 = 1.54i$

17.5 i

17.6 $-e^{-\pi^2} = -5.17\times 10^{-5}$ or $-e^{-\pi^2}\cdot e^{\pm 2n\pi^2}$

17.7 $e^{\pi/2} = 4.81$ or $e^{\pi/2}\cdot e^{\pm 2n\pi}$

17.8 -1

17.9 $\pi/2 \pm 2n\pi$

17.10 $\sqrt{3}-2$

17.11 i

17.12 $-1\pm\sqrt{2}$

17.13 $x=0,\ y=4$

17.14 Circle with center $(0,\ 2)$, radius 1

17.15 $|z|<1/e$

17.16 $y<-2$

17.26 1

17.27 (c) $e^{-2(x-t)^2}$

17.28 $1+\left[\dfrac{a^2+b^2}{2ab}\right]^2\sinh^2 b$

17.29 $\left(-1\pm i\sqrt{3}\right)/2$

17.30 $e^x\cos x = \displaystyle\sum_{\mathrm{n}=0}^{\infty}\frac{x^n 2^{n/2}\cos n\pi/4}{n!}$

$e^x\sin x = \displaystyle\sum_{\mathrm{n}=0}^{\infty}\frac{x^n 2^{n/2}\sin n\pi/4}{n!}$

Chapter 3

2.3 $\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \end{pmatrix}$, $x = -3$, $y = 5$)

2.4 $\begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1 \end{pmatrix}$, $x = (z+1)/2$, $y = 1$

2.5 $\begin{pmatrix} 1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, no solution

2.6 $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$, $x = 1$, $z = y$

2.7 $\begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$, $x = -4$, $y = 3$

2.8 $\begin{pmatrix} 1 & -1 & 0 & -11 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $x = y - 11$, $z = 7$

2.9 $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, inconsistent, no solution

2.10 $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, inconsistent, no solution

2.11 $\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{pmatrix}$, $x = 2$, $y = -1$, $z = -3$

2.12 $\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$, $x = -2$, $y = 1$, $z = 1$

2.13 $\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -2 & 5/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $x = -2$, $y = 2z + 5/2$

2.14 $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, inconsistent, no solution

2.15 $R = 2$ 2.16 $R = 3$
2.17 $R = 2$ 2.18 $R = 3$

3.1 -11 3.2 -721 3.3 1 3.4 2140
3.5 -544 3.6 4 3.11 0 3.12 16
3.16 $A = -(K + ik)/(K - ik)$, $|A| = 1$
3.17 $x = \gamma(x' + vt')$, $t = \gamma(t' + vx'/c^2)$
3.18 $D = 3b(a + b)(a^2 + ab + b^2)$, $z = 1$
(Also $x = a + 2b$, $y = a - b$; these were not required.)

4.11 $-3\mathbf{i} + 8\mathbf{j} - 6\mathbf{k}$, $\mathbf{i} - 10\mathbf{j} + 3\mathbf{k}$, $2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.
4.12 $\arccos(-1/\sqrt{2}) = 3\pi/4$
4.13 $-5/3$, -1, $\cos\theta = -1/3$
4.14 (a) $\arccos(1/3) = 70.5°$
(b) $\arccos(1/\sqrt{3}) = 54.7°$
(c) $\arccos\sqrt{2/3} = 35.3°$
4.15 (a) $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})/3$
(b) $8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}$
(c) Any combination of $\mathbf{i} + 2\mathbf{j}$ and $\mathbf{i} - \mathbf{k}$.
(d) Answer to (c) divided by its magnitude.
4.17 Legs = any two vectors with dot product = 0;
hypotenuse = their sum (or difference).
4.18 $2\mathbf{i} - 8\mathbf{j} - 3\mathbf{k}$ 4.19 $\mathbf{i} + \mathbf{j} + \mathbf{k}$
4.20 $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ 4.22 Law of cosines
4.24 A^2B^2

In the following answers, note that the point and vector used may be *any* point on the line and *any* vector along the line.
5.1 $\mathbf{r} = (2, -3) + (4, 3)t$ 5.2 $3/2$
5.3 $\mathbf{r} = (3, 0) + (1, 1)t$ 5.4 $\mathbf{r} = (1, 0) + (2, 1)t$
5.5 $\mathbf{r} = \mathbf{j}\,t$
5.6 $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z+5}{2}$; $\mathbf{r} = (1, -1, -5) + (1, -2, 2)t$
5.7 $\frac{x-2}{3} = \frac{y-3}{-2} = \frac{z-4}{-6}$; $\mathbf{r} = (2, 3, 4) + (3, -2, -6)t$
5.8 $\frac{x}{3} = \frac{z-4}{-5}$, $y = -2$; $\mathbf{r} = (0, -2, 4) + (3, 0, -5)t$
5.9 $x = -1$, $z = 7$; $\mathbf{r} = -\mathbf{i} + 7\mathbf{k} + \mathbf{j}t$
5.10 $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z+1}{6}$; $\mathbf{r} = (3, 4, -1) + (2, -3, 6)t$
5.11 $\frac{x-4}{1} = \frac{z-3}{-2}$, $y = -1$; $\mathbf{r} = (4, -1, 3) + (1, 0, -2)t$
5.12 $\frac{x-5}{5} = \frac{y+4}{-2} = \frac{z-2}{1}$; $\mathbf{r} = (5, -4, 2) + (5, -2, 1)t$
5.13 $x = 3$, $\frac{y}{-3} = \frac{z+5}{1}$; $\mathbf{r} = 3\mathbf{i} - 5\mathbf{k} + (-3\mathbf{j} + \mathbf{k})t$
5.14 $36x - 3y - 22z = 23$ 5.15 $5x + 6y + 3z = 0$
5.16 $5x - 2y + z = 35$ 5.17 $3y - z = 5$
5.18 $x + 6y + 7z + 5 = 0$ 5.19 $x + y + 3z + 12 = 0$
5.20 $x - 4y - z + 5 = 0$
5.21 $\cos\theta = 25/\left(7\sqrt{30}\right) = 0.652$, $\theta = 49.3°$
5.22 $\cos\theta = 2/\sqrt{6}$, $\theta = 35.3°$
5.23 $\cos\theta = 4/21$, $\theta = 79°$
5.24 $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + (\mathbf{j} + 2\mathbf{k})t$, $d = 2\sqrt{6/5}$
5.25 $\mathbf{r} = (1, -2, 0) + (4, 9, -1)t$, $d = \left(3\sqrt{3}\right)/7$
5.26 $\mathbf{r} = (8, 1, 7) + (14, 2, 15)t$, $d = \sqrt{2/17}$
5.27 $y + 2z + 1 = 0$ 5.28 $4x + 9y - z + 27 = 0$
5.29 $2/\sqrt{6}$ 5.30 1
5.31 $5/7$ 5.32 $10/\sqrt{27}$

5.33 $\sqrt{43/15}$

5.34 $\sqrt{11/10}$

5.35 $\sqrt{5}$

5.36 3

5.37 Intersect at $(1, -3, 4)$

5.38 $\arccos\sqrt{21/22} = 12.3°$

5.39 $t_1 = 1$, $t_2 = -2$, intersect at $(3, 2, 0)$, $\cos\theta = 5/\sqrt{60}$, $\theta = 49.8°$

5.40 $t_1 = -1$, $t_2 = 1$, intersect at $(4, -1, 1)$, $\cos\theta = 5/\sqrt{39}$, $\theta = 36.8°$

5.41 $\sqrt{14}$

5.42 $1/\sqrt{5}$

5.43 $20/\sqrt{21}$

5.44 $2/\sqrt{10}$

5.45 $d = \sqrt{2}$, $t = -1$

6.1 $\mathrm{AB} = \begin{pmatrix} -5 & 10 \\ 1 & 24 \end{pmatrix}$ $\quad \mathrm{BA} = \begin{pmatrix} -2 & 8 \\ 11 & 21 \end{pmatrix}$ $\quad \mathrm{A+B} = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$

$\mathrm{A-B} = \begin{pmatrix} 5 & -1 \\ 1 & 1 \end{pmatrix}$ $\quad \mathrm{A}^2 = \begin{pmatrix} 11 & 8 \\ 16 & 27 \end{pmatrix}$ $\quad \mathrm{B}^2 = \begin{pmatrix} 6 & 4 \\ 2 & 18 \end{pmatrix}$

$5\mathrm{A} = \begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix}$ $\quad 3\mathrm{B} = \begin{pmatrix} -6 & 6 \\ 3 & 12 \end{pmatrix}$ $\quad \det(5\mathrm{A}) = 5^2 \det \mathrm{A}$

6.2 $\mathrm{AB} = \begin{pmatrix} -2 & -2 \\ 1 & 2 \end{pmatrix}$ $\quad \mathrm{BA} = \begin{pmatrix} -6 & 17 \\ -2 & 6 \end{pmatrix}$ $\quad \mathrm{A+B} = \begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix}$

$\mathrm{A-B} = \begin{pmatrix} 3 & -9 \\ -1 & 1 \end{pmatrix}$ $\quad \mathrm{A}^2 = \begin{pmatrix} 9 & -25 \\ -5 & 14 \end{pmatrix}$ $\quad \mathrm{B}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 4 \end{pmatrix}$

$5\mathrm{A} = \begin{pmatrix} 10 & -25 \\ -5 & 15 \end{pmatrix}$ $\quad 3\mathrm{B} = \begin{pmatrix} -3 & 12 \\ 0 & 6 \end{pmatrix}$

6.3 $\mathrm{AB} = \begin{pmatrix} 7 & -1 & 0 \\ 3 & 1 & -1 \\ 3 & 9 & 5 \end{pmatrix}$ $\quad \mathrm{BA} = \begin{pmatrix} 4 & -1 & 2 \\ 6 & 3 & 1 \\ 0 & 1 & 6 \end{pmatrix}$ $\quad \mathrm{A+B} = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix}$

$\mathrm{A-B} = \begin{pmatrix} 0 & -1 & 2 \\ 3 & -3 & -1 \\ -3 & 6 & 1 \end{pmatrix}$ $\quad \mathrm{A}^2 = \begin{pmatrix} 1 & 10 & 4 \\ 0 & 1 & 6 \\ 15 & 0 & 1 \end{pmatrix}$ $\quad \mathrm{B}^2 = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 3 & 2 \\ 3 & 1 & -1 \end{pmatrix}$

$5\mathrm{A} = \begin{pmatrix} 5 & 0 & 10 \\ 15 & -5 & 0 \\ 0 & 25 & 5 \end{pmatrix}$ $\quad 3\mathrm{B} = \begin{pmatrix} 3 & 3 & 0 \\ 0 & 6 & 3 \\ 9 & -3 & 0 \end{pmatrix}$ $\quad \det(5\mathrm{A}) = 5^3 \det \mathrm{A}$

6.4 $\mathrm{BA} = \begin{pmatrix} 12 & 10 & 2 & 12 \\ 0 & 2 & 1 & -9 \\ 4 & 8 & 3 & -17 \end{pmatrix}$ $\quad \mathrm{C}^2 = \begin{pmatrix} 5 & 1 & 7 \\ 6 & 5 & 12 \\ -3 & -1 & -2 \end{pmatrix}$

$\mathrm{CB} = \begin{pmatrix} 14 & 4 \\ 1 & 19 \\ 1 & -5 \end{pmatrix}$ $\quad \mathrm{C}^3 = \begin{pmatrix} 7 & 4 & 20 \\ 20 & 1 & 20 \\ -8 & -2 & -9 \end{pmatrix}$

$\mathrm{C}^2 B = \begin{pmatrix} 32 & 12 \\ 53 & 7 \\ -13 & -9 \end{pmatrix}$ $\quad \mathrm{CBA} = \begin{pmatrix} 36 & 46 & 14 & -36 \\ 40 & 22 & 1 & 91 \\ -8 & -2 & 1 & -29 \end{pmatrix}$

6.5 $\mathrm{AA}^\mathrm{T} = \begin{pmatrix} 30 & -13 \\ -13 & 30 \end{pmatrix}$ $\quad \mathrm{A}^\mathrm{T}\mathrm{A} = \begin{pmatrix} 8 & 8 & 2 & 2 \\ 8 & 10 & 3 & -7 \\ 2 & 3 & 1 & -4 \\ 2 & -7 & -4 & 41 \end{pmatrix}$

$\mathrm{BB}^\mathrm{T} = \begin{pmatrix} 20 & -2 & 2 \\ -2 & 2 & 4 \\ 2 & 4 & 10 \end{pmatrix}$ $\quad \mathrm{B}^\mathrm{T} B = \begin{pmatrix} 14 & 4 \\ 4 & 18 \end{pmatrix}$

$\mathrm{CC}^\mathrm{T} = \begin{pmatrix} 14 & 1 & 1 \\ 1 & 21 & -6 \\ 1 & -6 & 2 \end{pmatrix}$ $\quad \mathrm{C}^\mathrm{T}\mathrm{C} = \begin{pmatrix} 21 & -2 & -3 \\ -2 & 2 & 5 \\ -3 & 5 & 14 \end{pmatrix}$

6.8 $5x^2 + 3y^2 = 30$

6.9 $\mathrm{AB} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathrm{BA} = \begin{pmatrix} 22 & 44 \\ -11 & -22 \end{pmatrix}$

6.10 $\mathrm{AC} = \mathrm{A\,D} = \begin{pmatrix} 11 & 12 \\ 33 & 36 \end{pmatrix}$

6.13 $\begin{pmatrix} 5/3 & -3 \\ -1 & 2 \end{pmatrix}$ 6.14 $\frac{1}{6}\begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix}$

6.15 $-\frac{1}{2}\begin{pmatrix} 4 & 5 & 8 \\ -2 & -2 & -2 \\ 2 & 3 & 4 \end{pmatrix}$ 6.16 $\frac{1}{8}\begin{pmatrix} -2 & 1 & 1 \\ 6 & -3 & 5 \\ 4 & 2 & 2 \end{pmatrix}$

6.17 $\mathrm{A}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 2 & -1 \\ 4 & 4 & -5 \\ 8 & 2 & -4 \end{pmatrix} \qquad \mathrm{B}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$

$\mathrm{B}^{-1}\mathrm{AB} = \begin{pmatrix} 3 & 1 & 2 \\ -2 & -2 & -2 \\ -2 & -1 & 0 \end{pmatrix} \qquad \mathrm{B}^{-1}\mathrm{A}^{-1}\mathrm{B} = \frac{1}{6}\begin{pmatrix} 2 & 2 & -2 \\ -4 & -4 & -2 \\ 2 & -1 & 4 \end{pmatrix}$

6.19 $\mathrm{A}^{-1} = \frac{1}{7}\begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}, \quad (x, y) = (5, 0)$

6.20 $\mathrm{A}^{-1} = \frac{1}{7}\begin{pmatrix} -4 & 3 \\ 5 & -2 \end{pmatrix}, \quad (x, y) = (4, -3)$

6.21 $\mathrm{A}^{-1} = \frac{1}{5}\begin{pmatrix} -1 & 2 & 2 \\ -2 & -1 & 4 \\ 3 & -1 & -1 \end{pmatrix}, \quad (x, y, z) = (-2, 1, 5)$

6.22 $\mathrm{A}^{-1} = \frac{1}{12}\begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}, \quad (x, y, z) = (1, -1, 2)$

6.30 $\sin k\mathrm{A} = \mathrm{A}\sin k = \begin{pmatrix} 0 & \sin k \\ \sin k & 0 \end{pmatrix}, \ \cos k\mathrm{A} = \mathrm{I}\cos k = \begin{pmatrix} \cos k & 0 \\ 0 & \cos k \end{pmatrix},$

$e^{k\mathrm{A}} = \begin{pmatrix} \cosh k & \sinh k \\ \sinh k & \cosh k \end{pmatrix}, \quad e^{ik\mathrm{A}} = \begin{pmatrix} \cos k & i\sin k \\ i\sin k & \cos k \end{pmatrix}$

6.32 $e^{i\theta\mathrm{B}} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

In the following, L = linear, N = not linear.

7.1	N	7.2	L	7.3	N	7.4	L	7.5	L
7.6	N	7.7	L	7.8	N	7.9	N	7.10	N
7.11	N	7.12	L	7.13	(a) L	(b) L			
7.14	N	7.15	L	7.16	N	7.17	N		

7.22 $D = 1$, rotation $\theta = -45°$ 7.23 $D = 1$, rotation $\theta = 210°$

7.24 $D = -1$, reflection line $x + y = 0$ 7.25 $D = -1$, reflection line $y = x\sqrt{2}$

7.26 $D = -1$, reflection line $x = 2y$ 7.27 $D = 1$, rotation $\theta = 135°$

7.28 $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \qquad \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

7.29 $\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

7.30 $R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$; R is a 90° rotation about the z axis; S is a 90° rotation about the x axis.

7.31 From problem 30, $RS = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $SR = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$;
RS is a 120° rotation about $\mathbf{i}+\mathbf{j}+\mathbf{k}$; SR is a 120° rotation about $\mathbf{i}-\mathbf{j}+\mathbf{k}$.

7.32 180° rotation about $\mathbf{i}-\mathbf{k}$

7.33 120° rotation about $\mathbf{i}-\mathbf{j}-\mathbf{k}$

7.34 Reflection through the plane $y+z=0$

7.35 Reflection through the (x, y) plane, and 90° rotation about the z axis.

8.1 In terms of basis $\mathbf{u} = \frac{1}{9}(9, 0, 7)$, $\mathbf{v} = \frac{1}{9}(0, -9, 13)$, the vectors are: $\mathbf{u}-4\mathbf{v}$, $5\mathbf{u}-2\mathbf{v}$, $2\mathbf{u}+\mathbf{v}$, $3\mathbf{u}+6\mathbf{v}$.

8.2 In terms of basis $\mathbf{u} = \frac{1}{3}(3, 0, 5)$, $\mathbf{v} = \frac{1}{3}(0, 3, -2)$, the vectors are: $\mathbf{u}-2\mathbf{v}$, $\mathbf{u}+\mathbf{v}$, $-2\mathbf{u}+\mathbf{v}$, $3\mathbf{u}$.

8.3 Basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

8.4 Basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

8.6 $\mathbf{V} = 3\mathbf{A} - \mathbf{B}$

8.7 $\mathbf{V} = \frac{3}{2}(1, -4) + \frac{1}{2}(5, 2)$

8.17 $x = 0$, $y = \frac{3}{2}z$

8.18 $x = -3y$, $z = 2y$

8.19 $x = y = z = w = 0$

8.20 $x = -z$, $y = z$

8.21 $\begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{pmatrix} = 0$

8.22 $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = 0$

8.23 For $\lambda = 3$, $x = 2y$; for $\lambda = 8$, $y = -2x$

8.24 For $\lambda = 7$, $x = 3y$; for $\lambda = -3$, $y = -3x$

8.25 For $\lambda = 2$: $x = 0$, $y = -3z$; for $\lambda = -3$: $x = -5y$, $z = 3y$; .
for $\lambda = 4$: $z = 3y$, $x = 2y$

8.26 $\mathbf{r} = (3, 1, 0) + (-1, 1, 1)z$

8.27 $\mathbf{r} = (0, 1, 2) + (1, 1, 0)x$

8.28 $\mathbf{r} = (3, 1, 0) + (2, 1, 1)z$

9.3 $A^\dagger = \begin{pmatrix} 1 & 2i & 1 \\ 0 & 2 & 1-i \\ -5i & 0 & 0 \end{pmatrix}$, $A^{-1} = \dfrac{1}{10}\begin{pmatrix} 0 & 5i-5 & -10i \\ 0 & -5i & 10 \\ -2i & -1-i & 2 \end{pmatrix}$

9.4 $A^\dagger = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, $A^{-1} = \dfrac{1}{6}\begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & i \\ -6 & 6i & -2 \end{pmatrix}$

9.14 $C^T B A^T$, $C^{-1} M^{-1} C$, H

10.1 (a) $d = 5$ (b) $d = 8$ (c) $d = \sqrt{56}$

10.2 The dimension of the space = the number of basis vectors listed. One possible basis is given; other bases consist of the same number of independent linear combinations of the vectors given.
(a) $(1, -1, 0, 0)$, $(-2, 0, 5, 1)$
(b) $(1, 0, 0, 5, 0, 1)$, $(0, 1, 0, 0, 6, 4)$, $(0, 0, 1, 0, -3, 0)$
(c) $(1, 0, 0, 0, -3)$, $(0, 2, 0, 0, 1)$, $(0, 0, 1, 0, -1)$, $(0, 0, 0, 1, 4)$

10.3 (a) Label the vectors **A**, **B**, **C**, **D**. Then $\cos(\mathbf{A},\mathbf{B}) = \frac{1}{\sqrt{15}}$, $\cos(\mathbf{A},\mathbf{C}) = \frac{\sqrt{2}}{3}$, $\cos(\mathbf{A},\mathbf{D}) = \frac{3}{\sqrt{23}}$, $\cos(\mathbf{B},\mathbf{C}) = \frac{2}{3\sqrt{15}}$, $\cos(\mathbf{B},\mathbf{D}) = \sqrt{\frac{17}{690}}$, $\cos(\mathbf{C},\mathbf{D}) = \frac{\sqrt{21}}{6\sqrt{23}}$.

(b) $(1,0,0,5,0,1)$ and $(0,0,1,0,-3,0)$

10.4 (a) $\mathbf{e}_1 = (0,1,0,0)$, $\mathbf{e}_2 = (1,0,0,0)$, $\mathbf{e}_3 = (0,0,3,4)/5$

(b) $\mathbf{e}_1 = (0,0,0,1)$, $\mathbf{e}_2 = (1,0,0,0)$, $\mathbf{e}_3 = (0,1,1,0)/\sqrt{2}$

(c) $\mathbf{e}_1 = (1,0,0,0)$, $\mathbf{e}_2 = (0,0,1,0)$, $\mathbf{e}_3 = (0,1,0,2)/\sqrt{5}$

10.5 (a) $\|\mathbf{A}\| = \sqrt{43}$, $\|\mathbf{B}\| = \sqrt{41}$, $|\text{Inner product of } \mathbf{A} \text{ and } \mathbf{B}| = \sqrt{74}$

(b) $\|\mathbf{A}\| = 7$, $\|\mathbf{B}\| = \sqrt{60}$, $|\text{Inner product of } \mathbf{A} \text{ and } \mathbf{B}| = \sqrt{5}$

11.5 $\theta = 1.1 = 63.4^\circ$

11.11 $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$, not orthogonal

In the following answers, for each eigenvalue, the components of a corresponding eigenvector are listed in parentheses.

11.12	4	$(1,1)$	11.13	3	$(2,1)$
	-1	$(3,-2)$		-2	$(-1,2)$
11.14	4	$(2,-1)$	11.15	1	$(0,0,1)$
	-1	$(1,2)$		-1	$(1,-1,0)$
				5	$(1,1,0)$
11.16	2	$(0,1,0)$	11.17	7	$(1,0,1)$
	3	$(2,0,1)$		3	$(1,0,-1)$
	-2	$(1,0,-2)$		3	$(0,1,0)$
11.18	4	$(2,1,3)$	11.19	3	$(0,1,-1)$
	2	$(0,-3,1)$		5	$(1,1,1)$
	-3	$(5,-1,-3)$		-1	$(2,-1,-1)$
11.20	3	$(0,-1,2)$	11.21	-1	$(-1,1,1)$
	4	$(1,2,1)$		2	$(2,1,1)$
	-2	$(-5,2,1)$		-2	$(0,-1,1)$
11.22	-4	$(-4,1,1)$			
	5	$(1,2,2)$			
	-2	$(0,-1,1)$			

11.23 18 (2,2,-1)
9, 9 $\left\{\right.$ Any two vectors orthogonal to (2,2,-1) and to each other, for example: (1,-1,0) and (1,1,4)

11.24 8 (2,1,2)
-1, -1 $\left\{\right.$ Any two vectors orthogonal to (2,1,2) and to each other, for example: (1,0,-1) and (1,-4,1)

11.25
1 $(-1,1,1)$
2 $(1,1,0)$
-2 $(1,-1,2)$

11.26 4 (1,1,1)
1, 1 $\left\{\right.$ Any two vectors orthogonal to (1,1,1) and to each other, for example: (1,-1,0) and (1,1,-2)

11.27 $\mathrm{D} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathrm{C} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

11.28 $\mathrm{D} = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, $\mathrm{C} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

11.29 $D = \begin{pmatrix} 11 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

11.30 $D = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}, \quad C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

11.31 $D = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

11.32 $D = \begin{pmatrix} 7 & 0 \\ 0 & 2 \end{pmatrix}, \quad C = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

11.41 $\lambda = 1,\ 3; \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

11.42 $\lambda = 1,\ 4; \quad U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ -1-i & 1 \end{pmatrix}$

11.43 $\lambda = 2,\ -3; \quad U = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -i \\ -i & 2 \end{pmatrix}$

11.44 $\lambda = 3,\ -7; \quad U = \frac{1}{5\sqrt{2}} \begin{pmatrix} 5 & -3-4i \\ 3-4i & 5 \end{pmatrix}$

11.47 $U = \frac{1}{2} \begin{pmatrix} -1 & i\sqrt{2} & 1 \\ -1 & -i\sqrt{2} & 1 \\ \sqrt{2} & 0 & \sqrt{2} \end{pmatrix}$

11.51 Reflection through the plane $3x - 2y - 3z = 0$, no rotation

11.52 60° rotation about $-\mathbf{i}\sqrt{2} + \mathbf{k}$ and reflection through the plane $z = x\sqrt{2}$

11.53 180° rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$

11.54 -120° (or 240°) rotation about $\mathbf{i}\sqrt{2} + \mathbf{j}$

11.55 Rotation -90° about $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, and reflection through the plane $x - 2y + 2z = 0$

11.56 45° rotation about $\mathbf{j} - \mathbf{k}$

11.58 $f(M) = \frac{1}{5} \begin{pmatrix} f(1) + 4f(6) & 2f(1) - 2f(6) \\ 2f(1) - 2f(6) & 4f(1) + f(6) \end{pmatrix}$

$M^4 = \frac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^4 & 2 - 2 \cdot 6^4 \\ 2 - 2 \cdot 6^4 & 4 + 6^4 \end{pmatrix} \qquad M^{10} = \frac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^{10} & 2 - 2 \cdot 6^{10} \\ 2 - 2 \cdot 6^{10} & 4 + 6^{10} \end{pmatrix}$

$e^M = \frac{e}{5} \begin{pmatrix} 1 + 4e^5 & 2(1 - e^5) \\ 2(1 - e^5) & 4 + e^5 \end{pmatrix}$

11.59 $M^4 = 2^3 \begin{pmatrix} 1 + 2^4 & 1 - 2^4 \\ 1 - 2^4 & 1 + 2^4 \end{pmatrix} \qquad M^{10} = 2^3 \begin{pmatrix} 1 + 2^{10} & 1 - 2^{10} \\ 1 - 2^{10} & 1 + 2^{10} \end{pmatrix},$

$e^M = e^3 \begin{pmatrix} \cosh 1 & -\sinh 1 \\ -\sinh 1 & \cosh 1 \end{pmatrix}$

12.2 $3x'^2 - 2y'^2 = 24$

12.3 $10x'^2 = 35$

12.4 $5x'^2 - 5y'^2 = 8$

12.5 $x'^2 + 3y'^2 + 6z'^2 = 14$

12.6 $3x'^2 + \sqrt{3}y'^2 - \sqrt{3}z'^2 = 12$

12.7 $3x'^2 + 5\,y'^2 - z'^2 = 60$

12.14 $y = x$ with $\omega = \sqrt{k/m}$; $y = -x$ with $\omega = \sqrt{5k/m}$

12.15 $y = 2x$ with $\omega = \sqrt{3k/m}$; $x = -2y$ with $\omega = \sqrt{8k/m}$

12.16 $y = 2x$ with $\omega = \sqrt{2k/m}$; $x = -2y$ with $\omega = \sqrt{7k/m}$

12.17 $x = -2y$ with $\omega = \sqrt{2k/m}$; $3x = 2y$ with $\omega = \sqrt{2k/(3m)}$

12.18 $y = x$ with $\omega = \sqrt{2k/m}$; $x = -5y$ with $\omega = \sqrt{16k/(5m)}$

12.19 $y = -x$ with $\omega = \sqrt{3k/m}$; $y = 2x$ with $\omega = \sqrt{3k/(2m)}$

12.21 $y = 2x$ with $\omega = \sqrt{k/m}$; $x = -2y$ with $\omega = \sqrt{6k/m}$

12.22 $y = -x$ with $\omega = \sqrt{2k/m}$; $y = 3x$ with $\omega = \sqrt{2k/(3m)}$

12.23 $y = -x$ with $\omega = \sqrt{k/m}$; $y = 2x$ with $\omega = \sqrt{k/(4m)}$

13.5 The 4's group 13.6 The cyclic group 13.7 The 4's group

13.10 If R = 90° rotation, P = reflection through the y axis, and Q = PR, then the 8 matrices of the symmetry group of the square are:

$\mathrm{I}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $\mathrm{R}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$, $\mathrm{R}^2=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}=-\mathrm{I}$,

$\mathrm{R}^3=\begin{pmatrix}0&1\\-1&0\end{pmatrix}=-\mathrm{R}$, $\mathrm{P}=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$, $\mathrm{PR}=\begin{pmatrix}0&1\\1&0\end{pmatrix}=\mathrm{Q}$,

$\mathrm{PR}^2=\begin{pmatrix}1&0\\0&-1\end{pmatrix}=-\mathrm{P}$, $\mathrm{PR}^3=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}=-\mathrm{Q}$,

with multiplication table:

	I	R	−I	−R	P	Q	−P	−Q
I	I	R	−I	−R	P	Q	−P	−Q
R	R	−I	−R	I	−Q	P	Q	−P
−I	−I	−R	I	R	−P	−Q	P	Q
−R	−R	I	R	−I	Q	−P	−Q	P
P	P	Q	−P	−Q	I	R	−I	−R
Q	Q	−P	−Q	P	−R	I	R	−I
−P	−P	−Q	P	Q	−I	−R	I	R
−Q	−Q	P	Q	−P	R	−I	−R	I

13.11 The 4 matrices of the symmetry group of the rectangle are

$\mathrm{I}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $\mathrm{P}=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$, $\begin{pmatrix}1&0\\0&-1\end{pmatrix}=-\mathrm{P}$, $\begin{pmatrix}-1&0\\0&-1\end{pmatrix}=-\mathrm{I}$

This group is isomorphic to the 4's group.

13.14

Class	I	±R	−I	±P	±Q
Character	2	0	−2	0	0

13.20 Not a group (no unit element)

13.21 SO(2) is Abelian; SO(3) is not Abelian.

For Problems 2 to 10, we list a possible basis.

14.2 e^x, $x\,e^x$, e^{-x}, or the three given functions

14.3 x, $\cos x$, $x\cos x$, $e^x\cos x$

14.4 1, x, x^3

14.5 1, $x+x^3$, x^2, x^4, x^5

14.6 Not a vector space

14.7 $(1+x^2+x^4+x^6)$, $(x+x^3+x^5+x^7)$

14.8 1, x^2, x^4, x^6

14.9 Not a vector space; the negative of a vector with positive coefficients does not have positive coefficients.

14.10 $(1+\frac{1}{2}x)$, $(x^2+\frac{1}{2}x^3)$, $(x^4+\frac{1}{2}x^5)$, $(x^6+\frac{1}{2}x^7)$, $(x^8+\frac{1}{2}x^9)$, $(x^{10}+\frac{1}{2}x^{11})$, $(x^{12}+\frac{1}{2}x^{13})$

15.3 (a) $\frac{x-4}{1}=\frac{y+1}{-2}=\frac{z-2}{-2}$, $\quad\mathbf{r}=(4,-1,2)+(1,-2,-2)t$

(b) $x-5y+3z=0$ (c) $5/7$

(d) $5\sqrt{2}/3$ (e) $\arcsin(19/21)=64.8°$

15.4 (a) $4x+2y+5z=10$ (b) $\arcsin(2/3)=41.8°$

(c) $2/\sqrt{5}$ (d) $2x+y-2z=5$

(e) $x=\frac{5}{2}$, $\frac{y}{2}=z$, $\quad\mathbf{r}=\frac{5}{2}\mathbf{i}+(2\mathbf{j}+\mathbf{k})t$

15.5 (a) $y=7$, $\frac{x-2}{3}=\frac{z+1}{4}$, $\quad\mathbf{r}=(2,7,-1)+(3,0,4)t$

(b) $x-4y-9z=0$ (c) $\arcsin\frac{33}{35\sqrt{2}}=41.8°$

(d) $\frac{12}{7\sqrt{2}}$ (e) $\frac{\sqrt{29}}{5}$

15.7 $\mathrm{A}^{\mathrm{T}} = \begin{pmatrix} 1 & 0 \\ -1 & i \end{pmatrix}$ $\quad \mathrm{A}^{-1} = \begin{pmatrix} 1 & -i \\ 0 & -i \end{pmatrix}$ $\quad \mathrm{AB} = \begin{pmatrix} 2 & -2 & -6 \\ 0 & 3i & 5i \end{pmatrix}$

$\overline{A} = \begin{pmatrix} 1 & -1 \\ 0 & -i \end{pmatrix}$ $\quad \mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}} = (\mathrm{AB})^{\mathrm{T}}$ $\quad \mathrm{B}^{\mathrm{T}}\mathrm{AC} = \begin{pmatrix} 2 & 2 \\ 1-3i & 1 \\ -1-5i & -1 \end{pmatrix}$

$\mathrm{A}^{\dagger} = \begin{pmatrix} 1 & 0 \\ -1 & -i \end{pmatrix}$ $\quad \mathrm{B}^{\mathrm{T}}\mathrm{C} = \begin{pmatrix} 0 & 2 \\ -3 & 1 \\ -5 & -1 \end{pmatrix}$ $\quad \mathrm{C}^{-1}\mathrm{A} = \begin{pmatrix} 0 & -i \\ 1 & -1 \end{pmatrix}$

$\mathrm{A}^{\mathrm{T}}\mathrm{B}^{\mathrm{T}}$, BA^{T}, ABC, $\mathrm{AB}^{\mathrm{T}}\mathrm{C}$, $\mathrm{B}^{-1}\mathrm{C}$, and CB^{T} are meaningless.

15.8 $\mathrm{A}^{\dagger} = \begin{pmatrix} 1 & -i & 1 \\ 0 & -3 & 0 \\ -2i & 0 & -i \end{pmatrix}$ $\quad \mathrm{A}^{-1} = \dfrac{1}{3i}\begin{pmatrix} -3i & 0 & 6i \\ 1 & -i & -2 \\ 3 & 0 & -3 \end{pmatrix}$

15.9 $\mathrm{A} = \begin{pmatrix} 1 + \frac{(n-1)d}{nR_2} & -(n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right] \\ \frac{d}{n} & 1 - \frac{(n-1)d}{nR_1} \end{pmatrix}, \quad \dfrac{1}{f} = -A_{12}$

15.10 $\mathrm{M} = \begin{pmatrix} 1 - \frac{d}{f_2} & -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} \\ d & 1 - \frac{d}{f_1} \end{pmatrix}, \quad \dfrac{1}{f} = \frac{f_1+f_2-d}{f_1 f_2}, \quad \det \mathrm{M} = 1$

15.13 Area $= \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = 7/2$

15.14 $x'' = -x$, $y'' = -y$, 180° rotation

15.15 $x'' = -y$, $y'' = x$, 90° rotation of vectors or -90° rotation of axes

15.16 $x'' = y$, $y'' = -x$, $z'' = z$, 90° rotation of (x, y) axes about the z axis, or -90° rotation of vectors about the z axis

15.17 $x'' = x$, $y'' = -y$, $z'' = -z$, rotation of π about the x axis

15.18 1 $(1,1)$; -2 $(0,1)$

15.19 6 $(1,1)$; 1 $(1,-4)$

15.20 1 $(1,1)$; 9 $(1,-1)$

15.21 0 $(1,-2)$; 5 $(2,1)$

15.22 1 $(1,0,1)$; 4 $(0,1,0)$; 5 $(1,0,-1)$

15.23 1 $(1,1,-2)$; 3 $(1,-1,0)$; 4 $(1,1,1)$

15.24 2 $(0,4,3)$; 7 $(5,-3,4)$; -3 $(5,3,-4)$

15.25 $\mathrm{C} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 1 & \sqrt{2} \end{pmatrix}, \quad \mathrm{C}^{-1} = \begin{pmatrix} \sqrt{2} & 0 \\ -1 & 1 \end{pmatrix}$

15.26 $\mathrm{C} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{17} \\ 1/\sqrt{2} & -4/\sqrt{17} \end{pmatrix}, \quad \mathrm{C}^{-1} = \dfrac{1}{5}\begin{pmatrix} 4\sqrt{2} & \sqrt{2} \\ \sqrt{17} & -\sqrt{17} \end{pmatrix}$

15.27 $3x'^2 - y'^2 - 5z'^2 = 15$, $d = \sqrt{5}$

15.28 $9x'^2 + 4y'^2 - z'^2 = 36$, $d = 2$

15.29 $3x'^2 + 6y'^2 - 4z'^2 = 54$, $d = 3$

15.30 $7x'^2 + 20y'^2 - 6z'^2 = 20$, $d = 1$

15.31 $\omega = (k/m)^{1/2}, \quad (7k/m)^{1/2}$

15.32 $\omega = 2(k/m)^{1/2}, \quad (3k/m)^{1/2}$

Chapter 4

1.1	$\partial u/\partial x = 2xy^2/(x^2+y^2)^2$, $\partial u/\partial y = -2x^2y/(x^2+y^2)^2$				
1.2	$\partial s/\partial t = ut^{u-1}$, $\partial s/\partial u = t^u \ln t$				
1.3	$\partial z/\partial u = u/(u^2+v^2+w^2)$				
1.4	At $(0,\ 0)$, both $= 0$; at $(-2/3,\ 2/3)$, both $= -4$				
1.5	At $(0,\ 0)$, both $= 0$; at $(1/4,\ \pm 1/2)$, $\partial^2 w/\partial x^2 = 6$, $\partial^2 w/\partial y^2 = 2$				
1.7	$2x$	1.8	$-2x$	1.9	$2x(1+2\tan^2\theta)$
1.10	$4y$	1.11	$2y$	1.12	$2y(\cot^2\theta+2)$
1.13	$4r^2\tan\theta$	1.14	$-2r^2\cot\theta$	1.15	$r^2\sin 2\theta$
1.16	$2r(1+\sin^2\theta)$	1.17	$4r$	1.18	$2r$
1.19	0	1.20	$8y\sec^2\theta$	1.21	$-4x\csc^2\theta$
1.22	0	1.23	$2r\sin 2\theta$	1.24	0
1.7′	$-2y^4/x^3$	1.8′	$-2r^4/x^3$	1.9′	$2x\tan^2\theta\sec^2\theta$
1.10′	$2y+4y^3/x^2$	1.11′	$2yr^4/(r^2-y^2)^2$	1.12′	$2y\sec^2\theta$
1.13′	$2x^2\sec^2\theta\tan\theta(\sec^2\theta+\tan^2\theta)$				
1.14′	$2y^2\sec^2\theta\tan\theta$	1.15′	$2r^2\tan\theta\sec^2\theta$	1.16′	$2r\tan^2\theta$
1.17′	$4r^3/x^2-2r$	1.18′	$-2ry^4/(r^2-y^2)^2$		
1.19′	$-8r^3y^3/(r^2-y^2)^3$	1.20′	$4x\tan\theta\sec^2\theta(\tan^2\theta+\sec^2\theta)$		
1.21′	$4y\sec^2\theta\tan\theta$	1.22′	$-8r^3/x^3$		
1.23′	$4r\tan\theta\sec^2\theta$	1.24′	$-8y^3/x^3$		

2.1	$y + y^3/6 - x^2y/2 + x^4y/24 - x^2y^3/12 + y^5/120 + \cdots$
2.2	$1 - (x^2+2xy+y^2)/2 + (x^4+4x^3y+6x^2y^2+4xy^3+y^4)/24 + \cdots$
2.3	$x - x^2/2 - xy + x^3/3 + x^2y/2 + xy^2 \cdots$
2.4	$1 + xy + x^2y^2/2 + x^3y^3/3! + x^4y^4/4! \cdots$
2.5	$1 + \frac{1}{2}xy - \frac{1}{8}x^2y^2 + \frac{1}{16}x^3y^3 - \frac{5}{128}x^4y^4 \cdots$
2.6	$1 + x + y + (x^2+2xy+y^2)/2 \cdots$
2.8	$e^x\cos y = 1 + x + (x^2-y^2)/2 + (x^3-3xy^2)/3! \cdots$
	$e^x\sin y = y + xy + (3x^2y-y^3)/3! \cdots$

4.2	2.5×10^{-13}	4.3	14.8	4.4	12.2	4.5	14.96
4.6	9%	4.7	15%	4.8	5%	4.10	4.28 nt
4.11	3.95	4.12	2.01	4.13	5/3	4.14	0.005
4.15	8×10^{23}						

5.1	$e^{-y}\sinh t + z\sin t$	5.2	$w = 1$, $dw/dp = 0$
5.3	$2r(q^2-p^2)$	5.4	$(4ut + 2v\sin t)/(u^2-v^2)$
5.6	$5(x+y)^4(1+10\cos 10x)$	5.7	$(1-2b-e^{2a})\cos(a-b)$

6.1	$dv/dp = -v/(ap)$, $d^2v/dp^2 = v(1+a)/(a^2p^2)$		
6.2	$y' = 1$, $y'' = 0$	6.3	$y' = 4(\ln 2 - 1)/(2\ln 2 - 1)$

6.4 $y' = y(x-1)/[x(y-1)]$, $y'' = (y-x)(y+x-2)y/[x^2(y-1)^3]$

6.5 $2x + 11y - 24 = 0$

6.6 $1800/11^3$

6.7 $y' = 1$, $x - y - 4 = 0$

6.8 $-8/3$

6.9 $y = x - 4\sqrt{2}$, $y = 0$, $x = 0$

6.10 $x + y = 0$

6.11 $y'' = 4$

7.1 $dx/dy = z - y + \tan(y+z)$, $d^2x/dy^2 = \frac{1}{2}\sec^3(y+z) + \frac{1}{2}\sec(y+z) - 2$

7.2 $[2e^r \cos t - r + r^2 \sin^2 t]/[(1-r)\sin t]$

7.3 $\partial z/\partial s = z \sin s$, $\partial z/\partial t = e^{-y} \sinh t$

7.4 $\partial w/\partial u = -2(rv+s)w$, $\partial w/\partial v = -2(ru+2s)w$

7.5 $\partial u/\partial s = (2y^2 - 3x^2 + xyt)u/(xy)$, $\partial u/\partial t = (2y^2 - 3x^2 + xys)u/(xy)$

7.6 $\partial^2 w/\partial r^2 = f_{xx}\cos^2\theta + 2f_{xy}\sin\theta\cos\theta + f_{yy}\sin^2\theta$

7.7 $(\partial y/\partial\theta)_r = x$, $(\partial y/\partial\theta)_x = r^2/x$, $(\partial\theta/\partial y)_x = x/r^2$

7.8 $\partial x/\partial s = -19/13$, $\partial x/\partial t = -21/13$, $\partial y/\partial s = 24/13$, $\partial y/\partial t = 6/13$

7.10 $\partial x/\partial s = 1/6$, $\partial x/\partial t = 13/6$, $\partial y/\partial s = 7/6$, $\partial y/\partial t = -11/6$

7.11 $\partial z/\partial s = 481/93$, $\partial z/\partial t = 125/93$

7.12 $\partial w/\partial s = w/(3w^3 - xy)$, $\partial w/\partial t = (3w-1)/(3w^3 - xy)$

7.13 $(\partial p/\partial q)_m = -p/q$, $(\partial p/\partial q)_a = 1/(a\cos p - 1)$,
$(\partial p/\partial q)_b = 1 - b\sin q$, $(\partial b/\partial a)_p = (\sin p)(b\sin q - 1)/\cos q$
$(\partial a/\partial q)_m = [q + p(a\cos p - 1)]/(q\sin p)$

7.14 13

7.15 $(\partial x/\partial u)_v = (2yv^2 - x^2)/(2yv + 2xu)$,
$(\partial x/\partial u)_y = (x^2u + y^2v)/(y^2 - 2xu^2)$

7.16 (a) $\dfrac{dw}{dt} = \dfrac{3(2x+y)}{3x^2+1} + \dfrac{4x}{4y^3+1} + \dfrac{10z}{5z^4+1}$

(b) $\dfrac{dw}{dx} = 2x + y - \dfrac{xy}{3y^2+x} + \dfrac{2z^2}{3z^2-x}$

(c) $\left(\dfrac{\partial w}{\partial x}\right)_y = 2x + y - \dfrac{2z(y^3+3x^2z)}{x^3+3yz^2}$

7.17 $(\partial p/\partial s)_t = -9/7$, $(\partial p/\partial s)_q = 3/2$

7.18 $(\partial b/\partial m)_n = a/(a-b)$, $(\partial m/\partial b)_a = 1$

7.19 $(\partial x/\partial z)_s = 7/2$, $(\partial x/\partial z)_r = 4$, $(\partial x/\partial z)_y = 3$

7.20 $(\partial u)/(\partial x)_y = 4/3$, $(\partial u/\partial x)_v = 14/5$, $(\partial x/\partial u)_y = 3/4$, $(\partial x/\partial u)_v = 5/14$

7.21 -1, -15, 2, $15/7$, $-5/2$, $-6/5$

7.26 $dy/dx = -(f_1g_3 - f_3g_1)/(f_2g_3 - g_2f_3)$

8.3 $(-1,\ 2)$ is a minimum point.

8.4 $(-1,\ -2)$ is a saddle point.

8.5 $(0,\ 1)$ is a maximum point.

8.6 $(0,\ 0)$ is a saddle point.
$(-2/3,\ 2/3)$ is a maximum point.

8.8 $\theta = \pi/3$; bend up 8 cm on each side.

8.9 $l = w = 2h$

8.10 $l = w = 2h/3$

8.11 $\theta = 30°$, $x = y\sqrt{3} = z/2$

8.12 $d = 3$

8.13 $(4/3,\ 5/3)$

8.15 $(1/2,\ 1/2,\ 0)$, $(1/3,\ 1/3,\ 1/3)$

8.16 $m = 5/2$, $b = 1/3$

8.17 (a) $y = 5 - 4x$ (b) $y = 0.5 + 3.35x$ (c) $y = -3 - 3.6x$

9.1 $s = l$, $\theta = 30°$ (regular hexagon)

9.2 $r : l : s = \sqrt{5} : (1 + \sqrt{5}) : 3$

9.3 36 in by 18 in by 18 in

9.4 $4/\sqrt{3}$ by $6/\sqrt{3}$ by $10/\sqrt{3}$

9.5 $(1/2,\ 3,\ 1)$

9.6 $V = 1/3$
9.7 $V = d^3/(27abc)$
9.8 $(8/13,\ 12/13)$
9.9 $A = 3ab\sqrt{3}/4$
9.10 $d = 5/\sqrt{2}$
9.11 $d = \sqrt{6}/2$
9.12 Let legs of right triangle be a and b, height of prism $= h$;
then $a = b$, $h = \left(2 - \sqrt{2}\right) a$.

10.1 $d = 1$
10.2 4, 2
10.3 2, $\sqrt{14}$
10.4 $d = 1$
10.5 $d = 1$
10.6 $d = 2$
10.7 $\frac{1}{2}\sqrt{11}$
10.8 $T = 8$
10.9 $\max T = 4$ at $(-1,\ 0)$
$\min T = -\frac{16}{5}$ at $\left(\frac{1}{5}, \pm\frac{2}{5}\sqrt{6}\right)$
10.10 (a) $\max T = 1/2$, $\min T = -1/2$
(b) $\max T = 1$, $\min T = -1/2$
(c) $\max T = 1$, $\min T = -1/2$
10.11 $\max T = 14$ at $(-1,\ 0)$
$\min T = 13/2$ at $(1/2,\ \pm 1)$
10.12 Largest sum $= 180°$
Smallest sum $= 3 \arccos \frac{1}{\sqrt{3}}$
$= 164.2°$
10.13 Largest sum $= 3 \arcsin(1/\sqrt{3}) = 105.8°$, smallest sum $= 90°$

11.1 $z = f(y + 2x) + g(y + 3x)$
11.2 $z = f(5x - 2y) + g(2x + y)$
11.3 $w = (x^2 - y^2)/4 + F(x + y) + G(x - y)$
11.6 $\dfrac{d^2y}{dz^2} + \dfrac{dy}{dz} - 5y = 0$
11.10 $f = u - Ts$ $\quad h = u + pv$ $\quad g = u + pv - Ts$
$df = -p\ dv - sdT$ $\quad dh = Tds + vdp$ $\quad dg = v\ dp - s\ dT$
11.11 $H = p\dot{q} - L$
11.13 (a) $(\partial s/\partial v)_T = (\partial p/\partial T)_v$ (b) $(\partial T/\partial p)_s = (\partial v/\partial s)_p$
(c) $(\partial v/\partial T)_p = -(\partial s/\partial p)_T$

12.1 $\dfrac{\sin x}{2\sqrt{x}}$
12.2 $\dfrac{\partial s}{\partial v} = \dfrac{1 - e^v}{v} \to -1$; $\dfrac{\partial s}{\partial u} = \dfrac{e^u - 1}{u} \to 1$
12.3 $dz/dx = -\sin(\cos x)\tan x - \sin(\sin x)\cot x$
12.4 $(\sin 2)/2$
12.5 $\partial u/\partial x = -4/\pi,\ \ \partial u/\partial y = 2/\pi,\ \ \partial y/\partial x = 2$
12.6 $\partial w/\partial x = 1/\ln 3,\ \ \partial w/\partial y = -6/\ln 3,\ \ \partial y/\partial x = 1/6$
12.7 $(\partial u/\partial x)_y = -e^4,\ \ (\partial u/\partial y)_x = e^4/\ln 2,\ \ (\partial y/\partial x)_u = \ln 2$
12.8 $dx/du = e^{x^2}$
12.9 $(\cos \pi x + \pi x \sin \pi x - 1)/x^2$
12.10 $dy/dx = (e^x - 1)/x$
12.11 $3x^2 - 2x^3 + 3x - 6$
12.12 $(2x + 1)/\ln(x + x^2) - 2/\ln(2x)$
12.13 0
12.14 $\pi/(4y^3)$
12.16 $n = 2$, $I = \frac{1}{4}\sqrt{\pi}\, a^{-3/2}$ $\quad n = 4$, $I = \dfrac{1 \cdot 3}{8}\sqrt{\pi}\, a^{-5/2}$
$n = 2m$, $I = \dfrac{1 \cdot 3 \cdot 5\ \cdots\ (2m - 1)}{2^{m+1}}\sqrt{\pi}\, a^{-(2m+1)/2}$

13.2 (a) and (b) $d = 4/\sqrt{13}$
13.3 $\sec^2\theta$
13.4 $-\csc\theta\cot\theta$
13.5 $-6x$, $2x^2\tan\theta\sec^2\theta$, $4x\tan\theta\sec^2\theta$
13.6 $2r\sin^2\theta$, $2r^2\sin\theta\cos\theta$, $4r\sin\theta\cos\theta$, 0
13.7 5%

13.8 π^{-1}ft $\cong$ 4 inches
13.9 $dz/dt = 1 + t(2 - x - y)/z,\ z \neq 0$
13.10 $(x \ln x - y^2/x)x^y$ where $x = r\cos\theta,\ y = r\sin\theta$
13.11 $\dfrac{dy}{dx} = -\dfrac{b^2x}{a^2y},\ \ \dfrac{d^2y}{dx^2} = -\dfrac{b^4}{a^2y^3}$
13.12 13
13.13 -1
13.14 $(\partial w/\partial x)_y = (\partial f/\partial x)_{s,\ t} + 2(\partial f/\partial s)_{x,\ t} + 2(\partial f/\partial t)_{x,\ s} = f_1 + 2f_2 + 2f_3$
13.15 $(\partial w/\partial x)_y = f_1 + 2xf_2 + 2yf_3$
13.17 $\sqrt{19}$
13.18 $\sqrt{26/3}$
13.19 $1/27$
13.20 At $x = -1,\ y = 20$; at $x = 1/2,\ y = -1/4$
13.21 $T(2) = 4,\ T(5) = -5$
13.22 $T(5,\ 0) = 10,\ T(2,\ \pm\sqrt{2}) = -4$
13.23 $t\cot t$
13.24 0
13.25 $-e^x/x$
13.26 $3\sin x^3/x$
13.29 $dt = 3.9$
13.30 $2f(x,\ x) + \int_0^x \frac{\partial}{\partial x}\,[f(x,\ u) + f(u,\ x)]\,du$

Chapter 5

2.1	3	2.2	-18	2.3	4	2.4	$8/3$
2.5	$\dfrac{e^2}{4}-\dfrac{5}{12}$	2.6	2.35	2.7	$5/3$	2.8	$1/2$
2.9	6	2.10	5π	2.11	36	2.12	2
2.13	$7/4$	2.14	$4-e(\ln 4)$	2.15	$3/2$	2.16	$(\ln 3)/6$
2.17	$(\ln 2)/2$	2.18	$(8\sqrt{2}-7)/3$	2.19	32	2.20	16
2.21	$131/6$	2.22	$5/3$	2.23	$9/8$	2.24	$9/2$
2.25	$3/2$	2.26	$4/3$	2.27	$32/5$	2.28	$1/3$
2.29	2	2.30	$1-e^{-2}$	2.31	6	2.32	$e-1$
2.33	$16/3$	2.34	$8192k/15$	2.35	$216k$	2.36	$1/6$
2.37	$7/6$	2.38	-20	2.39	70	2.40	$3/2$
2.41	5	2.42	4	2.43	$9/2$	2.44	$7k/3$
2.45	$46k/15$	2.46	$8k$	2.47	$16/3$	2.48	$16\pi/3$
2.49	$1/3$	2.50	$64/3$				

3.2	(a) ρl	(b) $Ml^2/12$	(c) $Ml^2/3$	
3.3	(a) $M=140$	(b) $\bar{x}=130/21$	(c) $I_m=6.92M$	(d) $I=150M/7$
3.4	(a) $M=3l/2$	(b) $\bar{x}=4l/9$	(c) $I_m=\frac{13}{162}Ml^2$	(d) $I=5Ml^2/18$
3.5	(a) $Ma^2/3$	(b) $Ma^2/12$	(c) $2Ma^2/3$	
3.6	(a) $(2,2)$	(b) $6M$	(c) $2M$	

3.7 (a) $M=9$ (b) $(\bar{x},\bar{y})=(2,4/3)$
(c) $I_x=2M$, $I_y=9M/2$ (d) $I_m=13M/18$

3.8 $2Ma^2/3$

3.9 (a) $1/6$ (b) $(1/4,1/4,1/4)$ (c) $M=1/24$, $\bar{z}=2/5$

3.10 (a) $s=2\sinh 1$ (b) $\bar{y}=(2+\sinh 2)/(4\sinh 1)=1.2$

3.11 (a) $M=(5\sqrt{5}-1)/6=1.7$
(b) $\bar{x}=0$, $M\bar{y}=(25\sqrt{5}+1)/60=0.95$, $\bar{y}=(313+15\sqrt{5}\,)/620=0.56$

3.14 $V=2\pi^2a^2b$, $A=4\pi^2ab$, where a = radius of revolving circle,
b = distance to axis from center of this circle.

3.15 For area, $(\bar{x},\bar{y})=(0,\frac{4}{3}r/\pi)$; for arc, $(\bar{x},\bar{y})=(0,2r/\pi)$

3.17 $4\sqrt{2}/3$ 3.18 $s=\left[3\sqrt{2}+\ln(1+\sqrt{2})\right]/2=2.56$

3.19 2π 3.20 $13\pi/3$

3.21 $s\bar{x}=\left[51\sqrt{2}-\ln(1+\sqrt{2})\right]/32=2.23$, $s\bar{y}=13/6$, s as in Problem 3.18;
then $\bar{x}=0.87$, $\bar{y}=0.85$

3.22 $(4/3,0,0)$

3.23 $(149/130,0,0)$

3.24 $2M/5$

3.25 I/M has the same numerical value as $\bar{x}$ in 3.21.

3.26 $2M/3$ 3.27 $\frac{149}{130}M$ 3.28 $13/6$ 3.29 2 3.30 $32/5$

4.1 (b) $\bar{x} = \bar{y} = 4a/(3\pi)$
(c) $I = Ma^2/4$
(e) $\bar{x} = \bar{y} = 2a/\pi$
4.2 (c) $\bar{y} = 4a/(3\pi)$
(d) $I_x = Ma^2/4$, $I_y = 5Ma^2/4$, $I_z = 3Ma^2/2$
(e) $\bar{y} = 2a/\pi$
(f) $\bar{x} = 6a/5$, $I_x = 48Ma^2/175$, $I_y = 288Ma^2/175$, $I_z = 48Ma^2/25$
(g) $A = (\frac{2}{3}\pi - \frac{1}{2}\sqrt{3})a^2$
4.3 (a), (b), or (c) $\frac{1}{2}Ma^2$
4.4 (a) $4\pi a^2$ (b) $(0, 0, a/2)$ (c) $2Ma^2/3$
(d) $4\pi a^3/3$ (e) $(0, 0, 3a/8)$
4.5 $7\pi/3$ 4.6 $\pi \ln 2$
4.7 (a) $V = 2\pi a^3(1 - \cos\alpha)/3$ (b) $\bar{z} = 3a(1 + \cos\alpha)/8$
4.8 $I_z = Ma^2/4$
4.10 (a) $V = 64\pi$ (b) $\bar{z} = 231/64$
4.11 12π
4.12 (c) $M = (16\rho/9)(3\pi - 4) = 9.64\rho$
$I = (128\rho/15^2)(15\pi - 26) = 12.02\rho = 1.25M$
4.13 (b) $\pi a^2(z_2 - z_1) - \pi(z_2^3 - z_1^3)/3$ (c) $\dfrac{\frac{1}{2}a^2(z_2^2 - z_1^2) - \frac{1}{4}(z_2^4 - z_1^4)}{a^2(z_2 - z_1) - (z_2^3 - z_1^3)/3}$
4.14 $\pi(1 - e^{-1})/4$ 4.16 $u^2 + v^2$
4.17 $a^2(\sinh^2 u + \sin^2 v)$ 4.19 $\pi/4$
4.20 $1/12$ 4.22 $12(1 + 36\pi^2)^{1/2}$
4.23 Length = $(R\sec\alpha)$ times change in latitude
4.24 $\rho G\pi a/2$
4.26 (a) $7Ma^2/5$ (b) $3Ma^2/2$
4.27 $2\pi ah$ (where h = distance between parallel planes)
4.28 $(0, 0, a/2)$

5.1 $9\pi\sqrt{30}/5$ 5.2 $\pi\sqrt{7/5}$
5.3 $\pi(37^{3/2} - 1)/6 = 117.3$ 5.4 $\pi/\sqrt{6}$
5.5 8π for each nappe 5.6 4
5.7 4 5.8 $\left[3\sqrt{6} + 9\ln(\sqrt{2} + \sqrt{3})\right]/16$
5.9 $\pi\sqrt{2}$ 5.10 $2\pi a^2(\sqrt{2} - 1)$
5.11 $(\bar{x}, \bar{y}, \bar{z}) = (1/3, 1/3, 1/3)$ 5.12 $M = \sqrt{3}/6$, $(\bar{x}, \bar{y}, \bar{z}) = (1/2, 1/4, 1/4)$
5.13 $\bar{z} = \dfrac{\pi}{4(\pi - 2)}$ 5.14 $M = \dfrac{\pi}{2} - \dfrac{4}{3}$
5.15 $I_z/M = \dfrac{2(3\pi - 7)}{9(\pi - 2)} = 0.472$ 5.16 $\bar{x} = 0$, $\bar{y} = 1$, $\bar{z} = \dfrac{32}{9\pi}\sqrt{\dfrac{2}{5}} = 0.716$

6.1 $7\pi(2 - \sqrt{2})/3$ 6.2 $45(2 + \sqrt{2})/112$ 6.3 $15\pi/8$
6.4 (a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
6.5 cone: $2\pi ab^2/3$; ellipsoid: $4\pi ab^2/3$; cylinder: $2\pi ab^2$
6.6 (a) $\dfrac{4\pi - 3\sqrt{3}}{6}$ (b) $\bar{x} = \dfrac{5}{4\pi - 3\sqrt{3}}$, $\bar{y} = \dfrac{6\sqrt{3}}{4\pi - 3\sqrt{3}}$
6.7 $\dfrac{8\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}M$ 6.8 (a) $5\pi/3$ (b) $27/20$
6.9 $(\bar{x}, \bar{y}) = (0, 3c/5)$
6.10 (a) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ (b) $\pi^2/2$ (c) $3M/8$
6.11 $\bar{z} = 3h/4$ 6.12 $(abc)^2/6$
6.13 $8a^2$ 6.14 $16a^3/3$
6.15 $I_x = 8Ma^2/15$, $I_y = 7Ma^2/15$ 6.16 $\bar{x} = \bar{y} = 2a/5$

6.17 $Ma^2/6$ 6.18 $(0, 0, 5h/6)$
6.19 $I_x = I_y = 20Mh^2/21$, $I_z = 10Mh^2/21$, $I_m = 65Mh^2/252$
6.20 (a) $\pi(5\sqrt{5} - 1)/6$ (b) $3\pi/2$
6.21 $\pi G\rho h(2 - \sqrt{2})$
6.22 $I_x = Mb^2/4$, $I_y = Ma^2/4$, $I_z = M(a^2 + b^2)/4$
6.23 (a) $(0, 0, 2c/3)$ (b) $(0, 0, 5c/7)$
6.24 $(0, 0, 2c/3)$
6.25 $\pi/2$
6.26 $\frac{1}{2}\sinh 1$
6.27 $e^2 - e - 1$

Chapter 6

3.1 $(\mathbf{A}\cdot\mathbf{B})\mathbf{C} = 6\mathbf{C} = 6(\mathbf{j}+\mathbf{k})$, $\mathbf{A}(\mathbf{B}\cdot\mathbf{C}) = -2\mathbf{A} = -2(2,-1,-1)$, $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = \mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = -8$, $(\mathbf{A}\times\mathbf{B})\times\mathbf{C} = 4(\mathbf{j}-\mathbf{k})$, $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = -4(\mathbf{i}+2\mathbf{k})$

3.2 $\mathbf{B}\cdot\mathbf{C} = -16$

3.3 $(\mathbf{A}+\mathbf{B})\cdot\mathbf{C} = -5$

3.4 $\mathbf{B}\times\mathbf{A} = -\mathbf{i}+7\mathbf{j}+3\mathbf{k}$, $|\mathbf{B}\times\mathbf{A}| = \sqrt{59}$, $(\mathbf{B}\times\mathbf{A})\cdot\mathbf{C}/|\mathbf{C}| = -8/\sqrt{26}$

3.5 $\boldsymbol{\omega} = 2\mathbf{A}/\sqrt{6}$, $\mathbf{v} = \boldsymbol{\omega}\times\mathbf{C} = \left(2/\sqrt{6}\right)(-3\mathbf{i}+5\mathbf{j}+\mathbf{k})$

3.6 $\mathbf{v} = \left(2/\sqrt{6}\right)(\mathbf{A}\times\mathbf{B}) = \left(2/\sqrt{6}\right)(\mathbf{i}-7\mathbf{j}-3\mathbf{k})$, $\mathbf{r}\times\mathbf{F} = (\mathbf{A}-\mathbf{C})\times\mathbf{B} = 3\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $\mathbf{n}\cdot\mathbf{r}\times\mathbf{F} = [(\mathbf{A}-\mathbf{C})\times\mathbf{B}]\cdot\mathbf{C}/|\mathbf{C}| = 8/\sqrt{26}$

3.7 (a) $11\mathbf{i}+3\mathbf{j}-13\mathbf{k}$ (b) 3 (c) 17

3.8 $4\mathbf{i}-8\mathbf{j}+4\mathbf{k}$, 4, -8, 4

3.9 $-9\mathbf{i}-23\mathbf{j}+\mathbf{k}$, $1/\sqrt{21}$

3.12 A^2B^2

3.15 $\mathbf{u}_1\cdot\mathbf{u} = -\mathbf{u}_3\cdot\mathbf{u}$, $n_1\mathbf{u}_1\times\mathbf{u} = n_2\mathbf{u}_2\times\mathbf{u}$

3.16 $\mathbf{L} = m[r^2\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\mathbf{r})\mathbf{r}]$
For $\mathbf{r}\perp\boldsymbol{\omega}$, $v = |\boldsymbol{\omega}\times\mathbf{r}| = \omega r$, $L = m|r^2\boldsymbol{\omega}| = mvr$

3.17 $\mathbf{a} = (\boldsymbol{\omega}\cdot\mathbf{r})\omega - \omega^2\mathbf{r}$; for $\mathbf{r}\perp\boldsymbol{\omega}$, $\mathbf{a} = -\omega^2\mathbf{r}$, $|\mathbf{a}| = v^2/r$.

3.19 (a) $16\mathbf{i}-2\mathbf{j}-5\mathbf{k}$ (b) $8/\sqrt{6}$

3.20 (a) 13/5 (b) 12

4.2 (a) $t = 2$
(b) $\mathbf{v} = 4\mathbf{i}-2\mathbf{j}+6\mathbf{k}$, $|\mathbf{v}| = 2\sqrt{14}$
(c) $(x-4)/4 = (y+4)/(-2) = (z-8)/6$, $2x-y+3z = 36$

4.3 $t = -1$, $\mathbf{v} = 3\mathbf{i}+3\mathbf{j}-5\mathbf{k}$, $(x-1)/3 = (y+1)/3 = (z-5)/(-5)$, $3x+3y-5z+25 = 0$

4.5 $|d\mathbf{r}/dt| = \sqrt{2}$; $|d^2\mathbf{r}/dt^2| = 1$; path is a helix.

4.8 $d\mathbf{r}/dt = \mathbf{e}_r(dr/dt) + \mathbf{e}_\theta(rd\theta/dt)$,
$$d^2\mathbf{r}/dt^2 = \mathbf{e}_r[d^2r/dt^2 - r(d\theta/dt)^2] + \mathbf{e}_\theta[rd^2\theta/dt^2 + 2(dr/dt)(d\theta/dt)].$$

4.10 $\mathbf{V}\times d\mathbf{V}/dt$

6.1 $-16\mathbf{i}-12\mathbf{j}+8\mathbf{k}$

6.2 $-\mathbf{i}$

6.3 0

6.4 $\pi e/(3\sqrt{5})$

6.5 $\boldsymbol{\nabla}\phi = \mathbf{i}-\mathbf{k}$; $-\boldsymbol{\nabla}\phi$; $d\phi/ds = 2/\sqrt{13}$

6.6 $6x+8y-z = 25$, $(x-3)/6 = (y-4)/8 = (z-25)/(-1)$

6.7 $5x-3y+2z+3 = 0$, $\mathbf{r} = \mathbf{i}+2\mathbf{j}-\mathbf{k}+(5\mathbf{i}-3\mathbf{j}+2\mathbf{k})t$

6.8 (a) 7/3 (b) $5x-z = 8$; $\frac{x-1}{5} = \frac{z+3}{-1}$, $y = \pi/2$

6.9 (a) $2\mathbf{i}-2\mathbf{j}-\mathbf{k}$ (b) $5/\sqrt{6}$ (c) $\mathbf{r} = (1,1,1)+(2,-2,-1)t$

6.10 $\mathbf{j}$, 1, $-4/5$

6.11 $\nabla\phi = 2x\mathbf{i} - 2y\mathbf{j}$, $\mathbf{E} = -2x\mathbf{i} + 2y\mathbf{j}$
6.12 (a) $2\sqrt{5}$, $-2\mathbf{i}+\mathbf{j}$ (b) $3\mathbf{i}+2\mathbf{j}$ (c) $\sqrt{10}$
6.13 (a) $\mathbf{i}+\mathbf{j}$, $|\nabla\phi| = e$ (b) $-1/2$ (c) $\mathbf{i}$, $|\mathbf{E}| = 1$ (d) e^{-1}
6.14 (b) Down, at the rate $11\sqrt{2}$
6.15 (a) $4\sqrt{2}$, up (b) 0, around the hill
(c) $-4/\sqrt{10}$, down (d) 8/5, up
6.17 $\mathbf{e}_r$ 6.18 $\mathbf{i}$ 6.19 $\mathbf{j}$ 6.20 $2r\mathbf{e}_r$

7.1 $\nabla\cdot\mathbf{r} = 3$, $\nabla\times\mathbf{r} = 0$ 7.2 $\nabla\cdot\mathbf{r} = 2$, $\nabla\times\mathbf{r} = 0$
7.3 $\nabla\cdot\mathbf{V} = 1$, $\nabla\times\mathbf{V} = 0$ 7.4 $\nabla\cdot\mathbf{V} = 0$, $\nabla\times\mathbf{V} = -(\mathbf{i}+\mathbf{j}+\mathbf{k})$
7.5 $\nabla\cdot\mathbf{V} = 2(x+y+z)$, $\nabla\times\mathbf{V} = 0$
7.6 $\nabla\cdot\mathbf{V} = 5xy$, $\nabla\times\mathbf{V} = \mathbf{i}xz - \mathbf{j}yz + \mathbf{k}(y^2 - x^2)$
7.7 $\nabla\cdot\mathbf{V} = 0$, $\nabla\times\mathbf{V} = x\mathbf{i} - y\mathbf{j} - x\cos y\mathbf{k}$
7.8 $\nabla\cdot\mathbf{V} = 2 + x\sinh z$, $\nabla\times\mathbf{V} = 0$ 7.9 $6y$
7.10 0 7.11 $-(x^2+y^2)/(x^2-y^2)^{3/2}$
7.12 $4(x+y)^{-3}$ 7.13 $2xy$
7.14 0 7.15 0
7.16 $2(x^2+y^2+z^2)^{-1}$ 7.18 $2\mathbf{k}$
7.19 $2/r$ 7.20 0

8.1 $-11/3$
8.2 (a) -4π (b) -16 (c) -8
8.3 (a) 5/3 (b) 1 (c) 2/3
8.4 (a) 3 (b) 8/3
8.5 (a) 86/3 (b) $-31/3$
8.6 (a) 3 (b) 3 (c) 3
8.7 (a) -2π (b) 0 (c) -2 (d) 2π
8.8 $yz - x$ 8.9 $3xy - x^3yz - z^2$
8.10 $\frac{1}{2}kr^2$ 8.11 $-y\sin^2 x$
8.12 $-(xy+z)$ 8.13 $-z^2\cosh y$
8.14 $-\arcsin xy$ 8.15 $-(x^2+1)\cos^2 y$
8.16 (a) $\mathbf{F}_1$; $\phi_1 = y^2z - x^2$
(b) For $\mathbf{F}_2$: (1) $W = 0$ (2) $W = -4$ (3) $W = 2\pi$
8.17 $\mathbf{F}_2$ conservative, $W = 0$; for $\mathbf{F}_1$, $W = 2\pi$
8.18 (a) $\pi + \pi^2/2$ (b) $\pi^2/2$
8.20 $\phi = mgz$, $\phi = -C/r$

9.2 40 9.3 14/3 9.4 $-3/2$
9.5 20 9.7 πab 9.8 24π
9.9 $(\overline{x},\overline{y}) = (1,1)$ 9.10 -20 9.11 2
9.12 29/3

10.1 4π 10.2 3 10.3 9π
10.4 36π 10.5 $4\pi\cdot 5^5$ 10.6 1
10.7 48π 10.8 80π 10.9 16π
10.10 27π

10.12 $$\phi = \begin{cases} 0, r \le R_1 \\ (k/2\pi\epsilon_0)\ln(R_1/r), & R_1 \le r \le R_2 \\ (k/2\pi\epsilon_0)\ln(R_1/R_2), & r \ge R_2 \end{cases}$$

11.1 $-3\pi a^2$	11.2 $2ab^2$	11.3 0
11.4 -12	11.5 36	11.6 45π
11.7 0	11.8 0	11.9 $32\pi/3$
11.10 -6π	11.11 24	11.12 18π
11.13 0	11.14 -8π	11.15 $-2\pi\sqrt{2}$

In the answers for Problems 18 to 22, u is arbitrary.

11.18 $\mathbf{A} = (xz - yz^2 - y^2/2)\mathbf{i} + (x^2/2 - x^2z + yz^2/2 - yz)\mathbf{j} + \boldsymbol{\nabla} u$
11.19 $\mathbf{A} = (y^2z - xy^2/2)\mathbf{i} + xz^2\mathbf{j} + x^2y\mathbf{k} + \boldsymbol{\nabla} u$
11.20 $\mathbf{A} = \mathbf{i}\sin zx + \mathbf{j}\cos zx + \mathbf{k}e^{zy} + \boldsymbol{\nabla} u$
11.21 $\mathbf{A} = \mathbf{i}y + \boldsymbol{\nabla} u$
11.22 $\mathbf{A} = \mathbf{i}(xz - y^3/3) + \mathbf{j}(-yz + x^3/3) + \mathbf{k}(x+y)z + \boldsymbol{\nabla} u$

12.1 $\sin\theta\cos\theta\,\mathbf{C}$
12.2 $\frac{1}{2}|\mathbf{B}\times\mathbf{A}|$
12.5 (a) -4π (b) -16 (c) -8
12.6 $5\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$
12.7 (a) $9\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ (b) $29/3$
12.8 $2/\sqrt{5}$
12.9 24
12.10 (a) $2\mathbf{i} + \mathbf{j}$ (b) $11/5$ (c) $2x + y = 4$
12.11 (a) $\operatorname{grad}\phi = -3y\mathbf{i} - 3x\mathbf{j} + 2z\mathbf{k}$
(b) $-\sqrt{3}$
(c) $2x + y - 2z + 2 = 0$, $\mathbf{r} = (1,2,3) + (2,1,-2)t$
12.12 (a) $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
(b) 3
(c) $3x + 2y + z = 4$, $\mathbf{r} = (0,1,2) + (3,2,1)t$
(d) $(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})/\sqrt{14}$
12.13 (a) $6\mathbf{i} - \mathbf{j} - 4\mathbf{k}$
(b) $53^{-1/2}(6\mathbf{i} - \mathbf{j} - 4\mathbf{k})$
(c) same as (a)
(d) $53^{1/2}$
(e) $53^{1/2}$
12.14 (a) $-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$
(b) 0
12.15 $\phi = -y^2\cosh^2 xz$
12.16 (a) $\mathbf{F}_1$ is conservative.
(b) $W_1 = x^2z + \frac{1}{2}y^2$
(c) and (d) 1
12.17 6
12.18 Not conservative
(a) 1/2
(b) 4/3
12.19 (a) $\mathbf{F}_1$ is conservative; $\mathbf{F}_2$ is not conservative ($\boldsymbol{\nabla}\times\mathbf{F}_2 = \mathbf{k}$)
(b) 2π
(c) For $\mathbf{F}_1$, $V_1 = 2xy - yz - \frac{1}{2}z^2$
(d) $W_1 = 45/2$
(e) $W_2 = 2\pi$

12.20 π	12.21 4	12.22 108π	12.23 192π
12.24 54π	12.25 -18π	12.26 0	12.27 4
12.28 -2π	12.29 10	12.30 4	12.31 $29/3$

Chapter 7

		amplitude	period	frequency	velocity amplitude
2.1		3	$2\pi/5$	$5/(2\pi)$	15
2.2		2	$\pi/2$	$2/\pi$	8
2.3		$1/2$	2	$1/2$	$\pi/2$
2.4		5	2π	$1/(2\pi)$	5
2.5	$s = \sin 6t$	1	$\pi/3$	$3/\pi$	6
2.6	$s = 6\cos\frac{\pi}{8}\sin 2t$	$6\cos\frac{\pi}{8} = 5.54$	π	$1/\pi$	$12\cos\frac{\pi}{8} = 11.1$
2.7		5	2π	$1/(2\pi)$	5
2.8		2	4π	$1/(4\pi)$	1
2.9		2	2	$1/2$	2π
2.10		4	π	$1/\pi$	8
2.11	q	3	$1/60$	60	
	I	360π	$1/60$	60	
2.12	q	4	$1/15$	15	
	I	120π	$1/15$	15	

2.13 $A =$ maximum value of θ, $\omega = \sqrt{g/l}$.

2.14 $t = 12$

2.15 $t = 3\pi$

2.16 $t \cong 4.91 \cong 281°$

2.18 $A = 2,\ T = 1,\ f = 1,\ v = 3,\ \lambda = 3$

2.19 $A = 1,\ T = 4,\ f = 1/4,\ v = 1/4,\ \lambda = 1$

2.20 $A = 3,\ T = 4,\ f = 1/4,\ v = 1/2,\ \lambda = 2$

2.21 $y = 20\sin\frac{\pi}{2}(x - 6t)$, $\frac{\partial y}{\partial t} = -60\pi\cos\frac{\pi}{2}(x - 6t)$

2.22 $y = 4\sin 2\pi(\frac{x}{3} - \frac{t}{6})$

2.23 $y = \sin 880\pi(\frac{x}{350} - t)$

2.24 $y = \sin\frac{200\pi}{153}(x - 1530t)$

2.25 $y = 10\sin\frac{\pi}{250}(x - 3 \cdot 10^8 t)$

3.6 $\sin(2x + \frac{\pi}{3})$

3.7 $-\sqrt{2}\sin(\pi x - \frac{\pi}{4})$

4.3	0	4.4	e^{-1}	4.5	$1/\pi + 1/2$	4.6	$2/\pi$
4.7	$\pi/12 - 1/2$	4.8	0	4.9	$1/2$	4.10	0
4.11	$1/2$	4.12	$1/2$	4.14	(a) $2\pi/3$	(b) π	
4.15	(a) $3/2$	(b) $3/2$		4.16	(a) π/ω	(b) 1	

5.1 to 5.11 The answers for Problems 5.1 to 5.11 are the sine-cosine series in Problems 7.1 to 7.11.

$x \to$	-2π	$-\pi$	$-\pi/2$	0	$\pi/2$	π	2π
6.1	1/2	1/2	1	1/2	0	1/2	1/2
6.2	1/2	0	0	1/2	1/2	0	1/2
6.3	0	1/2	0	0	1/2	1/2	0
6.4	−1	0	−1	−1	0	0	−1
6.5	−1/2	1/2	0	−1/2	0	1/2	−1/2
6.6	1/2	1/2	1/2	1/2	1/2	1/2	1/2
6.7	0	$\pi/2$	0	0	$\pi/2$	$\pi/2$	0
6.8	1	1	$1-\frac{\pi}{2}$	1	$1+\frac{\pi}{2}$	1	1
6.9	0	π	$\pi/2$	0	$\pi/2$	π	0
6.10	π	0	$\pi/2$	π	$\pi/2$	0	π
6.11	0	0	0	0	1	0	0

6.13 and 6.14 At $x = \pi/2$, same series as in the example.

7.1 $$f(x) = \frac{1}{2} + \frac{i}{\pi}\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{1}{n}\, e^{inx} = \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n}\sin nx$$

7.2 $$a_n = \tfrac{1}{n\pi}\sin\tfrac{n\pi}{2},\ \ b_n = \tfrac{1}{n\pi}\left(1-\cos\tfrac{n\pi}{2}\right),\ \ a_0/2 = c_0 = \tfrac{1}{4},$$
$$c_n = \tfrac{i}{2n\pi}(e^{-in\pi/2}-1),\ \ n>0;\ \ c_{-n} = \bar{c}_n$$
$$f(x) = \tfrac{1}{4} + \tfrac{1}{2\pi}\Big[(1-i)e^{ix} + (1+i)e^{-ix} - \tfrac{2i}{2}(e^{2ix}-e^{-2ix})$$
$$-\tfrac{1+i}{3}e^{3ix} - \tfrac{1-i}{3}e^{-3ix} + \tfrac{1-i}{5}e^{5ix} + \tfrac{1+i}{5}e^{-5ix}\cdots\Big]$$
$$= \tfrac{1}{4} + \tfrac{1}{\pi}\left(\cos\ x - \tfrac{1}{3}\cos 3x + \tfrac{1}{5}\cos 5x\cdots\right)$$
$$+\tfrac{1}{\pi}\left(\sin x + \tfrac{2}{2}\sin 2x + \tfrac{1}{3}\sin 3x + \tfrac{1}{5}\sin 5x + \tfrac{2}{6}\sin 6x\cdots\right)$$

7.3 $$a_n = -\tfrac{1}{n\pi}\sin\tfrac{n\pi}{2},\ a_0/2 = c_0 = \tfrac{1}{4}$$
$$b_n = \tfrac{1}{n\pi}\left(\cos\tfrac{n\pi}{2} - \cos n\pi\right) = \tfrac{1}{n\pi}\{1,-2,1,0,\text{ and repeat}\}$$
$$c_n = \tfrac{i}{2n\pi}\left(e^{-in\pi} - e^{-in\pi/2}\right),\ \ n>0;\ \ c_{-n} = \bar{c}_n$$
$$f(x) = \tfrac{1}{4} + \tfrac{1}{2\pi}\Big[-(1+i)\ e^{ix} - (1-i)\ e^{-ix} + \tfrac{2i}{2}(e^{2ix}-e^{-2ix})$$
$$+\tfrac{1-i}{3}\ e^{3ix} + \tfrac{1+i}{3}\ e^{-3ix} - \tfrac{1+i}{5}\ e^{5ix} - \tfrac{(1-i)}{5}\ e^{-5ix}\cdots\Big]$$
$$= \tfrac{1}{4} - \tfrac{1}{\pi}\left(\cos\ x - \tfrac{1}{3}\cos 3x + \tfrac{1}{5}\cos 5x\cdots\right)$$
$$+\tfrac{1}{\pi}\left(\sin x - \tfrac{2}{2}\sin 2x + \tfrac{1}{3}\sin 3x + \tfrac{1}{5}\sin 5x\cdots\right)$$

7.4 $c_0 = a_0/2 = -1/2$; for $n \neq 0$, coefficients are 2 times the coefficients in Problem 7.3.
$$f(x) = -\tfrac{1}{2} - \tfrac{1}{\pi}\Big[(1+i)e^{ix} + (1-i)e^{-ix} - \tfrac{2i}{2}(e^{2ix}-e^{-2ix})$$
$$-\tfrac{1-i}{3}e^{3ix} - \tfrac{1+i}{3}e^{-3ix} + \tfrac{1+i}{5}e^{5ix} + \tfrac{1-i}{5}e^{-5ix}\cdots\Big]$$
$$= -\tfrac{1}{2} - \tfrac{2}{\pi}\left(\cos\ x - \tfrac{1}{3}\cos 3x + \tfrac{1}{5}\cos 5x\cdots\right)$$
$$+\tfrac{2}{\pi}\left(\sin x - \tfrac{2}{2}\sin 2x + \tfrac{1}{3}\sin 3x + \tfrac{1}{5}\sin 5x - \tfrac{2}{6}\sin 6x\cdots\right)$$

7.5 $a_n = -\frac{2}{n\pi}\sin\frac{n\pi}{2}$ $\qquad a_0/2 = c_0 = 0$

$b_n = \frac{1}{n\pi}\left(2\cos\frac{n\pi}{2} - 1 - \cos n\pi\right) = -\frac{4}{n\pi}\{0, 1, 0, 0, \text{ and repeat}\}$

$c_n = \frac{1}{2in\pi}\left(2e^{-in\pi/2} - 1 - e^{-in\pi}\right) = \frac{1}{n\pi}\{-1, 2i, 1, 0, \text{ and repeat }\},\ n > 0$

$c_{-n} = \overline{c}_n$

$$f(x) = -\frac{1}{\pi}\Big[e^{ix} + e^{-ix} - \frac{2i}{2}\left(e^{2ix} - e^{-2ix}\right) - \frac{1}{3}(e^{3ix} + e^{-3ix}) + \frac{1}{5}(e^{5ix} + e^{-5ix}) - \frac{2i}{6}(e^{6ix} - e^{-6ix})\cdots\Big]$$
$$= -\frac{2}{\pi}\left(\cos\ x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x\cdots\right) - \frac{4}{\pi}\left(\frac{1}{2}\sin 2x + \frac{1}{6}\sin 6x + \frac{1}{10}\sin 10x\cdots\right)$$

7.6 $f(x) = \frac{1}{2} + \frac{2}{i\pi}\sum \frac{1}{n}e^{inx}$ $\qquad (n = \pm 2,\ \pm 6,\ \pm 10, \cdots)$

$= \frac{1}{2} + \frac{4}{\pi}\sum \frac{1}{n}\sin\ nx$ $\qquad (n = 2,\ 6,\ 10, \cdots)$

7.7
$$f(x) = \frac{\pi}{4} - \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\left(\frac{1}{n^2\pi} + \frac{i}{2n}\right)e^{inx} + \sum_{\substack{-\infty \\ \text{even } n\neq 0}}^{\infty}\frac{i}{2n}e^{inx}$$
$$= \frac{\pi}{4} - \sum_{1}^{\infty}\frac{(-1)^n}{n}\sin\ nx\ -\ \frac{2}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{1}{n^2}\cos\ nx$$

7.8
$$f(x) = 1 + \sum_{\substack{-\infty \\ n\neq 0}}^{\infty}(-1)^n\frac{i}{n}\,e^{inx} = 1 + 2\sum_{1}^{\infty}(-1)^{n+1}\,\frac{1}{n}\sin\ nx$$

7.9
$$f(x) = \frac{\pi}{2} - \frac{2}{\pi}\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\frac{e^{inx}}{n^2} = \frac{\pi}{2} - \frac{4}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{\cos\ nx}{n^2}$$

7.10
$$f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\frac{e^{inx}}{n^2} = \frac{\pi}{2} + \frac{4}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{\cos\ nx}{n^2}$$

7.11
$$f(x) = \frac{1}{\pi} + \frac{e^{ix} - e^{-ix}}{4i} - \frac{1}{\pi}\sum_{\substack{-\infty \\ \text{even } n\neq 0}}^{\infty}\frac{e^{inx}}{n^2 - 1}$$
$$= \frac{1}{\pi} + \frac{1}{2}\sin\ x - \frac{2}{\pi}\sum_{\substack{2 \\ \text{even } n}}^{\infty}\frac{\cos nx}{n^2 - 1}$$

7.13 $a_n = 2\,\mathrm{Re}\,c_n,\ b_n = -2\,\mathrm{Im}\,c_n,\ c_n = \frac{1}{2}(a_n - ib_n),\ c_{-n} = \frac{1}{2}(a_n + ib_n)$

8.1
$$f(x) = \frac{1}{2} + \frac{i}{\pi}\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\frac{1}{n}e^{in\pi x/l} = \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{l}$$

8.2 $a_n = \frac{1}{n\pi}\sin\frac{n\pi}{2},\ \ b_n = \frac{1}{n\pi}\left(1 - \cos\frac{n\pi}{2}\right),\ \ a_0/2 = c_0 = \frac{1}{4}$

$c_n = \frac{i}{2n\pi}(e^{-in\pi/2} - 1)$

$= \frac{1}{2n\pi}\ \{1 - i,\ -2i,\ -(1 + i),\ 0,\ \text{and repeat}\},\ \ n > 0; c_{-n} = \overline{c}_n$

$$f(x) = \frac{1}{4} + \frac{1}{2\pi}\Big[(1 - i)e^{i\pi x/l} + (1 + i)e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) - \frac{1+i}{3}e^{3i\pi x/l} - \frac{1-i}{3}e^{-3i\pi x/l} + \frac{1-i}{5}e^{5i\pi x/l} + \frac{1+i}{5}e^{-5i\pi x/l}\cdots\Big]$$
$$= \frac{1}{4} + \frac{1}{\pi}\left(\cos\frac{\pi x}{l} - \frac{1}{3}\cos\frac{3\pi x}{l} + \frac{1}{5}\cos\frac{5\pi x}{l}\cdots\right) + \frac{1}{\pi}\left(\sin\frac{\pi x}{l} + \frac{2}{2}\sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l} + \frac{1}{5}\sin\frac{5\pi x}{l} + \frac{2}{6}\sin\frac{6\pi x}{l}\cdots\right)$$

8.3 $a_n = -\frac{1}{n\pi}\sin\frac{n\pi}{2},\ \ a_0/2 = c_0 = \frac{1}{4}$
$b_n = \frac{1}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right) = \frac{1}{n\pi}\{1, -2, 1, 0\text{ , and repeat}\}$
$c_n = \frac{i}{2n\pi}(e^{-in\pi} - e^{-in\pi/2})$
$= \frac{1}{2n\pi}\ \{-(1+i), 2i, 1-i, 0, \text{ and repeat}\},\ \ n > 0; c_{-n} = \overline{c}_n$
$f(x) = \frac{1}{4} + \frac{1}{2\pi}\Big[-(1+i)e^{i\pi x/l} - (1-i)e^{-i\pi x/l} + \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l})$
$+\frac{1-i}{3}e^{3i\pi x/l} + \frac{1+i}{3}e^{-3i\pi x/l} - \frac{1+i}{5}e^{5i\pi x/l} - \frac{1-i}{5}e^{-5i\pi x/l}\cdots\Big]$
$= \frac{1}{4} - \frac{1}{\pi}\left(\cos\frac{\pi x}{l} - \frac{1}{3}\cos\frac{3\pi x}{l} + \frac{1}{5}\cos\frac{5\pi x}{l}\cdots\right)$
$+\frac{1}{\pi}\left(\sin\frac{\pi x}{l} - \frac{2}{2}\sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l} + \frac{1}{5}\sin\frac{5\pi x}{l} - \frac{2}{6}\sin\frac{6\pi x}{l}\cdots\right)$

8.4 $c_0 = a_0/2 = -1/2$; for $n \neq 0$, coefficients are 2 times the coefficients in Problem 8.3.
$f(x) = -\frac{1}{2} - \frac{1}{\pi}\Big[(1+i)e^{i\pi x/l} + (1-i)e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l})$
$-\frac{1-i}{3}e^{3i\pi x/l} - \frac{1+i}{3}e^{-3i\pi x/l} + \frac{1+i}{5}e^{5i\pi x/l} + \frac{1-i}{5}e^{-5i\pi x/l}\cdots\Big]$
$= -\frac{1}{2} - \frac{2}{\pi}\left(\cos\frac{\pi x}{l} - \frac{1}{3}\cos\frac{3\pi x}{l} + \frac{1}{5}\cos\frac{5\pi x}{l}\cdots\right)$
$+\frac{2}{\pi}\left(\sin\frac{\pi x}{l} - \frac{2}{2}\sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l} + \frac{1}{5}\sin\frac{5\pi x}{l} - \frac{2}{6}\sin\frac{6\pi x}{l}\cdots\right)$

8.5 $a_n = -\frac{2}{n\pi}\sin\frac{n\pi}{2},\quad a_0 = 0,$
$b_n = \frac{1}{n\pi}(2\cos\frac{n\pi}{2} - 1 - \cos n\pi) = -\frac{4}{n\pi}\{0, 1, 0, 0, \text{ and repeat}\}$
$c_n = \frac{1}{2in\pi}(2e^{-in\pi/2} - 1 - e^{-in\pi}) = \frac{1}{n\pi}\{-1, 2i, 1, 0, \text{ and repeat }\},\ n > 0$
$c_{-n} = \overline{c}_n,\ c_0 = 0$
$f(x) = -\frac{1}{\pi}\Big[e^{i\pi x/l} + e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l})$
$-\frac{1}{3}(e^{3i\pi x/l} + e^{-3i\pi x/l})$
$+\frac{1}{5}(e^{5i\pi x/l} + e^{-5i\pi x/l}) - \frac{2i}{6}(e^{6i\pi x/l} - e^{-6i\pi x/l})\cdots\Big]$
$= -\frac{2}{\pi}\left(\cos\ \frac{\pi x}{l} - \frac{1}{3}\cos\frac{3\pi x}{l} + \frac{1}{5}\cos\frac{5\pi x}{l}\cdots\right)$
$-\frac{4}{\pi}\left(\frac{1}{2}\sin\frac{2\pi x}{l} + \frac{1}{6}\sin\frac{6\pi x}{l} + \frac{1}{10}\sin\frac{10\pi x}{l}\cdots\right)$

8.6 $f(x) = \frac{1}{2} + \frac{2}{i\pi}\sum\frac{1}{n}e^{in\pi x/l}\qquad (n = \pm 2,\ \pm 6,\ \pm 10,\ \cdots)$
$= \frac{1}{2} + \frac{4}{\pi}\sum\frac{1}{n}\sin\frac{n\pi x}{l}\qquad (n = 2,\ 6,\ 10,\ \cdots)$

8.7 $$f(x) = \frac{l}{4} + \frac{il}{2\pi}\sum_{\substack{-\infty\\ n\neq 0}}^{\infty}\frac{(-1)^n}{n}e^{in\pi x/l} - \sum_{\substack{-\infty\\ \text{odd } n}}^{\infty}\frac{l}{n^2\pi^2}e^{in\pi x/l}$$
$$= \frac{l}{4} - \frac{2l}{\pi^2}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{1}{n^2}\cos\frac{n\pi x}{l} - \frac{l}{\pi}\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{l}$$

8.8 $$f(x) = 1 + \frac{il}{\pi}\sum_{\substack{-\infty\\ n\neq 0}}^{\infty}\frac{(-1)^n}{n}e^{in\pi x/l} = 1 - \frac{2l}{\pi}\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{l}$$

8.9 $$f(x) = \frac{l}{2} - \frac{2l}{\pi^2}\sum_{\substack{-\infty\\ \text{odd } n}}^{\infty}\frac{e^{in\pi x/l}}{n^2} = \frac{l}{2} - \frac{4l}{\pi^2}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{\cos n\pi x/l}{n^2}$$

8.10 (a) $$f(x) = i\sum_{\substack{-\infty\\ n\neq 0}}^{\infty}\frac{(-1)^n}{n}e^{inx} = -2\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin nx$$

(b) $$f(x) = \pi + \sum_{\substack{-\infty\\ n\neq 0}}^{\infty}\frac{i}{n}e^{inx} = \pi - 2\sum_{1}^{\infty}\frac{\sin nx}{n}$$

8.11 (a) $f(x) = \dfrac{\pi^2}{3} + 2\sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \dfrac{(-1)^n}{n^2} e^{inx} = \dfrac{\pi^2}{3} + 4\sum_{1}^{\infty} \dfrac{(-1)^n}{n^2} \cos nx$

(b) $f(x) = \dfrac{4\pi^2}{3} + 2\sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \left(\dfrac{1}{n^2} + \dfrac{i\pi}{n}\right) e^{inx} = \dfrac{4\pi^2}{3} + 4\sum_{1}^{\infty} \dfrac{\cos nx}{n^2} - 4\pi \sum_{1}^{\infty} \dfrac{\sin nx}{n}$

8.12 (a) $f(x) = \dfrac{\sinh \pi}{\pi} \sum_{-\infty}^{\infty} \dfrac{(-1)^n(1+in)}{1+n^2} e^{inx}$

$= \dfrac{\sinh \pi}{\pi} + \dfrac{2\sinh \pi}{\pi} \sum_{1}^{\infty} \dfrac{(-1)^n}{1+n^2} (\cos\ nx - n\ \sin\ nx)$

(b) $f(x) = \dfrac{e^{2\pi}-1}{2\pi} \sum_{-\infty}^{\infty} \dfrac{1+in}{1+n^2} e^{inx}$

$= \dfrac{e^{2\pi}-1}{\pi} \left[\dfrac{1}{2} + \sum_{1}^{\infty} \dfrac{1}{1+n^2} (\cos\ nx - n\ \sin\ nx)\right]$

8.13 (a) $f(x) = 2 + \dfrac{2}{i\pi} \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \dfrac{(-1)^n}{n} e^{in\pi x/2} = 2 + \dfrac{4}{\pi} \sum_{1}^{\infty} \dfrac{(-1)^n}{n} \sin \dfrac{n\pi x}{2}$

(b) $f(x) = \dfrac{2}{i\pi} \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \dfrac{1}{n} e^{in\pi x/2} = \dfrac{4}{\pi} \sum_{1}^{\infty} \dfrac{1}{n} \sin \dfrac{n\pi x}{2}$

8.14 (a) $f(x) = \dfrac{8}{\pi} \sum_{1}^{\infty} \dfrac{n(-1)^{n+1}}{4n^2-1} \sin 2n\pi x = \dfrac{4i}{\pi} \sum_{-\infty}^{\infty} \dfrac{n(-1)^n}{4n^2-1} e^{2in\pi x}$

(b) $f(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi} \sum_{1}^{\infty} \dfrac{\cos 2n\pi x}{4n^2-1} = -\dfrac{2}{\pi} \sum_{-\infty}^{\infty} \dfrac{1}{4n^2-1} e^{2in\pi x}$

8.15 (a) $f(x) = \dfrac{i}{\pi} \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \dfrac{(-1)^n}{n} e^{in\pi x} = \dfrac{2}{\pi} \sum_{1}^{\infty} (-1)^{n+1} \dfrac{\sin n\pi x}{n}.$

(b) $f(x) = \dfrac{4}{\pi^2} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2} e^{in\pi x} = \dfrac{8}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{\cos n\pi x}{n^2}.$

(c) $f(x) = \dfrac{-4i}{\pi^3} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \dfrac{1}{n^3} e^{in\pi x} = \dfrac{8}{\pi^3} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{\sin n\pi x}{n^3}.$

8.16 $f(x) = 1 - \dfrac{2}{\pi} \sum_{1}^{\infty} \dfrac{\sin n\pi x}{n} = 1 - \dfrac{1}{i\pi} \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \dfrac{1}{n} e^{in\pi x}$

8.17 $f(x) = \frac{3}{4} - \frac{1}{\pi}(\cos \frac{\pi x}{2} - \frac{1}{3}\cos \frac{3\pi x}{2} + \frac{1}{5}\cos \frac{5\pi x}{2} \cdots)$
$+\frac{1}{\pi}(\sin \frac{\pi x}{2} + \frac{2}{2}\sin \pi x + \frac{1}{3}\sin \frac{3\pi x}{2} + \frac{1}{5}\sin \frac{5\pi x}{2} + \frac{2}{6}\sin 3\pi x \cdots)$

8.18 $f(x) = \dfrac{100}{3} + \dfrac{100}{\pi^2} \sum_{1}^{\infty} \dfrac{1}{n^2} \cos \dfrac{n\pi x}{5} - \dfrac{100}{\pi} \sum_{1}^{\infty} \dfrac{1}{n} \sin \dfrac{n\pi x}{5}$

$= \dfrac{100}{3} + 50 \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \left(\dfrac{1}{n^2\pi^2} - \dfrac{1}{in\pi}\right) e^{in\pi x/5}$

8.19 $f(x) = \dfrac{1}{8} - \dfrac{1}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2} \cos 2n\pi x + \dfrac{1}{2\pi} \sum_{1}^{\infty} \dfrac{(-1)^{n+1}}{n} \sin 2n\pi x$

8.20 $f(x)=\frac{2}{3}+\sum_{1}^{\infty} a_n\cos\frac{2n\pi x}{3}+\sum_{1}^{\infty} b_n\sin\frac{2n\pi x}{3}$, where

$$a_n=\begin{cases}0, & n=3k\\ \dfrac{-9}{8n^2\pi^2}, & \text{otherwise}\end{cases}\qquad b_n=\begin{cases}-\dfrac{1}{n\pi}, & n=3k\\ -\dfrac{1}{n\pi}-\dfrac{3\sqrt{3}}{8n^2\pi^2}, & n=3k+1\\ -\dfrac{1}{n\pi}+\dfrac{3\sqrt{3}}{8n^2\pi^2}, & n=3k+2\end{cases}$$

9.1 (a) $\cos nx+i\sin nx$ (b) $x\sinh x+x\cosh x$

9.2 (a) $\frac{1}{2}\ln|1-x^2|+\frac{1}{2}\ln|\frac{1-x}{1+x}|$ (b) $(\cos x+x\sin x)+(\sin x+x\cos x)$

9.3 (a) $(-x^4-1)+(x^5+x^3)$ (b) $(1+\cosh x)+\sinh x$

9.5 $f(x)=\frac{4}{\pi}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{1}{n}\sin nx$

9.6 $f(x)=\frac{4}{\pi}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{l}$

9.7 $a_n=\frac{2}{n\pi}\sin\frac{n\pi}{2}$, $a_0/2=1/2$
$f(x)=\frac{1}{2}+\frac{2}{\pi}(\cos\frac{\pi x}{2}-\frac{1}{3}\cos\frac{3\pi x}{2}+\frac{1}{5}\cos\frac{5\pi x}{2}\cdots)$

9.8 $f(x)=\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n}\sin 2nx$

9.9 $f(x)=\frac{1}{12}+\frac{1}{\pi^2}\sum_{1}^{\infty}\frac{(-1)^n}{n^2}\cos 2n\pi x$

9.10 $f(x)=\frac{\pi}{4}-\frac{2}{\pi}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{1}{n^2}\cos 2nx$

9.11 $f(x)=\frac{2\sinh\pi}{\pi}\left(\frac{1}{2}+\sum_{1}^{\infty}\frac{(-1)^n}{n^2+1}\cos nx\right)$

9.12 $f(x)=-\frac{2}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin n\pi x$

9.15 $f_c(x)=\frac{1}{2}-\frac{4}{\pi^2}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{\cos n\pi x}{n^2}$ $\qquad f_s(x)=\frac{2}{\pi}\sum_{1}^{\infty}\frac{(-1)^{n+1}\sin n\pi x}{n}$

9.16 $f_s=\frac{8}{\pi}\sum_{\text{odd } n=1}^{\infty}\frac{\sin nx}{n(4-n^2)}$ $\qquad f_c=f_p=(1-\cos 2x)/2$

9.17 $f_c(x)=\frac{4}{\pi}(\cos\pi x-\frac{1}{3}\cos 3\pi x+\frac{1}{5}\cos 5\pi x\cdots)$
$f_s(x)=f_p(x)=\frac{4}{\pi}\sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{1}{n}\sin 2n\pi x$

9.18 Even function: $a_0/2=1/3$,
$a_n=\frac{2}{n\pi}\sin\frac{n\pi}{3}=\frac{\sqrt{3}}{n\pi}\{1,1,0,-1,-1,0,\text{ and repeat}\}$
$f_c(x)=\frac{1}{3}+\frac{\sqrt{3}}{\pi}(\cos\frac{\pi x}{3}+\frac{1}{2}\cos\frac{2\pi x}{3}-\frac{1}{4}\cos\frac{4\pi x}{3}-\frac{1}{5}\cos\frac{5\pi x}{3}+\frac{1}{7}\cos\frac{7\pi x}{3}\cdots)$
Odd function: $b_n=\frac{2}{n\pi}(1-\cos\frac{n\pi}{3})=\frac{1}{n\pi}\{1,3,4,3,1,0,\text{ and repeat}\}$
$f_s(x)=\frac{1}{\pi}(\sin\frac{\pi x}{3}+\frac{3}{2}\sin\frac{2\pi x}{3}+\frac{4}{3}\sin\frac{3\pi x}{3}$
$+\frac{3}{4}\sin\frac{4\pi x}{3}+\frac{1}{5}\sin\frac{5\pi x}{3}+\frac{1}{7}\sin\frac{7\pi x}{3}\cdots)$

9.18 continued

Function of period 3:

$a_n = \frac{1}{n\pi}\sin\frac{2n\pi}{3} = \frac{\sqrt{3}}{2n\pi}\{1, -1, 0, \text{and repeat}\}, \quad a_0/2 = 1/3$

$b_n = \frac{1}{n\pi}(1-\cos\frac{2n\pi}{3}) = \frac{3}{2n\pi}\{1, 1, 0, \text{and repeat}\}$

$f_p(x) = \frac{1}{3} + \frac{\sqrt{3}}{2\pi}(\cos\frac{2\pi x}{3} - \frac{1}{2}\cos\frac{4\pi x}{3} + \frac{1}{4}\cos\frac{8\pi x}{3} - \frac{1}{5}\cos\frac{10\pi x}{3}\cdots)$
$\quad + \frac{3}{2\pi}(\sin\frac{2\pi x}{3} + \frac{1}{2}\sin\frac{4\pi x}{3} + \frac{1}{4}\sin\frac{8\pi x}{3} + \frac{1}{5}\sin\frac{10\pi x}{3}\cdots)$

9.19 $f_c(x) = f_p(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi}\displaystyle\sum_1^\infty \dfrac{(-1)^n\cos 2nx}{4n^2-1}$

For f_s, $b_n = \frac{2}{\pi}\begin{cases} 0, & n \text{ even} \\ \frac{2}{n+1}, & n = 1+4k \\ \frac{2}{n-1}, & n = 3+4k \end{cases}$

$f_s(x) = \frac{2}{\pi}(\sin x + \sin 3x + \frac{1}{3}\sin 5x + \frac{1}{3}\sin 7x + \frac{1}{5}\sin 9x + \frac{1}{5}\sin 11x\cdots)$

9.20 $f_c(x) = \dfrac{1}{3} + \dfrac{4}{\pi^2}\displaystyle\sum_1^\infty \dfrac{(-1)^n}{n^2}\cos n\pi x$

$f_s(x) = \dfrac{2}{\pi}\displaystyle\sum_1^\infty \dfrac{(-1)^{n+1}}{n}\sin n\pi x - \dfrac{8}{\pi^3}\sum_{\substack{1\\ \text{odd } n}}^\infty \dfrac{1}{n^3}\sin n\pi x$

$f_p(x) = \dfrac{1}{3} + \dfrac{1}{\pi^2}\displaystyle\sum_1^\infty \dfrac{1}{n^2}\cos 2n\pi x - \dfrac{1}{\pi}\sum_1^\infty \dfrac{1}{n}\sin 2n\pi x$

9.21 $f_c(x) = f_p(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\displaystyle\sum_{\substack{1\\ \text{odd } n}}^\infty \dfrac{1}{n^2}\cos n\pi x$

$f_s(x) = \dfrac{8}{\pi^2}\left(\sin\dfrac{\pi x}{2} - \dfrac{1}{3^2}\sin\dfrac{3\pi x}{2} + \dfrac{1}{5^2}\sin\dfrac{5\pi x}{2}\cdots\right); \quad b_n = \dfrac{8}{n^2\pi^2}\sin\dfrac{n\pi}{2}$

9.22 Even function: $a_n = -\frac{20}{n\pi}\sin\frac{n\pi}{2}$

$f_c(x) = 15 - \frac{20}{\pi}(\cos\frac{\pi x}{20} - \frac{1}{3}\cos\frac{3\pi x}{20} + \frac{1}{5}\cos\frac{5\pi x}{20}\cdots)$

Odd function:

$b_n = \frac{20}{n\pi}(\cos\frac{n\pi}{2} + 1 - 2\cos n\pi) = \frac{20}{n\pi}\{3, -2, 3, 0, \text{and repeat}\}$

$f_s(x) = \frac{20}{\pi}(3\sin\frac{\pi x}{20} - \frac{2}{2}\sin\frac{2\pi x}{20} + \frac{3}{3}\sin\frac{3\pi x}{20} + \frac{3}{5}\sin\frac{5\pi x}{20}\cdots)$

Function of period 20:

$f_p(x) = 15 - \dfrac{20}{\pi}\displaystyle\sum_{\substack{1\\ \text{odd } n}}^\infty \dfrac{1}{n}\sin\dfrac{n\pi x}{10}$

9.23 $f(x,0) = \dfrac{8h}{\pi^2}\left(\sin\dfrac{\pi x}{l} - \dfrac{1}{3^2}\sin\dfrac{3\pi x}{l} + \dfrac{1}{5^2}\sin\dfrac{5\pi x}{l}\cdots\right)$

9.24 $f(x,0) = \dfrac{8h}{\pi^2}\displaystyle\sum_1^\infty \dfrac{\lambda_n}{n^2}\sin\dfrac{n\pi x}{l}$ where

$\lambda_1 = \sqrt{2}-1$, $\lambda_2 = 2$, $\lambda_3 = \sqrt{2}+1$, $\lambda_4 = 0$, $\lambda_5 = -(\sqrt{2}+1)$,
$\lambda_6 = -2$, $\lambda_7 = -\sqrt{2}+1$, $\lambda_8 = 0$, ..., $\lambda_n = 2\sin\frac{n\pi}{4} - \sin\frac{n\pi}{2}$

9.26 $f(x) = \dfrac{1}{2} - \dfrac{48}{\pi^4}\displaystyle\sum_{\substack{1\\ \text{odd } n}}^\infty \dfrac{\cos n\pi x}{n^4}$

9.27 $f(x) = \dfrac{8\pi^4}{15} - 48\displaystyle\sum_1^\infty (-1)^n\dfrac{\cos nx}{n^4}$

10.1 $p(t) = \sum_{1}^{\infty} a_n \cos 220n\pi t,\ a_0 = 0$

$a_n = \frac{2}{n\pi}(\sin \frac{n\pi}{3} + \sin\frac{2n\pi}{3}) = \frac{2}{n\pi}\{\sqrt{3},\ 0,\ 0,\ 0,\ -\sqrt{3},\ 0, \text{ and repeat}\}$

Relative intensities $= 1 : 0 : 0 : 0 : \frac{1}{25} : 0 : \frac{1}{49} : 0 : 0 : 0$

10.2 $p(t) = \sum_{1}^{\infty} b_n \sin 262n\pi t$, where

$b_n = \frac{2}{n\pi}(1 - \cos \frac{n\pi}{3} - 3\cos n\pi + 3\cos\frac{2n\pi}{3})$

$= \frac{2}{n\pi}\{2, -3, 8, -3, 2, 0, \text{ and repeat}\}$

Relative intensities $= 4 : \frac{9}{4} : \frac{64}{9} : \frac{9}{16} : \frac{4}{25} : 0$

10.3 $p(t) = \sum b_n \sin 220n\pi t$

$b_n = \frac{2}{n\pi}(3 - 5\cos\frac{n\pi}{2} + 2\cos n\pi) = \frac{2}{n\pi}\{1, 10, 1, 0, \text{ and repeat}\}$

Relative intensities $= 1 : 25 : \frac{1}{9} : 0 : \frac{1}{25} : \frac{25}{9} : \frac{1}{49} : 0 : \frac{1}{81} : 1$

10.4 $V(t) = \frac{200}{\pi}\left[1 + \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{2}{1-n^2}\cos 120n\pi t\right]$

Relative intensities $= 0 : 1 : 0 : \frac{1}{25} : 0 : (\frac{3}{35})^2$

10.5 $I(t) = \frac{5}{\pi}\left[1 + \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{2}{1-n^2}\cos 120n\pi t\right] + \frac{5}{2}\sin 120\pi t$

Relative intensities $= (\frac{5}{2})^2 : (\frac{10}{3\pi})^2 : 0 : (\frac{2}{3\pi})^2 : 0 : (\frac{2}{7\pi})^2$

$= 6.25 : 1.13 : 0 : 0.045 : 0 : 0.008$

10.6 $V(t) = 50 - \frac{400}{\pi^2}\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2}\cos 120n\pi t$

Relative intensities $= 1 : 0 : \left(\frac{1}{3}\right)^4 : 0 : \left(\frac{1}{5}\right)^4$

10.7 $I(t) = -\frac{20}{\pi}\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin 120n\pi t$

Relative intensities $= 1 : \frac{1}{4} : \frac{1}{9} : \frac{1}{16} : \frac{1}{25}$

10.8 $I(t) = \frac{5}{2} - \frac{20}{\pi^2}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{1}{n^2}\cos 120n\pi t - \frac{10}{\pi}\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin 120n\pi t$

Relative intensities $= \left(1 + \frac{4}{\pi^2}\right) : \frac{1}{4} : \frac{1}{9}\left(1 + \frac{4}{9\pi^2}\right) : \frac{1}{16} : \frac{1}{25}\left(1 + \frac{4}{25\pi^2}\right)$

$= 1.4 : 0.25 : 0.12 : 0.06 : 0.04$

10.9 $V(t) = \frac{400}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{1}{n}\sin 120n\pi t$

Relative intensities $= 1 : 0 : \frac{1}{9} : 0 : \frac{1}{25}$

10.10 $V(t) = 75 - \frac{200}{\pi^2}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{1}{n^2}\cos 120n\pi t - \frac{100}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin 120n\pi t$

Relative intensities as in problem 10.8

11.5 $\pi^2/8$ 11.6 $\pi^4/90$ 11.7 $\pi^2/6$

11.8 $\pi^4/96$ 11.9 $\frac{\pi^2}{16} - \frac{1}{2}$

12.2 $f_s(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{1-\cos\alpha}{\alpha}\sin\alpha x\,d\alpha$

12.3 $f(x) = \displaystyle\int_{-\infty}^\infty \frac{1-\cos\alpha\pi}{i\alpha\pi}e^{i\alpha x}\,d\alpha$

12.4 $f(x) = \displaystyle\int_{-\infty}^\infty \frac{\sin\alpha\pi - \sin(\alpha\pi/2)}{\alpha\pi}e^{i\alpha x}\,d\alpha$

12.5 $f(x) = \displaystyle\int_{-\infty}^\infty \frac{1-e^{-i\alpha}}{2\pi i\alpha}e^{i\alpha x}\,d\alpha$

12.6 $f(x) = \displaystyle\int_{-\infty}^\infty \frac{\sin\alpha - \alpha\cos\alpha}{i\pi\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.7 $f(x) = \displaystyle\int_{-\infty}^\infty \frac{\cos\alpha + \alpha\sin\alpha - 1}{\pi\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.8 $f(x) = \displaystyle\int_{-\infty}^\infty \frac{(i\alpha+1)e^{-i\alpha} - 1}{2\pi\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.9 $f(x) = \dfrac{2}{\pi}\displaystyle\int_{-\infty}^\infty \frac{i - \cos\alpha a}{\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.10 $f(x) = 2\displaystyle\int_{-\infty}^\infty \frac{\alpha a - \sin\alpha a}{i\pi\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.11 $f(x) = \dfrac{1}{\pi}\displaystyle\int_{-\infty}^\infty \frac{\cos(\alpha\pi/2)}{1-\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.12 $f(x) = \dfrac{1}{\pi i}\displaystyle\int_{-\infty}^\infty \frac{\alpha\cos(\alpha\pi/2)}{1-\alpha^2}\,e^{i\alpha x}\,d\alpha$

12.13 $f_c(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\sin\alpha\pi - \sin(\alpha\pi/2)}{\alpha}\cos\alpha x\,d\alpha$

12.14 $f_c(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\cos\alpha + \alpha\sin\alpha - 1}{\alpha^2}\cos\alpha x\,d\alpha$

12.15 $f_c(x) = \dfrac{4}{\pi}\displaystyle\int_0^\infty \frac{1-\cos\alpha a}{\alpha^2}\cos\alpha x\,d\alpha$

12.16 $f_c(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\cos(\alpha\pi/2)}{1-\alpha^2}\cos\alpha x\,d\alpha$

12.17 $f_s(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{1-\cos\pi\alpha}{\alpha}\sin\alpha x\,d\alpha$

12.18 $f_s(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\sin\alpha - \alpha\cos\alpha}{\alpha^2}\sin\alpha x\,d\alpha$

12.19 $f_s(x) = \dfrac{4}{\pi}\displaystyle\int_0^\infty \frac{\alpha a - \sin\alpha a}{\alpha^2}\sin\alpha x\,d\alpha$

12.20 $f_s(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\alpha\cos(\alpha\pi/2)}{1-\alpha^2}\sin\alpha x\,d\alpha$

12.21 $g(\alpha) = \dfrac{\sigma}{\sqrt{2\pi}}\,e^{-\alpha^2\sigma^2/2}$

12.24 (c) $g_c(\alpha) = \sqrt{\dfrac{\pi}{2}}e^{-|\alpha|}$

12.25 (a) $f(x) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^\infty \frac{1+e^{-i\alpha\pi}}{1-\alpha^2}e^{i\alpha x}\,d\alpha$

12.27 (a) $f_c(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\sin(\alpha\pi/2)}{\alpha}\cos\alpha x\,d\alpha$

(b) $f_s(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{1-\cos(\alpha\pi/2)}{\alpha}\sin\alpha x\,d\alpha$

12.28 (a) $f_c(x) = \dfrac{4}{\pi}\displaystyle\int_0^\infty \frac{\cos 3\alpha\sin\alpha}{\alpha}\cos\alpha x\,d\alpha$

(b) $f_s(x) = \dfrac{4}{\pi}\displaystyle\int_0^\infty \frac{\sin 3\alpha\sin\alpha}{\alpha}\sin\alpha x\,d\alpha$

12.29 (a) $f_c(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin 3\alpha - 2\sin 2\alpha}{\alpha}\cos\alpha x\,d\alpha$

(b) $f_s(x) = \frac{2}{\pi}\int_0^\infty \frac{2\cos 2\alpha - \cos 3\alpha - 1}{\alpha}\sin\alpha x\,d\alpha$

12.30 (a) $f_c(x) = \frac{1}{\pi}\int_0^\infty \frac{1-\cos 2\alpha}{\alpha^2}\cos\alpha x\,d\alpha$

(b) $f_s(x) = \frac{1}{\pi}\int_0^\infty \frac{2\alpha - \sin 2\alpha}{\alpha^2}\sin\alpha x\,d\alpha$

13.2 $f(x) = \frac{i}{2\pi}\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\frac{1}{n}e^{2in\pi x}$

13.4 (c) $q\ (t) = CV\left[1 - 2(1-e^{-1/2})\sum_{-\infty}^{\infty}(1+4in\pi)^{-1}e^{4in\pi t/(RC)}\right]$

13.6 $f(t) = \sum_{-\infty}^{\infty}\frac{(-1)^n\sin\omega\pi}{\pi(\omega-n)}e^{int}$

13.7 $f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\frac{1}{n^2}\cos nx$

13.8 (a) 1/2 (b) 1

13.9 (b) −1/2, 0, 0, 1/2 (c) 13/6

13.10 (c) 0, −1/2, −2, −2 (d) −1, −1/2, −2, −1

13.11 Cosine series: $a_0/2 = -3/4$,

$$a_n = \frac{4}{n^2\pi^2}\left(\cos\frac{n\pi}{2} - 1\right) + \frac{6}{n\pi}\sin\frac{n\pi}{2}$$
$$= \frac{4}{n^2\pi^2}\{-1,-2,-1,0,\text{and repeat}\} + \frac{6}{n\pi}\{1,0,-1,0,\text{and repeat}\}$$
$$f_c(x) = -\frac{3}{4} + \left(-\frac{4}{\pi^2}+\frac{6}{\pi}\right)\cos\frac{\pi x}{2} - \frac{2}{\pi^2}\cos\pi x$$
$$-\left(\frac{4}{9\pi^2}+\frac{2}{\pi}\right)\cos\frac{3\pi x}{2} + \left(\frac{-4}{25\pi^2}+\frac{6}{5\pi}\right)\cos\frac{5\pi x}{2}\cdots$$

Sine series:

$$b_n = \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{1}{n\pi}\left(4\cos n\pi - 6\cos\frac{n\pi}{2}\right)$$
$$= \frac{4}{n^2\pi^2}\{1,0,-1,0,\text{and repeat}\} + \frac{1}{n\pi}\{-4,10,-4,-2,\text{and repeat}\}$$
$$f_s(x) = \left(\frac{4}{\pi^2}-\frac{4}{\pi}\right)\sin\frac{\pi x}{2} + \frac{5}{\pi}\sin\pi x - \left(\frac{4}{9\pi^2}+\frac{4}{3\pi}\right)\sin\frac{3\pi x}{2}$$
$$-\frac{1}{2\pi}\sin 2\pi x + \left(\frac{4}{25\pi^2}-\frac{4}{5\pi}\right)\sin\frac{5\pi x}{2} + \frac{5}{3\pi}\sin 3\pi x\cdots$$

Exponential series of period 2:

$$f_p(x) = -\frac{3}{4} - \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\left(\frac{1}{n^2\pi^2}+\frac{5i}{2n\pi}\right)e^{in\pi x} + \frac{i}{2\pi}\sum_{\substack{-\infty \\ \text{even } n\neq 0}}^{\infty}\frac{1}{n}e^{in\pi x}$$

13.12 $f = 90$

13.13 (a) $f_s(x) = \sum_1^\infty \frac{\sin nx}{n}$ (b) $\pi^2/6$

13.14 (a) $f(x) = \frac{1}{3} + \frac{4}{\pi^2}\sum_1^\infty\frac{\cos n\pi x}{n^2}$ (b) $\pi^4/90$

13.15 $g(\alpha) = \dfrac{\cos 2\alpha - 1}{i\pi\alpha}, \quad f(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \dfrac{\cos 2\alpha - 1}{\alpha} \sin \alpha x \, d\alpha, \quad -\pi/4$

13.16 $f(x) = \dfrac{8}{\pi}\displaystyle\int_0^\infty \dfrac{\cos\alpha \sin^2(\alpha/2)}{\alpha^2} \cos \alpha x \, d\alpha, \quad \pi/8$

13.19 $\displaystyle\int_0^\infty |f(x)|^2 \, dx = \int_0^\infty |g_c(\alpha)|^2 \, d\alpha = \int_0^\infty |g_s(\alpha)|^2 \, d\alpha$

13.20 $g(\alpha) = \dfrac{2}{\pi}\dfrac{\sin^2 \alpha a}{\alpha^2}, \quad \pi a^3/3$ 13.23 $\pi^2/8$

Chapter 8

1.4	$x = k^{-1}gt + k^{-2}g(e^{-kt} - 1)$		
1.5	$x = -A\omega^{-2}\sin\omega t + v_0 t + x_0$		
1.6	(a) 15 months (b) $t = 30(1 - 2^{-1/3}) = 6.19$ months		
1.7	$x = (c/F)[(m^2c^2 + F^2t^2)^{1/2} - mc]$		
2.1	$y = mx$, $m = 3/2$	2.2	$(1-x^2)^{1/2} + (1-y^2)^{1/2} = C$, $C = \sqrt{3}$
2.3	$\ln y = A(\csc x - \cot x)$, $A = \sqrt{3}$	2.4	$x^2(1+y^2) = K$, $K = 25$
2.5	$y = axe^x$, $a = 1/e$	2.6	$2y^2 + 1 = A(x^2-1)^2$, $A = 1$
2.7	$y^2 = 8 + e^{K-x^2}$, $K = 1$	2.8	$y(x^2 + C) = 1$, $C = -3$
2.9	$ye^y = ae^x$, $a = 1$	2.10	$y + 1 = ke^{x^2/2}$, $k = 2$
2.11	$(y-2)^2 = (x+C)^3$, $C = 0$	2.12	$xye^y = K$, $K = e$
2.13	$y \equiv 1$, $y \equiv -1$, $x \equiv 1$, $x \equiv -1$	2.14	$y \equiv 0$
2.15	$y \equiv 2$	2.16	$4y = (x+C)^2$, $C = 0$
2.17	$x = (t - t_0)^2/4$		

2.19 (a) $I/I_0 = e^{-0.5} = 0.6$ for $s = 50$ft
Half value thickness $= (\ln 2)/\mu = 69.3$ft
(b) Half life $T = (\ln 2)/\lambda$

2.20 (a) $q = q_0 e^{-t/(RC)}$ (b) $I = I_0 e^{-(R/L)t}$ (c) $\tau = RC$, $\tau = L/R$
Corresponding quantities are a, $\lambda = (\ln 2)/T$, μ, $1/\tau$.

2.21 $N = N_0 e^{Kt}$

2.22 $N = N_0 e^{Kt} - (R/K)(e^{Kt} - 1)$ where N_0 = number of bacteria at $t = 0$,
KN = rate of increase, R = removal rate.

2.23 $T = 100[1 - (\ln r)/(\ln 2)]$

2.24 $T = 100(2r^{-1} - 1)$

2.26 (a) k = weight divided by terminal speed.
(b) $t = g^{-1} \cdot$ (terminal speed) $\cdot (\ln 100)$; typical terminal
speeds are 0.02 to 0.1 cm/sec, so t is of the order of 10^{-4} sec.

2.27 $t = 10(\ln \frac{5}{13})/(\ln \frac{3}{13}) = 6.6$ min

2.28	66°	2.29	$t = 100 \ln \frac{9}{4} = 81.1$min
2.30	$A = Pe^{It/100}$	2.31	$ay = bx$
2.32	$x^2 + 2y^2 = C$	2.33	$x^2 + ny^2 = C$
2.34	$x^2 - y^2 = C$	2.35	$x(y-1) = C$
3.1	$y = \frac{1}{2}e^x + Ce^{-x}$	3.2	$y = 1/(2x) + C/x^3$
3.3	$y = (\frac{1}{2}x^2 + C)e^{-x^2}$	3.4	$y = \frac{1}{3}x^{5/2} + Cx^{-1/2}$
3.5	$y(\sec x + \tan x) = x - \cos x + C$	3.6	$y = (x+C)/(x + \sqrt{x^2+1})$
3.7	$y = \frac{1}{3}(1+e^x) + C(1+e^x)^{-2}$	3.8	$y = \frac{1}{2}\ln x + C/\ln x$
3.9	$y(1-x^2)^{1/2} = x^2 + C$	3.10	$y \cosh x = \frac{1}{2}e^{2x} + x + C$
3.11	$y = 2(\sin x - 1) + Ce^{-\sin x}$	3.12	$x = (y+C)\cos y$

3.13 $x = \frac{1}{2}e^y + Ce^{-y}$

3.14 $x = y^{2/3} + Cy^{-1/3}$

3.15 $S = \frac{1}{2} \times 10^7 \left[(1 + 3t/10^4) + (1 + 3t/10^4)^{-1/3}\right]$, where S = number of pounds of salt, and t is in hours.

3.16 $I = Ae^{-Rt/L} + V_0(R^2 + \omega^2 L^2)^{-1}(R\cos\omega t + \omega L \sin\omega t)$

3.17 $I = Ae^{-t/(RC)} - V_0\omega C(\sin\omega t - \omega RC\cos\omega t)/(1 + \omega^2 R^2 C^2)$

3.18 RL circuit: $I = Ae^{-Rt/L} + V_0(R + i\omega L)^{-1}e^{i\omega t}$

RC circuit: $I = Ae^{-t/RC} + i\omega V_0 C(1 + i\omega RC)^{-1}e^{i\omega t}$

3.19 $N_2 = N_0\lambda te^{-\lambda t}$

3.20 $N_3 = c_1e^{-\lambda_1 t} + c_2e^{-\lambda_2 t} + c_3e^{-\lambda_3 t}$, where

$c_1 = \dfrac{\lambda_1\lambda_2 N_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)}$, $c_2 = \dfrac{\lambda_1\lambda_2 N_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)}$, $c_3 = \dfrac{\lambda_1\lambda_2 N_0}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)}$

3.21 $N_n = c_1e^{-\lambda_1 t} + c_2e^{-\lambda_2 t} + \cdots$, where

$c_1 = \dfrac{\lambda_1\lambda_2 ... \lambda_{n-1} N_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)...(\lambda_n-\lambda_1)}$, $c_2 = \dfrac{\lambda_1\lambda_2 ... \lambda_{n-1} N_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)...(\lambda_n-\lambda_2)}$,

etc. (all $\lambda' s$ different).

3.22 $y = x + 1 + Ke^x$

3.23 $x = 2\pi^{-1/2}e^{-y^2}\displaystyle\int_k^y e^{u^2}\,du$

4.1 $y^{1/3} = x - 3 + Ce^{-x/3}$

4.2 $y^{1/2} = \frac{1}{3}x^{5/2} + Cx^{-1/2}$

4.3 $y^3 = 1/3 + Cx^{-3}$

4.4 $x^2e^{3y} + e^x - \frac{1}{3}y^3 = C$

4.5 $x^2 - y^2 + 2x(y+1) = C$

4.6 $4\,\sin x\cos y + 2x - \sin 2x - 2y - \sin 2y = C$

4.7 $x = y(\ln x + C)$

4.8 $y^2 = 2Cx + C^2$

4.9 $y^2 = Ce^{-x^2/y^2}$

4.10 $xy = Ce^{x/y}$

4.11 $\tan\frac{1}{2}(x+y) = x + C$

4.12 $x\sin(y/x) = C$

4.13 $y^2 = -\sin^2 x + C\sin^4 x$

4.14 $y = -x^{-2} + K(x-1)^{-1}$

4.15 $y = -x^{-1}\ln(C - x)$

4.16 $y^2 = C(C \pm 2x)$

4.17 $3x^2y - y^3 = C$

4.18 $x^2 + (y-k)^2 = k^2$

4.19 $r = Ae^{-\theta}$, $r = Be^{\theta}$

4.25 (a) $y = \dfrac{C + x^2}{x^2(C - x^2)}$ (b) $y = \dfrac{x(C + e^{4x})}{C - e^{4x}}$ (c) $y = \dfrac{e^x(C - e^{2x})}{C + e^{2x}}$

5.1 $y = Ae^x + Be^{-2x}$

5.2 $y = (Ax + B)e^{2x}$

5.3 $y = Ae^{3ix} + Be^{-3ix}$ or other forms as in (5.24)

5.4 $y = e^{-x}(Ae^{ix} + Be^{-ix})$ or equivalent forms (5.17), (5.18)

5.5 $y = (Ax + B)e^x$

5.6 $y = Ae^{4ix} + Be^{-4ix}$ or other forms as in (5.24)

5.7 $y = Ae^{3x} + Be^{2x}$

5.8 $y = A + Be^{-5x}$

5.9 $y = Ae^{2x}\sin(3x + \gamma)$

5.10 $y = A + Be^{2x}$

5.11 $y = (A + Bx)e^{-3x/2}$

5.12 $y = Ae^{-x} + Be^{x/2}$

5.19 $y = Ae^{-ix} + Be^{-(1+i)x}$

5.20 $y = Ae^{-x} + Be^{ix}$

5.22 $y = Ae^x + Be^{-3x} + Ce^{-5x}$

5.23 $y = Ae^{ix} + Be^{-ix} + Ce^x + De^{-x}$

5.24 $y = Ae^{-x} + Be^{x/2}\sin\left(\frac{1}{2}x\sqrt{3} + \gamma\right)$

5.25 $y = A + Be^{2x} + Ce^{-3x}$

5.26 $y = Ae^{5x} + (Bx + C)e^{-x}$

5.27 $y = Ax + B + (Cx + D)e^x + (Ex^2 + Fx + G)e^{-2x}$

5.28 $y = e^x(A\sin x + B\cos x) + e^{-x}(C\sin x + D\cos x)$

5.29 $y = (A + Bx)e^{-x} + Ce^{2x} + De^{-2x} + E\sin(2x + \gamma)$

5.30 $y = (Ax + B)\sin x + (Cx + D)\cos x + (Ex + F)e^x + (Gx + H)e^{-x}$
5.34 $\theta = \theta_0 \cos\omega t$, $\omega = \sqrt{g/l}$
5.35 $T = 2\pi\sqrt{R/g} \cong 85$ min.
5.36 $\omega = 1/\sqrt{LC}$
5.38 overdamped: $R^2C > 4L$; critically damped: $R^2C = 4L$; underdamped: $R^2C < 4L$.
5.40 $\ddot{y} + \frac{16\pi}{15}\dot{y} + \frac{4\pi^2}{9}y = 0$, $y = e^{-8\pi t/15}\left(A\sin\frac{2\pi t}{5} + B\cos\frac{2\pi t}{5}\right)$

6.1 $y = Ae^{2x} + Be^{-2x} - \frac{5}{2}$
6.2 $y = (A + Bx)e^{2x} + 4$
6.3 $y = Ae^x + Be^{-2x} + \frac{1}{4}e^{2x}$
6.4 $y = Ae^{-x} + Be^{3x} + 2e^{-3x}$
6.5 $y = Ae^{ix} + Be^{-ix} + e^x$
6.6 $y = (A + Bx)e^{-3x} + 3e^{-x}$
6.7 $y = Ae^{-x} + Be^{2x} + xe^{2x}$
6.8 $y = Ae^{4x} + Be^{-4x} + 5xe^{4x}$
6.9 $y = (Ax + B + x^2)e^{-x}$
6.10 $y = (A + Bx)e^{3x} + 3x^2e^{3x}$
6.11 $y = e^{-x}(A\sin 3x + B\cos 3x) + 8\sin 4x - 6\cos 4x$
6.12 $y = e^{-2x}[A\sin(2\sqrt{2}\ x) + B\cos(2\sqrt{2}\ x)] + 5(\sin 2x - \cos 2x)$
6.13 $y = (Ax + B)e^x - \sin x$
6.14 $y = e^{-2x}(A\sin 3x + B\cos 3x) - 3\cos 5x$
6.15 $y = e^{-6x/5}[A\sin(8x/5) + B\cos(8x/5)] - 5\cos 2x$
6.16 $y = A\sin 3x + B\cos 3x - 5x\cos 3x$
6.17 $y = A\sin 4x + B\cos 4x + 2x\sin 4x$
6.18 $y = e^{-x}(A\sin 4x + B\cos 4x) + 2e^{-4x}\cos 5x$
6.19 $y = e^{-x/2}(A\sin x + B\cos x) + e^{-3x/2}(2\cos 2x - \sin 2x)$
6.20 $y = Ae^{-2x}\sin(2x + \gamma) + 4e^{-x/2}\sin(5x/2)$
6.21 $y = e^{-3x/5}[A\sin(x/5) + B\cos(x/5)] + (x^2 - 5)/2$
6.22 $y = A + Be^{-x/2} + x^2 - 4x$
6.23 $y = A\sin x + B\cos x + (x - 1)e^x$
6.24 $y = (A + Bx + 2x^3)e^{3x}$
6.25 $y = Ae^{3x} + Be^{-x} - (\frac{4}{3}x^3 + x^2 + \frac{1}{2}x)e^{-x}$
6.26 $y = A\sin x + B\cos x - 2x^2\cos x + 2x\sin x$
6.33 $y = A\sin(x + \gamma) + x^3 - 6x - 1 + x\sin x + (3 - 2x)e^x$
6.34 $y = Ae^{3x} + Be^{2x} + e^x + x$
6.35 $y = A\sinh x + B\cosh x + \frac{1}{2}x\cosh x$
6.36 $y = A\sin x + B\cos x + x^2\sin x$
6.37 $y = (A + Bx)e^x + 2x^2e^x + (3 - x)e^{2x} + x + 1$
6.38 $y = A + Be^{2x} + (3x + 4)e^{-x} + x^3 + 3(x^2 + x)/2 + 2xe^{2x}$
6.41 $y = e^{-x}(A\cos x + B\sin x) + \frac{1}{4}\pi + \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{4(n^2 - 2)\cos nx - 8n\sin nx}{\pi n^2(n^4 + 4)}$
6.42 $y = A\cos 3x + B\sin 3x + \frac{1}{36} + \frac{2}{\pi^2}\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x}{n^2(n^2\pi^2 - 9)} + \frac{1}{\pi}\sum_{\substack{1 \\ \text{all } n}}^{\infty} \frac{(-1)^n \sin n\pi x}{n(n^2\pi^2 - 9)}$

7.1 $y = 2A\tanh(Ax + B)$, or $y = 2A\tan(B - Ax)$, or $y(x + a) = 2$, or $y = C$.
(a) $y \equiv 5$
(b) $y(x + 1) = 2$
(c) $y = \tan(\frac{\pi}{4} - \frac{x}{2}) = \sec x - \tan x$
(d) $y = 2\tanh x$
7.3 $y = a(x + b)^2$, or $y = C$
7.4 $x^2 + (y - b)^2 = a^2$, or $y = C$
7.5 $y = b + k^{-1}\cosh k(x - a)$
7.8 $v = (v_0^2 - 2gR + 2gR^2/r)^{1/2}$, $r_{\max} = 2gR^2/(2gR - v_0^2)$, escape velocity $= \sqrt{2gR}$

7.10 $x=\sqrt{1+t^2}$
7.11 $x=(1-3t)^{1/3}$
7.12 $t=\int_1^x u^2(1-u^4)^{-1/2}du$
7.13 $t=(\omega\sqrt{2}\,)^{-1}\int(\cos\theta)^{-1/2}d\theta$
7.16 (a) $y=Ax+Bx^{-3}$ (b) $y=Ax^2+Bx^{-2}$
(c) $y=(A+B\ln x)/x^3$ (d) $y=Ax\cos(\sqrt{5}\ln x)+Bx\sin(\sqrt{5}\ln x)$
7.17 $y=Ax^4+Bx^{-4}+x^4\ln x$
7.18 $y=Ax+Bx^{-1}+\frac{1}{2}\left(x+x^{-1}\right)\ln x$
7.19 $y=x^3(A+B\ln x)+x^3(\ln x)^2$
7.20 $y=x^2(A+B\ln x)+x^2(\ln x)^3$
7.21 $y=A\sqrt{x}\sin\left(\frac{\sqrt{3}}{2}\ln x+\gamma\right)+x^2$
7.22 $y=A\cos\ln x+B\sin\ln x+x$
7.23 $R=Ar^n+Br^{-n},\ n\neq 0;\ R=A\ln r+B,\ n=0$
$R=Ar^l+Br^{-l-1}$
7.25 $x^{-1}-1$ 7.26 x^2-1 7.27 x^3e^x
7.28 $x^{1/3}e^x$ 7.29 $xe^{1/x}$ 7.30 $(x-1)\ln x-4x$

8.8 $e^{-2t}-te^{-2t}$
8.9 e^t-3e^{-2t}
8.10 $\frac{1}{3}e^t\sin 3t+2e^t\cos 3t$
8.11 $\frac{4}{7}e^{-2t}+\frac{3}{7}e^{t/3}$
8.12 $3\cosh 5t+2\sinh 5t$
8.13 $e^{-2t}(2\sin 4t-\cos 4t)$
8.17 $2a(3p^2-a^2)/(p^2+a^2)^3$
8.21 $2b(p+a)/[(p+a)^2+b^2]^2$
8.22 $[(p+a)^2-b^2]/[(p+a)^2+b^2]^2$
8.23 $y=te^{-2t}(\cos t-\sin t)$
8.25 $e^{-p\pi/2}/(p^2+1)$
8.26 $\cos(t-\pi),\ t>\pi;\ 0,\ t<\pi$
8.27 $-v(p^2+v^2)^{-1}e^{-px/v}$

9.2 $y=e^t(3+2t)$
9.3 $y=e^{-2t}(4t+\frac{1}{2}t^2)$
9.4 $y=\cos t+\frac{1}{2}(\sin t-t\cos t)$
9.5 $y=-\frac{1}{2}t\cos t$
9.6 $y=\frac{1}{6}t^3e^{3t}+5te^{3t}$
9.7 $y=1-e^{2t}$
9.8 $y=t\sin 4t$
9.9 $y=(t+2)\sin 4t$
9.10 $y=3t^2e^{2t}$
9.11 $y=te^{2t}$
9.12 $y=\frac{1}{2}(t^2e^{-t}+3e^t-e^{-t})$
9.13 $y=\sinh 2t$
9.14 $y=te^{2t}$
9.15 $y=2\sin 3t+\frac{1}{6}t\sin 3t$
9.16 $y=\frac{1}{6}t\sin 3t+2\cos 3t$
9.17 $y=2$
9.18 $y=2e^{-2t}-e^{-t}$
9.19 $y=e^{2t}$
9.20 $y=2t+1$
9.21 $y=e^{3t}+2e^{-2t}\sin t$
9.22 $y=2\cos t+\sin t$
9.23 $y=\sin t+2\cos t-2e^{-t}\cos 2t$
9.24 $y=(5-6t)e^t-\sin t$
9.25 $y=(3+t)e^{-2t}\sin t$
9.26 $y=te^{-t}\cos 3t$
9.27 $y=t+(1-e^{4t})/4,\ z=\frac{1}{3}+e^{4t}$
9.28 $\begin{cases} y=t\cos t-1\\ z=\cos t+t\sin t\end{cases}$
9.29 $\begin{cases} y=e^t\\ z=t+e^t\end{cases}$
9.30 $y=t-\sin 2t$
$z=\cos 2t$
9.31 $y=t$
$z=e^t$
9.32 $\begin{cases} y=\sin 2t\\ z=\cos 2t-1\end{cases}$
9.33 $\begin{cases} y=\sin t-\cos t\\ z=\sin t\end{cases}$
9.34 $3/13$
9.35 $10/26^2$
9.36 $\arctan(2/3)$
9.37 $15/8$
9.38 $4/5$
9.39 $\ln 2$
9.40 1
9.41 $\arctan(1/\sqrt{2})$
9.42 $\pi/4$

10.3 $\frac{1}{2}t\sinh t$

10.4 $\dfrac{e^{-at}+e^{-bt}[(a-b)t-1]}{(b-a)^2}$

10.5 $\dfrac{b(b-a)te^{-bt}+a[e^{-bt}-e^{-at}]}{(b-a)^2}$

10.6 $\dfrac{e^{-at}-\cosh bt+(a/b)\sinh bt}{a^2-b^2}$

10.7 $\dfrac{a\cosh bt-b\sinh bt-ae^{-at}}{a^2-b^2}$

10.8 $\dfrac{e^{-at}}{(b-a)(c-a)}+\dfrac{e^{-bt}}{(a-b)(c-b)}+\dfrac{e^{-ct}}{(a-c)(b-c)}$

10.9 $(2t^2-2t+1-e^{-2t})/4$

10.10 $(1-\cos at-\frac{1}{2}at\sin at)/a^4$

10.11 $\dfrac{\cos at-\cos bt}{b^2-a^2}$

10.12 $\dfrac{1}{b^2-a^2}\left(\dfrac{\cos bt}{b^2}-\dfrac{\cos at}{a^2}\right)+\dfrac{1}{a^2b^2}$

10.13 $(e^{-t}+\sin t-\cos t)/2$

10.14 $e^{-3t}+(t-1)e^{-2t}$

10.15 $\dfrac{1}{14}e^{3t}+\dfrac{1}{35}e^{-4t}-\dfrac{1}{10}e^{t}$

10.17 $y=\begin{cases}(\cosh at-1)/a^2, & t>0\\ 0, & t<0\end{cases}$

11.1 $y=\begin{cases}t-2, & t>2\\ 0, & t<2\end{cases}$

11.7 $y=\begin{cases}(t-t_0)e^{-(t-t_0)}, & t>t_0\\ 0, & t<t_0\end{cases}$

11.8 $y=\begin{cases}e^{-2(t-t_0)}\sin(t-t_0), & t>t_0\\ 0, & t<t_0\end{cases}$

11.9 $y=\begin{cases}\frac{1}{3}e^{-(t-t_0)}\sin 3(t-t_0), & t>t_0\\ 0, & t<t_0\end{cases}$

11.10 $y=\begin{cases}\frac{1}{3}\sinh 3(t-t_0), & t>t_0\\ 0, & t<t_0\end{cases}$

11.11 $y=\begin{cases}\frac{1}{2}[\sinh(t-t_0)-\sin(t-t_0)], & t>t_0\\ 0, & t<t_0\end{cases}$

11.13 (a) $5\delta(x-2)+3\delta(x+7)$ (b) $3\delta(x+5)-4\delta(x-10)$

11.15 (a) 1 (b) 0 (c) -3 (d) $\cosh 1$

11.21 (a) 8 (b) $\phi(|a|)/(2|a|)$ (c) $1/2$ (d) 1

11.23 (a) $\delta(x+5)\delta(y-5)\delta(z)$, $\delta(r-5\sqrt{2})\delta(\theta-\frac{3\pi}{4})\delta(z)/r$,
$\delta(r-5\sqrt{2})\delta(\theta-\frac{\pi}{2})\delta(\phi-\frac{3\pi}{4})/(r\sin\theta)$

(b) $\delta(x)\delta(y+1)\delta(z+1)$, $\delta(r-1)\delta(\theta-\frac{3\pi}{2})\delta(z+1)/r$,
$\delta(r-\sqrt{2})\delta(\theta-\frac{3\pi}{4})\delta(\phi-\frac{3\pi}{2})/(r\sin\theta)$

(c) $\delta(x+2)\delta(y)\delta(z-2\sqrt{3})$, $\delta(r-2)\delta(\theta-\pi)\delta(z-2\sqrt{3})/r$,
$\delta(r-4)\delta(\theta-\frac{\pi}{6})\delta(\phi-\pi)/(r\sin\theta)$

(d) $\delta(x-3)\delta(y+3)\delta(z+\sqrt{6})$, $\delta(r-3\sqrt{2})\delta(\theta-\frac{7\pi}{4})\delta(z+\sqrt{6})/r$,
$\delta(r-2\sqrt{6})\delta(\theta-\frac{2\pi}{3})\delta(\phi-\frac{7\pi}{4})/(r\sin\theta)$

11.25 (a) and (b) $F''(x)=\delta(x)-2\delta'(x)$ (c) $G''(x)=\delta(x)+5\delta'(x)$

12.2 $y=\dfrac{\sin\omega t-\omega t\cos\omega t}{2\omega^2}$

12.3 $y=\dfrac{\sin\omega t-\omega\cos\omega t+\omega e^{-t}}{\omega(1+\omega^2)}$

12.6 $y=(\cosh at-1)/a^2,\ t>0$

12.7 $y=\dfrac{a(\cosh at-e^{-t})-\sinh at}{a(a^2-1)}$

12.8 $y=\begin{cases}1-e^{-t}-te^{-t}, & 0<t<a\\ (t+1-a)e^{a-t}-(t+1)e^{-t}, & t>a\end{cases}$

12.11 $y=-\frac{1}{3}\sin 2x$

12.12 $y=\cos x\ln\cos x+(x-\frac{\pi}{2})\sin x$

12.13 $y = \begin{cases} x - \sqrt{2}\,\sin x, & x < \pi/4 \\ \frac{\pi}{2} - x - \sqrt{2}\,\cos x, & x > \pi/4 \end{cases}$

12.15 $y = x\sinh x - \cosh x \ln\cosh x$

12.16 $y = -x\ln x - x - x(\ln x)^2/2$

12.17 $y = -\frac{1}{4}\sin^2 x$

12.18 $y = x^2/2 + x^4/6$

13.1	$y = -\frac{1}{3}x^{-2} + Cx$	linear 1st order
13.2	$(\ln y)^2 - (\ln x)^2 = C$	separable
13.3	$y = A + Be^{-x}\sin(x+\gamma)$	3rd order linear
13.4	$r = (A + Bt)e^{3t}$	2nd order linear, $a = b$
13.5	$x^2 + y^2 - y\sin^2 x = C$	exact
13.6	$y = Ae^{-x}\sin(x+\gamma)$ $+2e^x + 3xe^{-x}\sin x$	2nd order linear, complex a, b, c
13.7	$3x^2y^3 + 1 = Ax^3$	Bernoulli, or integrating factor $1/x^4$
13.8	$y = x(A + B\ln x) + \frac{1}{2}x(\ln x)^2$	Cauchy
13.9	$y(e^{3x} + Ce^{-2x}) + 5 = 0$	Bernoulli
13.10	$u - \ln u + \ln v + v^{-1} = C$	separable
13.11	$y = 2x\ln x + Cx$	linear 1st order, or homogeneous
13.12	$y = A\ln x + B + x^2$	y missing, or Cauchy
13.13	$y = Ae^{-2x}\sin(x+\gamma) + e^{3x}$	2nd order linear, complex a, b
13.14	$y = Ae^{-2x}\sin(x+\gamma)$ $+xe^{-2x}\sin x$	2nd order linear, complex a, b, c
13.15	$y = (A + Bx)e^{2x} + 3x^2e^{2x}$	2nd order linear, $c = a = b$
13.16	$y = Ae^{2x} + Be^{3x} - xe^{2x}$	2nd order linear, $c = a \neq b$
13.17	$y^2 + 4xy - x^2 = C$	exact, or homogeneous
13.18	$x = (y + C)e^{-\sin y}$	linear 1st order for $x(y)$
13.19	$(x + y)\sin^2 x = K$	separable with $u = x + y$
13.20	$y = Ae^x\sin(2x+\gamma)$ $+x+\frac{2}{5} + e^x(1 - x\cos 2x)$	2nd order linear, complex a, b, c
13.21	$x^2 + \ln(1 - y^2) = C$	separable, or Bernoulli
13.22	$y = (A + Bx)e^{2x} + C\sin(3x+\gamma)$	4th order linear
13.23	$r = \sin\theta[C + \ln(\sec\theta + \tan\theta)]$	1st order linear
13.24	$y^2 = ax^2 + b$	separable after substitution

13.25 $x^3y = 2$

13.26 $y = x^2 + x$

13.27 $y = 2e^{2x} - 1$

13.28 $y^2 + 4(x-1)^2 = 9$

13.29 62 min more

13.30 $y = \frac{1}{6}g[(1+t)^2 + 2(1+t)^{-1} - 3]$; at $t = 1$, $y = g/3$, $v = 7g/12$, $a = 5g/12$

13.31 $v = \sqrt{2k/(ma)}$

13.32 1:23 p.m.

13.33 In both (a) and (b), the temperature of the mixture at time t is $T_a(1 - e^{-kt}) + (n + n')^{-1}(nT_0 + n'T_0')e^{-kt}$.

13.36 (a) $v = u\ln(m_0/m)$

13.38 $\ln\sqrt{a^2 + p^2} - \ln p$

13.39 $2pa/(p^2 - a^2)^2$

13.40 5/27

13.41 $(\tanh 1 - \operatorname{sech}^2 1)/4 = 0.0854$

13.42 $te^{-at}(1 - \frac{1}{2}at)$

13.43 $(\sin at + at\cos at)/(2a)$

13.44 $(3\sin at - 3at\cos at - a^2t^2\sin at)/(8a^5)$

13.46 e^{-x}: $g_s(\alpha) = \sqrt{\frac{2}{\pi}}\frac{\alpha}{1+\alpha^2}$, $g_c(\alpha) = \sqrt{\frac{2}{\pi}}\frac{1}{1+\alpha^2}$

xe^{-x} : $g_s(\alpha) = \sqrt{\frac{2}{\pi}}\frac{2\alpha}{(1+\alpha^2)^2}$, $g_c(\alpha) = \sqrt{\frac{2}{\pi}}\frac{1-\alpha^2}{(1+\alpha^2)^2}$

13.47 $y = A\sin t + B\cos t + \sin t\ \ln(\sec t + \tan t) - 1$

13.48 $y = A\sin t + B\cos t + (t\sin t - t^2\cos t)/4$

Chapter 9

2.1 $(y-b)^2 = 4a^2(x-a^2)$ 2.2 $x^2+(y-b)^2 = a^2$
2.3 $ax = \sinh(ay+b)$ 2.4 $ax = \cosh(ay+b)$
2.5 $y = ae^x + be^{-x}$ or $y = A\cosh(x+B)$, etc.
2.6 $x+a = \frac{4}{3}(y^{1/2}-2b)(b+y^{1/2})^{1/2}$
2.7 $e^x\cos(y+b) = C$ 2.8 $K^2x^2-(y-b)^2 = K^4$
2.9 $x = ay^2+b$ 2.10 $y = Ax^{3/2} - \ln x + B$

3.1 $dx/dy = C(y^3-C^2)^{-1/2}$ 3.2 $dx/dy = Cy^2(1-C^2y^4)^{-1/2}$
3.3 $x^4y'^2 = C^2(1+x^2y'^2)^3$ 3.4 $\dfrac{dx}{dy} = \dfrac{C}{y(y^4-C^2)^{1/2}}$
3.5 $y^2 = ax+b$ 3.6 $x = ay^{3/2} - \frac{1}{2}y^2 + b$
3.7 $y = K\sinh(x+C) = ae^x + be^{-x}$, etc., as in Problem 2.5
3.8 $r\cos(\theta+\alpha) = C$ 3.9 $\cot\theta = A\cos(\phi-\alpha)$
3.10 $s = be^{at}$ 3.11 $a(x+1) = \cosh(ay+b)$
3.12 $(x-a)^2+y^2 = C^2$ 3.13 $(x-a)^2 = 4K^2(y-K^2)$
3.14 $r = be^{c\theta}$
3.15 $r\cos(\theta+\alpha) = C$, in polar coordinates; or, in rectangular coordinates, the straight line $x\cos\alpha - y\sin\alpha = C$.
3.17 Intersection of the cone with $r\cos\left(\dfrac{\theta+C}{\sqrt{2}}\right) = K$
3.18 Geodesics on the sphere: $\cot\theta = A\cos(\phi-\alpha)$. (See Problem 3.9)
Intersection of $z = ax+by$ with the sphere: $\cot\theta = a\cos\phi + b\sin\phi$.
4.4 42.2 min; 5.96 min
4.5 $x = a(1-\cos\theta)$, $y = a(\theta-\sin\theta)+C$
4.6 $x = a(\theta-\sin\theta)+C$, $y = 1+a(1-\cos\theta)$
4.7 $x = a(1-\cos\theta)-\frac{5}{2}$, $y = a(\theta-\sin\theta)+C$

5.2 $L = \frac{1}{2}m(\dot r^2 + r^2\dot\theta^2 + \dot z^2) - V(r,\theta,z)$
$m(\ddot r - r\dot\theta^2) = -\partial V/\partial r$
$m(r\ddot\theta + 2\dot r\dot\theta) = -(1/r)(\partial V/\partial\theta)$
$m\ddot z = -\partial V/\partial z$
Note: The equations in 5.2 and 5.3 are in the form
$m\mathbf{a} = \mathbf{F} = -\boldsymbol{\nabla} V$.
5.3 $L = \frac{1}{2}m(\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\,\dot\phi^2) - V(r,\theta,\phi)$
$m(\ddot r - r\dot\theta^2 - r\sin^2\theta\,\dot\phi^2) = -\partial V/\partial r$
$m(r\ddot\theta + 2\dot r\dot\theta - r\sin\theta\cos\theta\,\dot\phi^2) = -(1/r)(\partial V/\partial\theta)$
$m(r\sin\theta\,\ddot\phi + 2r\cos\theta\,\dot\theta\dot\phi + 2\sin\theta\,\dot r\dot\phi) = -(1/r\sin\theta)(\partial V/\partial\phi)$
5.4 $L = \frac{1}{2}ml^2\dot\theta^2 - mgl(1-\cos\theta)$
$l\ddot\theta + g\sin\theta = 0$

5.5 $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
$m\ddot{x} + kx = 0$

5.6 $L = \frac{1}{2}m(r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2) - mgr\cos\theta$
$\begin{cases} a\ddot{\theta} - a\sin\theta\cos\theta\,\dot{\phi}^2 - g\sin\theta = 0 \\ (d/dt)(\sin^2\theta\,\dot{\phi}) = 0 \end{cases}$

5.8 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$
$2\ddot{r} - r\dot{\theta}^2 + g = 0$
$(d/dt)(r^2\dot{\theta}) = 0$

5.9 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$
$2\ddot{r} - r\dot{\theta}^2 + g = 0$
$r^2\dot{\theta} = \text{const.}$

5.10 $L = \frac{1}{2}m_1(4\dot{r}^2 + r^2\dot{\theta}^2) + 2m_2\dot{r}^2 - m_1 gr\sqrt{3} + m_2 g(l - 2r)$
$4(m_1 + m_2)\ddot{r} - m_1 r\dot{\theta}^2 + m_1 g\sqrt{3} + 2m_2 g = 0$
$r^2\dot{\theta} = \text{const.}$

5.11 $L = \frac{1}{2}(m + Ia^{-2})\dot{z}^2 - mgz$ (If z is taken as positive down,
$(ma^2 + I)\ddot{z} + mga^2 = 0$ change the signs of z and $\ddot{z}$.)

5.12 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \left[\frac{1}{2}k(r - r_0)^2 - mgr\cos\theta\right]$
$\ddot{r} - r\dot{\theta}^2 + \frac{k}{m}(r - r_0) - g\cos\theta = 0$, $\frac{d}{dt}(r^2\dot{\theta}) + gr\sin\theta = 0$

5.13 $L = \frac{1}{2}m[\dot{r}^2(1 + 4r^2) + r^2\dot{\theta}^2] - mgr^2$
$\ddot{r}(1 + 4r^2) + 4r\dot{r}^2 - r\dot{\theta}^2 + 2gr = 0$, $r^2\dot{\theta} = \text{const.}$
If $z = \text{const.}$, then $r = \text{const.}$, so $\dot{\theta} = \sqrt{2g}$

5.14 $L = M\dot{x}^2 + Mgx\sin\alpha$, $\quad 2M\ddot{x} - Mg\sin\alpha = 0$

5.15 $L = \frac{1}{2}(M + Ia^{-2})\dot{x}^2 + Mgx\sin\alpha$, $(M + Ia^{-2})\ddot{x} - Mg\sin\alpha = 0$
Since smaller I means greater acceleration, objects reach the bottom in order of increasing I.

5.16 $L = \frac{1}{2}m(l + a\theta)^2\dot{\theta}^2 - mg[a\sin\theta - (l + a\theta)\cos\theta]$
$(l + a\theta)\ddot{\theta} + a\dot{\theta}^2 + g\sin\theta = 0$

5.17 $L = \frac{1}{2}(M + m)\dot{X}^2 + \frac{1}{2}m(l^2\dot{\theta}^2 + 2l\cos\theta\,\dot{X}\dot{\theta}) + mgl\cos\theta$
$(M + m)\dot{X} + ml\cos\theta\,\dot{\theta} = \text{const.}$
$\frac{d}{dt}(l\dot{\theta} + \cos\theta\,\dot{X}) + g\sin\theta = 0$

5.18 $x + y = x_0 + y_0 + a\theta$, $L = m(\dot{x}^2 + \dot{y}^2 + \dot{x}\dot{y}) + mgy$
$\dot{x} = -\frac{1}{3}gt$, $\dot{y} = \frac{2}{3}gt$, $a\dot{\theta} = \frac{1}{3}gt$

5.19 $x = y$ with $\omega = \sqrt{g/l}$; $\quad x = -y$ with $\omega = \sqrt{3g/l}$

5.20 $x = y$ with $\omega = \sqrt{g/l}$; $\quad x = -y$ with $\omega = \sqrt{7g/l}$

5.21 $L = ml^2[\dot{\theta}^2 + \frac{1}{2}\dot{\phi}^2 + \dot{\theta}\dot{\phi}\cos(\theta - \phi)] + mgl(2\cos\theta + \cos\phi)$

$$2\ddot{\theta} + \ddot{\phi}\cos(\theta - \phi) + \dot{\phi}^2\sin(\theta - \phi) + \frac{2g}{l}\sin\theta = 0$$

$$\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$$

5.22 $L = l^2[\frac{1}{2}M\dot{\theta}^2 + \frac{1}{2}m\dot{\phi}^2 + m\dot{\theta}\dot{\phi}\cos(\theta - \phi)] + gl(M\cos\theta + m\cos\phi)$

$$M\ddot{\theta} + m\phi\cos(\theta - \phi) + m\dot{\phi}^2\sin(\theta - \phi) + \frac{Mg}{l}\sin\theta = 0$$

$$\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$$

5.23 $\phi = 2\theta$ with $\omega = \sqrt{2g/(3l)}$; $\quad \phi = -2\theta$ with $\omega = \sqrt{2g/l}$

5.24 $\phi = \frac{3}{2}\theta$ with $\omega = \sqrt{3g/(5l)}$; $\quad \phi = -\frac{3}{2}\theta$ with $\omega = \sqrt{3g/l}$

5.25 $\phi = \sqrt{M/m}\,\theta \quad$ with $\quad \omega^2 = \dfrac{g}{l}\dfrac{1}{1 + \sqrt{m/M}}$

$\phi = -\sqrt{M/m}\,\theta \quad$ with $\quad \omega^2 = \dfrac{g}{l}\dfrac{1}{1 - \sqrt{m/M}}$

6.1	catenary	6.2	circle	6.3	circular cylinder
6.4	catenary	6.5	circle	6.6	circle

8.2 $I = \int \frac{x^2 y'^2}{\sqrt{1+y'^2}}\,dx, \quad x^2(2y' + y'^3) = K(1+y'^2)^{3/2}$

8.3 $I = \int \frac{y\,dy}{\sqrt{x'^2+1}}, \quad x'^2 y^2 = C^2(1+x'^2)^3$

8.4 $I = \int \sqrt{r^2 + r^4\theta'^2}\,dr, \quad \frac{dr}{d\theta} = Kr\sqrt{r^4 - K^2}$

8.5 $y = ae^{bx}$

8.6 $(x-a)^2 + (y+1)^2 = C^2$

8.7 $(y-b)^2 = 4a^2(x+1-a^2)$

8.8 Intersection of $r = 1 + \cos\theta$ with $z = a + b\sin(\theta/2)$

8.9 Intersection of the cone with $r\cos(\theta\sin\alpha + C) = K$

8.10 Intersection of $y = x^2$ with $az = b[2x\sqrt{4x^2+1} + \sinh^{-1} 2x] + c$

8.11 $r = K\sec^2\frac{\theta+c}{2}$

8.12 $e^y\cos(x-a) = K$

8.13 $(x+\frac{3}{2})^2 + (y-b)^2 = c^2$

8.14 $(x-a)^2 = 4K^2(y + 2 - K^2)$

8.15 $y + c = \frac{3}{2}K\left[x^{1/3}\sqrt{x^{2/3} - K^2} + K^2\cosh^{-1}(x^{1/3}/K)\right]$

8.16 Hyperbola: $r^2\cos(2\theta + \alpha) = K$ or $(x^2 - y^2)\cos\alpha - 2xy\sin\alpha = K$

8.17 $K\ln r = \cosh(K\theta + C)$

8.18 Parabola: $(x - y - C)^2 = 4K^2(x + y - K^2)$

8.19 $m(\ddot{r} - r\dot{\theta}^2) + kr = 0$, $r^2\dot{\theta} = \text{const.}$

8.20 $m(\ddot{r} - r\dot{\theta}^2) + K/r^2 = 0$, $r^2\dot{\theta} = \text{const.}$

8.21 $\ddot{r} - r\dot{\theta}^2 = 0$, $r^2\dot{\theta} = \text{const.}$, $\ddot{z} + g = 0$

8.22 $\frac{1}{r}\cdot m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - r^2\sin\theta\cos\theta\;\dot{\phi}^2) = -\frac{1}{r}\frac{\partial V}{\partial\theta} = F_\theta = ma_\theta$
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\;\dot{\phi}^2$

8.23 $L = \frac{1}{2}ma^2\dot{\theta}^2 - mga(1-\cos\theta)$, $a\ddot{\theta} + g\sin\theta = 0$,
θ measured from the downward direction.

8.25 $l = 2\sqrt{\pi A}$

8.26 $r = Ae^{b\theta}$

8.27 $\frac{dr}{d\theta} = r\sqrt{K^2(1+\lambda r)^2 - 1}$

8.28 $r^2\dot{\theta} = \text{const.}$, $|\mathbf{r}\times m\mathbf{v}| = mr^2\dot{\theta} = \text{const.}$, $\frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = \text{const.}$

Chapter 10

4.4 $I = \frac{2}{15}\begin{pmatrix} \pi & -1 & 0 \\ -1 & \pi & 0 \\ 0 & 0 & \pi \end{pmatrix}$ Principal moments: $\frac{2}{15}(\pi-1, \pi, \pi+1)$; principal axes along the vectors: $(1,1,0), (0,0,1), (1,-1,0)$.

4.5 $I = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -2 & 4 \end{pmatrix}$ Principal moments: $(2,4,6)$; principal axes along the vectors: $(0,1,1), (1,0,0), (0,1,-1)$.

4.6 $I = \begin{pmatrix} 9 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 9 \end{pmatrix}$ Principal moments: $(6,6,12)$; principal axes along the vectors: $(1,0,-1)$ and any two orthogonal vectors in the plane $z = x$, say $(0,1,0)$ and $(1,0,1)$.

4.7 $I = \frac{1}{120}\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ Principal moments: $\left(\frac{1}{60}, \frac{1}{24}, \frac{1}{24}\right)$; principal axes along the vectors: $(1,1,1)$ and any two orthogonal vectors in the plane $x + y + z = 0$, say $(1,-1,0)$ and $(1,1,-2)$.

5.5 1 if $j\ k = m\ n$ (6 cases); -1 if $j\ k = n\ m$ (6 cases); 0 otherwise
5.6 (a) 3 (b) 0 (c) 2 (d) -2 (e) -1 (f) -1
5.7 (a) $\delta_{kq}\delta_{ip} - \delta_{kp}\delta_{iq}$ (b) $\delta_{ap}\delta_{bq} - \delta_{aq}\delta_{bp}$

6.9 to 6.14 $\mathbf{r}$, $\mathbf{v}$, $\mathbf{F}$, $\mathbf{E}$ are vectors; $\boldsymbol{\omega}$, $\boldsymbol{\tau}$, $\mathbf{L}$, $\mathbf{B}$ are pseudovectors; T is a scalar.
6.15 (a) vector (b) pseudovector (c) vector
6.16 vector (if $\mathbf{V}$ is a vector); pseudovector (if $\mathbf{V}$ is a pseudovector)

8.1 $h_r = 1, \quad h_\theta = r, \quad h_\phi = r\sin\theta$
$d\mathbf{s} = \mathbf{e}_r dr + \mathbf{e}_\theta r d\theta + \mathbf{e}_\phi r\sin\theta d\phi$
$dV = r^2 \sin\theta dr d\theta d\phi$
$\mathbf{a}_r = \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta = \mathbf{e}_r$
$\mathbf{a}_\theta = \mathbf{i}r\cos\theta\cos\phi + \mathbf{j}r\cos\theta\sin\phi - \mathbf{k}r\sin\theta = r\mathbf{e}_\theta$
$\mathbf{a}_\phi = -\mathbf{i}r\sin\theta\sin\phi + \mathbf{j}r\sin\theta\cos\phi = r\sin\theta\mathbf{e}_\phi$

8.2 $d^2\mathbf{s}/dt^2 = \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2) + \mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta}) + \mathbf{e}_z\ddot{z}$

8.3 $d\mathbf{s}/dt = \mathbf{e}_r\dot{r} + \mathbf{e}_\theta r\dot{\theta} + \mathbf{e}_\phi r\sin\theta\dot{\phi}$
$d^2\mathbf{s}/dt^2 = \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2)$
$+\mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\dot{\phi}^2)$
$+\mathbf{e}_\phi(r\sin\theta\,\ddot{\phi} + 2r\cos\theta\,\dot{\theta}\dot{\phi} + 2\sin\theta\,\dot{r}\dot{\phi})$

8.4 $\mathbf{V} = -r\mathbf{e}_\theta + \mathbf{k}$

8.5 $\mathbf{V} = \mathbf{e}_r \cos\theta - \mathbf{e}_\theta \sin\theta - \mathbf{e}_\phi r \sin\theta$

8.6 $h_u = h_v = (u^2+v^2)^{1/2}, \quad h_z = 1$

$d\mathbf{s} = (u^2+v^2)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + \mathbf{e}_z dz$

$dV = (u^2+v^2)\ du\ dv\ dz$

$\mathbf{a}_u = \mathbf{i}u + \mathbf{j}v = (u^2+v^2)^{1/2}\mathbf{e}_u$

$\mathbf{a}_v = -\mathbf{i}v + \mathbf{j}u = (u^2+v^2)^{1/2}\mathbf{e}_v$

$\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$

8.7 $h_u = h_v = a(\sinh^2 u + \sin^2 v)^{1/2},\ h_z = 1$

$d\mathbf{s} = a(\sinh^2 u + \sin^2 v)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + \mathbf{e}_z dz$

$dV = a^2(\sinh^2 u + \sin^2 v)\ du\ dv\ dz$

$\mathbf{a}_u = \mathbf{i}a \sinh u \cos v + \mathbf{j}a \cosh u \sin v = h_u \mathbf{e}_u$

$\mathbf{a}_v = -\mathbf{i}a \cosh u \sin v + \mathbf{j}a \sinh u \cos v = h_v \mathbf{e}_v$

$\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$

8.8 $h_u = h_v = (u^2+v^2)^{1/2},\ \ h_\phi = uv$

$d\mathbf{s} = (u^2+v^2)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + uv\mathbf{e}_\phi d\phi$

$dV = \ uv\ (u^2+v^2)\ du\ dv\ d\phi$

$\mathbf{a}_u = \mathbf{i}v\cos\phi + \mathbf{j}v\sin\phi + \mathbf{k}u = h_u\mathbf{e}_u$

$\mathbf{a}_v = \mathbf{i}u\cos\phi + \mathbf{j}u\sin\phi - \mathbf{k}v = h_v\mathbf{e}_v$

$\mathbf{a}_\phi = -\mathbf{i}uv\sin\phi + \mathbf{j}uv\cos\phi = h_\phi\mathbf{e}_\phi$

8.9 $h_u = h_v = \ a(\cosh u + \cos v)^{-1}$

$d\mathbf{s} = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u du + \mathbf{e}_v dv)$

$dA = a^2(\cosh u + \cos v)^{-2}\ du\ dv$

$\mathbf{a}_u = (h_u^2/a)[\mathbf{i}(1+\cos v\cosh u) - \mathbf{j}\sin v \sinh u] = h_u\mathbf{e}_u$

$\mathbf{a}_v = (h_v^2/a)[\mathbf{i}\sinh u \sin v + \mathbf{j}(1+\cos v \cosh u)] = h_v\mathbf{e}_v$

8.11 $d\mathbf{e}_u/dt = (u^2+v^2)^{-1}(uv - vu)\mathbf{e}_v$

$d\mathbf{e}_v/dt = (u^2+v^2)^{-1}(v\dot{u} - u\dot{v})\mathbf{e}_u$

$d\mathbf{s}/dt = (u^2+v^2)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_z\dot{z}$

$$d^2\mathbf{s}/dt^2 = \mathbf{e}_u(u^2+v^2)^{-1/2}[(u^2+v^2)\ddot{u} + u(\dot{u}^2-\dot{v}^2) + 2v\dot{u}\dot{v}]$$
$$+\mathbf{e}_v(u^2+v^2)^{-1/2}[(u^2+v^2)\ddot{v} + v(\dot{v}^2-\dot{u}^2) + 2u\dot{u}\dot{v}] + \mathbf{e}_z\ddot{z}$$

8.12 $d\mathbf{e}_u/dt = (\sinh^2 u + \sin^2 v)^{-1}(\dot{v}\sinh u\cosh u - \dot{u}\sin v\cos v)\mathbf{e}_v$

$d\mathbf{e}_v/dt = (\sinh^2 u + \sin^2 v)^{-1}(\dot{u}\sin v\cos v - \dot{v}\sinh u\cosh u)\mathbf{e}_u$

$d\mathbf{s}/dt = a(\sinh^2 u + \sin^2 v)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_z\dot{z}$

$$d^2\mathbf{s}/dt^2 = \mathbf{e}_u a(\sinh^2 u + \sin^2 v)^{-1/2}$$
$$\times[(\sinh^2 u + \sin^2 v)\ddot{u} + (\dot{u}^2-\dot{v}^2)\sinh u\cosh u + 2\dot{u}\dot{v}\sin v\cos v]$$
$$+\mathbf{e}_v a(\sinh^2 u + \sin^2 v)^{-1/2}[(\sinh^2 u + \sin^2 v)\ddot{v}$$
$$+(\dot{v}^2-\dot{u}^2)\sin v\cos v + 2\dot{u}\dot{v}\sinh u\cosh u] + \mathbf{e}_z\ddot{z}$$

8.13 $d\mathbf{e}_u/dt = (u^2+v^2)^{-1}(u\dot{v} - v\dot{u})\mathbf{e}_v + (u^2+v^2)^{-1/2}v\dot{\phi}\mathbf{e}_\phi$

$d\mathbf{e}_v/dt = (u^2+v^2)^{-1}(v\dot{u} - u\dot{v})\mathbf{e}_u + (u^2+v^2)^{-1/2}u\dot{\phi}\mathbf{e}_\phi$

$d\mathbf{e}_\phi/dt = -(u^2+v^2)^{-1/2}(v\mathbf{e}_u + u\mathbf{e}_v)\dot{\phi}$

$d\mathbf{s}/dt = (u^2+v^2)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_\phi uv\dot{\phi}$

$$d^2\mathbf{s}/dt^2 = \mathbf{e}_u(u^2+v^2)^{-1/2}[(u^2+v^2)\ddot{u} + u(\dot{u}^2-\dot{v}^2) + 2v\dot{u}\dot{v} - uv^2\dot{\phi}^2]$$
$$+\mathbf{e}_v(u^2+v^2)^{-1/2}[(u^2+v^2)\ddot{v} + v(\dot{v}^2-\dot{u}^2) + 2u\dot{u}\dot{v} - u^2v\dot{\phi}^2]$$
$$+\mathbf{e}_\phi(uv\ddot{\phi} + 2v\dot{u}\dot{\phi} + 2u\dot{v}\dot{\phi})$$

8.14 $d\mathbf{e}_u/dt = -(\cosh u + \cos v)^{-1}(\dot{u}\sin v + \dot{v}\sinh u)\mathbf{e}_v$

$d\mathbf{e}_v/dt = (\cosh u + \cos v)^{-1}(\dot{u}\sin v + \dot{v}\sinh u)\mathbf{e}_u$

$d\mathbf{s}/dt = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v})$

$$d^2\mathbf{s}/dt^2 = \mathbf{e}_u a(\cosh u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{u} + (\dot{v}^2-\dot{u}^2)\sinh u + 2\dot{u}\dot{v}\sin v]$$
$$+\mathbf{e}_v a(\cosh\ u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{v} + (\dot{v}^2-\dot{u}^2)\sin v - 2\dot{u}\dot{v}\sinh u]$$

9.3 See 8.2

9.5 $\boldsymbol{\nabla} U = \mathbf{e}_r \frac{\partial U}{\partial r} + \mathbf{e}_\theta \left(\frac{1}{r}\frac{\partial U}{\partial \theta}\right) + \mathbf{e}_\phi \left(\frac{1}{r\sin\theta}\frac{\partial U}{\partial \phi}\right)$

$$\boldsymbol{\nabla}\cdot\mathbf{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, V_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

$$\nabla^2 U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial\phi^2}$$

$$\boldsymbol{\nabla}\times\mathbf{V} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, V_\phi) - \frac{\partial V_\theta}{\partial\phi}\right]\mathbf{e}_r + \frac{1}{r\sin\theta}\left[\frac{\partial V_r}{\partial\phi} - \sin\theta\frac{\partial}{\partial r}(rV_\phi)\right]\mathbf{e}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta}\right]\mathbf{e}_\phi$$

9.6 See 8.11 9.7 See 8.12 9.8 See 8.13 9.9 See 8.14

9.10 Let $h = (u^2+v^2)^{1/2}$ represent the u and v scale factors.

$$\boldsymbol{\nabla} U = h^{-1}\left(\mathbf{e}_u\frac{\partial U}{\partial u} + \mathbf{e}_v\frac{\partial U}{\partial v}\right) + \mathbf{k}\frac{\partial U}{\partial z}$$

$$\boldsymbol{\nabla}\cdot\mathbf{V} = h^{-2}\left[\frac{\partial}{\partial u}(hV_u) + \frac{\partial}{\partial v}(hV_v)\right] + \frac{\partial V_z}{\partial z}$$

$$\nabla^2 U = h^{-2}\left(\frac{\partial^2 U}{\partial u^2} + \frac{\partial^2 U}{dv^2}\right) + \frac{\partial^2 U}{\partial z^2}$$

$$\boldsymbol{\nabla}\times\mathbf{V} = \left(h^{-1}\frac{\partial V_z}{\partial v} - \frac{\partial V_v}{\partial z}\right)\mathbf{e}_u + \left(\frac{\partial V_u}{\partial z} - h^{-1}\frac{\partial V_z}{\partial u}\right)\mathbf{e}_v + h^{-2}\left[\frac{\partial}{\partial u}(hV_v) - \frac{\partial}{\partial v}(hV_u)\right]\mathbf{e}_z$$

9.11 Same as 9.10 with $h = a(\sinh^2 u + \sin^2 v)^{1/2}$

9.12 Let $h = (u^2+v^2)^{1/2}$

$$\boldsymbol{\nabla} U = h^{-1}\left(\mathbf{e}_u\frac{\partial U}{\partial u} + \mathbf{e}_v\frac{\partial U}{\partial v}\right) + (uv)^{-1}\frac{\partial U}{\partial\phi}\mathbf{e}_\phi$$

$$\boldsymbol{\nabla}\cdot\mathbf{V} = \frac{1}{uh^2}\frac{\partial}{\partial u}(uhV_u) + \frac{1}{vh^2}\frac{\partial}{\partial v}(vhV_v) + \frac{1}{uv}\frac{\partial V_\phi}{\partial\phi}$$

$$\nabla^2 U = \frac{1}{h^2 u}\frac{\partial}{\partial u}\left(u\frac{\partial U}{\partial u}\right) + \frac{1}{h^2 v}\frac{\partial}{\partial v}\left(v\frac{\partial U}{\partial v}\right) + \frac{1}{u^2v^2}\frac{\partial^2 U}{\partial\phi^2}$$

$$\boldsymbol{\nabla}\times\mathbf{V} = \left[\frac{1}{hv}\frac{\partial}{\partial v}(vV_\phi) - \frac{1}{uv}\frac{\partial V_v}{\partial\phi}\right]\mathbf{e}_u + \left[\frac{1}{uv}\frac{\partial V_u}{\partial\phi} - \frac{1}{hu}\frac{\partial}{\partial u}(uV_\phi)\right]\mathbf{e}_v + \frac{1}{h^2}\left[\frac{\partial}{\partial u}(hV_v) - \frac{\partial}{\partial v}(hv_u)\right]\mathbf{e}_\phi$$

9.13 Same as 9.10 if $h = a(\cosh\ u + \cos v)^{-1}$ and terms involving either z derivatives or V_z are omitted. Note, however, that $\boldsymbol{\nabla}\times\mathbf{V}$ has only a z component if $\mathbf{V} = \mathbf{e}_u V_u + \mathbf{e}_v V_v$ where V_u and V_v are functions of u and v.

9.14 $h_u = [(u+v)/u]^{1/2},\ \ h_v = [(u+v)/v]^{1/2}$

$\mathbf{e}_u = h_u^{-1}\mathbf{i} + h_v^{-1}\mathbf{j},\ \ \mathbf{e}_v = -h_v^{-1}\mathbf{i} + h_u^{-1}\mathbf{j}$

$m\ [h_u\ddot{u} - h_u^{-1}(u\dot{v} - v\dot{u})^2/(2u^2v)] = -h_u^{-1}\partial V/\partial u = F_u$

$m\ [h_v\ddot{v} - h_v^{-1}(u\dot{v} - v\dot{u})^2/(2uv^2)] = -h_v^{-1}\partial V/\partial v = F_v$

9.15 $h_u = 1,\ h_v = u(1-v^2)^{-1/2}$

$\mathbf{e}_u = \mathbf{i}v + \mathbf{j}(1-v^2)^{1/2},\ \mathbf{e}_v = \mathbf{i}(1-v^2)^{1/2} - \mathbf{j}v$

$m[\ddot{u} - u\dot{v}^2/(1-v^2)] = -\partial V/\partial u = F_u$

$m[(u\ddot{v} + 2\dot{u}\dot{v})(1-v^2)^{-1/2} + uv\dot{v}^2(1-v^2)^{-3/2}] = -h_v^{-1}\partial V/\partial v = F_v$

9.16 $r^{-1},\ \ 0,\ \ 0,\ \ r^{-1}\mathbf{e}_z$

9.17 $2r^{-1},\ \ r^{-1}\cot\theta,\ \ r^{-1}\mathbf{e}_\phi,\ \ r^{-1}(\mathbf{e}_r\cot\theta - \mathbf{e}_\theta)$

9.18 $-r^{-1}\mathbf{e}_\theta,\;\; r^{-1}\mathbf{e}_r,\;\; 3$

9.19 $2\mathbf{e}_\phi,\;\; \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta,\;\; 3$

9.20 $r^{-1},\quad r^{-3},\;\; 0$

9.21 $2r^{-1},\quad 6,\quad 2r^{-4},\quad -k^2e^{ikr\cos\theta}$

11.4 Vector

11.5 $ds^2 = du^2 + h_v^2 dv^2,\ h_u = 1,\ h_v = u(2v - v^2)^{-1/2},$
$dA = u(2v-v^2)^{-1/2}du\,dv,\ d\mathbf{s} = \mathbf{e}_u\ du + h_v\mathbf{e}_v dv,$
$\mathbf{e}_u = \mathbf{i}(1-v) + \mathbf{j}(2v-v^2)^{1/2},\ \ \mathbf{e}_v = -\mathbf{i}(2v-v^2)^{1/2} + \mathbf{j}(1-v)$
$\mathbf{a}_u = \mathbf{e}_u = \mathbf{a}^u,\quad \mathbf{a}_v = h_v\mathbf{e}_v,\quad \mathbf{a}^v = \mathbf{e}_v/h_v$

11.6 $$m\left(\ddot{u} - \frac{u\dot{v}^2}{v(2-v)}\right) = -\frac{\partial V}{\partial u} = F_u$$
$$m\left(\frac{u\ddot{v} + 2\dot{u}\dot{v}}{[v(2-v)]^{1/2}} + \frac{u\dot{v}^2(v-1)}{[v(2-v)]^{3/2}}\right) = -u^{-1}[v(2-v)]^{1/2}\frac{\partial V}{\partial v} = F_v$$

11.7 $\boldsymbol{\nabla}U = \mathbf{e}_u\partial U/\partial u + \mathbf{e}_v u^{-1}\sqrt{v(2-v)}\,\partial U/\partial v$
$\boldsymbol{\nabla}\cdot\mathbf{V} = u^{-1}\partial(uV_u)/\partial u + u^{-1}\sqrt{v(2-v)}\,\partial V_v/\partial v$
$$\nabla^2 U = \frac{1}{u}\frac{\partial}{\partial u}\left(u\frac{\partial U}{\partial u}\right) + \frac{1}{u^2}\sqrt{v(2-v)}\frac{\partial}{\partial V}\left(\sqrt{v(2-v)}\frac{\partial U}{\partial V}\right)$$

11.8 $u^{-1},\quad u^{-1}\mathbf{k},\quad 0$

Chapter 11

3.2	$3/2$	3.3	$9/10$	3.4	$25/14$
3.5	$32/35$	3.6	72	3.7	8
3.8	$\Gamma(5/3)$	3.9	$\Gamma(5/4)$	3.10	$\Gamma(3/5)$
3.11	1	3.12	$\Gamma(2/3)/3$	3.13	$3^{-4}\Gamma(4) = 2/27$
3.14	$-\Gamma(4/3)$	3.15	$\Gamma(2/3)/4$	3.17	$\Gamma(p)$

7.1 $\frac{1}{2}B(5/2, 1/2) = 3\pi/16$

7.2 $\frac{1}{2}B(5/4, 3/4) = \pi\sqrt{2}/8$

7.3 $\frac{1}{3}B(1/3, 1/2)$

7.4 $\frac{1}{2}B(3/2, 5/2) = \pi/32$

7.5 $B(3, 3) = 1/30$

7.6 $\frac{1}{3}B(2/3, 4/3) = 2\pi\sqrt{3}/27$

7.7 $\frac{1}{2}B(1/4, 1/2)$

7.8 $4\sqrt{2}B(3, 1/2) = 64\sqrt{2}/15$

7.10 $\frac{4}{3}B(1/3, 4/3)$

7.11 $2B(2/3, 4/3)/B(1/3, 4/3)$

7.12 $(8\pi/3)B(5/3, 1/3) = 32\pi^2\sqrt{3}/27$

7.13 $I_y/M = 8B(4/3, 4/3)/B(5/3, 1/3)$

8.1 $B(1/2, 1/4)\sqrt{2l/g} = 7.4163\sqrt{l/g}$ $\left(\text{Compare } 2\pi\sqrt{l/g}\,.\right)$

8.2 $\frac{1}{4}\sqrt{35/11}\,B(1/2, 1/4) = 2.34$ sec

8.3 $t = \pi\sqrt{a/g}$

10.2 $\Gamma(p, x) \sim x^{p-1}e^{-x}[1 + (p-1)x^{-1} + (p-1)(p-2)x^{-2} + \cdots]$

10.3 $\operatorname{erfc}(x) = \Gamma\left(1/2, x^2\right)/\sqrt{\pi}$

10.5 (a) $E_1(x) = \Gamma(0, x)$

(b) $\Gamma(0, x) \sim x^{-1}e^{-x}[1 - x^{-1} + 2x^{-2} - 3!\,x^{-3} + \cdots]$

10.6 (a) $\operatorname{Ei}(\ln x)$ (b) $\operatorname{Ei}(x)$ (c) $-\operatorname{Ei}(\ln x)$

11.4 $1/\sqrt{\pi}$ 11.5 1 11.10 e^{-1}

12.1 $K = F(\pi/2, k) = (\pi/2)\left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \cdots\right]$

$E = E\,(\pi/2, k) = (\pi/2)\left[1 - \left(\frac{1}{2}\right)^2 k^2 - \left(\frac{1}{2\cdot 4}\right)^2 \cdot 3k^4 - \left(\frac{1\cdot 3}{2\cdot 4\cdot 6}\right)^2 \cdot 5k^6 \cdots\right]$

Caution : For the following answers, see the text warning about elliptic integral notation just after equations (12.3) and in Example 1.

12.4 $K(1/2) \cong 1.686$

12.5 $E(1/3) \cong 1.526$

12.6 $\frac{1}{3}F\left(\frac{\pi}{3}, \frac{1}{3}\right) \cong 0.355$

12.7 $5E\left(\frac{5\pi}{4}, \frac{1}{5}\right) \cong 19.46$

12.8 $7E\left(\frac{\pi}{3}, \frac{2}{7}\right) \cong 7.242$

12.9 $F\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right) \cong 0.542$

12.10 $\frac{1}{2}F\left(\frac{\pi}{4}, \frac{1}{2}\right) \cong 0.402$

12.11 $F\left(\frac{3\pi}{8}, \frac{3}{\sqrt{10}}\right) + K\left(\frac{3}{\sqrt{10}}\right) \cong 4.097$

12.12 $10E\left(\frac{\pi}{6}, \frac{1}{10}\right) \cong 5.234$

12.13 $3E\left(\frac{\pi}{6}, \frac{2}{3}\right) + 3E\left(\arcsin\frac{3}{4}, \frac{2}{3}\right) \cong 3.96$

12.14 $12E\left(\frac{\sqrt{5}}{3}\right) \cong 15.86$

12.15 $2\left[E\left(\frac{\sqrt{3}}{2}\right) - E\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)\right] \cong 0.585$

12.16 $2\sqrt{2}\,E\left(1/\sqrt{2}\,\right) \cong 3.820$

12.23 $T = 8\sqrt{\frac{a}{5g}}\,K\left(1/\sqrt{5}\,\right)$; for small vibrations, $T \cong 2\pi\sqrt{\frac{2a}{3g}}$

13.7 $\Gamma(4) = 3!$

13.8 $\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(1)$

13.9 $2\ E\left(\sqrt{3}/2\right) \simeq 2.422$

13.10 $\sqrt{2}\ K\left(2^{-1/2}\right) \simeq 2.622$

13.11 $\frac{1}{5}F\left(\arcsin\frac{3}{4}, 4/5\right) \cong 0.1834$

13.12 $2^{-1/2}K\left(2^{-1/2}\right) \cong 1.311$

13.13 $-\,\mathrm{sn}\,u\,\mathrm{dn}\,u$

13.14 $\sqrt{\pi}/2\,\mathrm{erfc}(1/\sqrt{2})$

13.15 $\Gamma(7/2) = 15\sqrt{\pi}/8$

13.16 $\sqrt{\pi}$

13.17 $\frac{1}{2}B(5/4, 7/4) = 3\pi\sqrt{2}/64$

13.18 $\Gamma(3/4)$

13.19 $\frac{1}{2}\sqrt{\pi}\,\mathrm{erfc}\,5$

13.20 $\frac{1}{2}\ B(1/2, 7/4)$

13.21 $5^4B(2/3, 13/3) = (5/3)^5\left(14\pi/\sqrt{3}\,\right)$

13.22 $4E(1/2) - 2\ E(\pi/8, 1/2) \cong 5.089$

13.23 $109!!\sqrt{\pi}/2^{55}$

13.24 $-2^{55}\sqrt{\pi}/109!!$

13.25 $2^{28}\sqrt{\pi}/55!!$

Chapter 12

1.1	$y = a_1 x e^x$	1.2	$y = a_0 e^{x^3}$
1.3	$y = a_1 x$	1.4	$y = a_0 \cos 2x + a_1 \sin 2x$
1.5	$y = a_0 \cosh x + a_1 \sinh x$	1.6	$y = (A + Bx)e^x$
1.7	$y = Ax + Bx^3$	1.8	$y = a_0(1 + x) + a_2 x^2$
1.9	$y = a_0(1 - x^2) + a_1 x$	1.10	$y = (A + Bx)e^{x^2}$
2.1	See Problem 5.3	2.4	$Q_0 = \frac{1}{2} \ln \frac{1+x}{1-x}, \quad Q_1 = \frac{x}{2} \ln \frac{1+x}{1-x} - 1$
3.2	$xe^x + 10e^x$	3.3	$(30 - x^2) \sin x + 12x \cos x$
3.4	$-x \sin x + 25 \cos x$	3.5	$(x^2 - 200x + 9900)e^{-x}$
4.3	See Problem 5.3		
5.1	See Problem 5.3		

5.3
$P_0(x) = 1$ $\qquad P_4(x) = (35x^4 - 30x^2 + 3)/8$
$P_1(x) = x$ $\qquad P_5(x) = (63x^5 - 70x^3 + 15x)/8$
$P_2(x) = (3x^2 - 1)/2$ $\qquad P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16$
$P_3(x) = (5x^3 - 3x)/2$

5.8	$5P_0 - 2P_1$	5.9	$2P_2 + P_1$
5.10	$\frac{8}{35}P_4 + \frac{4}{7}P_2 + \frac{1}{5}P_0$	5.11	$\frac{2}{5}(P_1 - P_3)$
5.12	$\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$	5.13	$\frac{8}{63}P_5 + \frac{4}{9}P_3 + \frac{3}{7}P_1$
8.1	$N = \sqrt{\pi/2}, \quad \sqrt{2/\pi}\, \cos nx$	8.2	$N = \sqrt{2/5}, \quad \sqrt{5/2}\, P_2(x)$
8.3	$N = 2^{1/2}, \quad 2^{-1/2} x e^{-x/2}$	8.4	$N = \pi^{1/4}, \quad \pi^{-1/4} e^{-x^2/2}$
8.5	$N = \frac{1}{2}\pi^{1/4}, \quad 2\pi^{-1/4} x e^{-x^2/2}$		
9.1	$\frac{3}{2}P_1 - \frac{7}{8}P_3 + \frac{11}{16}P_5 \cdots$	9.2	$\frac{1}{4}P_0 + \frac{1}{2}P_1 + \frac{5}{16}P_2 - \frac{3}{32}P_4 \cdots$
9.3	$5P_2 + P_0$	9.4	$\frac{\pi}{8}(3P_1 + \frac{7}{16}P_3 + \frac{11}{64}P_5 \cdots)$
9.5	$\frac{1}{2}P_0 - \frac{5}{8}P_2 + \frac{3}{16}P_4 \cdots$	9.6	$P_0 + \frac{3}{8}P_1 - \frac{20}{9}P_2 \cdots$
9.7	$\frac{1}{3}P_0 + \frac{2}{5}P_1 - \frac{11}{42}P_2 \cdots$		

9.8 $\frac{1}{2}(1 - a)P_0 + \frac{3}{4}(1 - a^2)P_1 + \frac{5}{4}a(1 - a^2)P_2 + \frac{7}{16}(1 - a^2)(5a^2 - 1)P_3 \cdots$

9.9 $P_n' = \sum(2l + 1)P_l$, where the sum is over odd l from 1 to $n - 1$ when n is even, and over even l from 0 to $n - 1$, when n is odd.

9.10	$2P_2 + P_1$	9.11	$\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$
9.12	$\frac{2}{5}(P_1 - P_3)$	9.13	$\frac{1}{5}P_0 + \frac{4}{7}P_2 = \frac{3}{35}(10x^2 - 1)$
9.14	$\frac{1}{2}P_0 + \frac{5}{8}P_2 = \frac{3}{16}(5x^2 + 1)$	9.15	$-\frac{15}{\pi^2}P_2 = -\frac{15}{2\pi^2}(3x^2 - 1)$

10.4 $\sin\theta$

10.5 $\sin\theta\,(35\cos^3\theta - 15\cos\theta)/2$

10.6 $15\sin^2\theta\cos\theta$

11.1 $y = b_0\cos x/x^2$

11.2 $y = Ax^{-3} + Bx^3$

11.3 $y = Ax^{-3} + Bx^2$

11.4 $y = Ax^{-2} + Bx^3$

11.5 $y = A\cos(2x^{1/2}) + B\sin(2x^{1/2})$

11.6 $y = Ae^{-x} + Bx^{2/3}[1 - 3x/5 + (3x)^2/(5\cdot 8) - (3x)^3/(5\cdot 8\cdot 11) + \cdots]$

11.7 $y = Ax^2(1 + x^2/10 + x^4/280 + \cdots) + Bx^{-1}(1 - x^2/2 - x^4/8 - \cdots)$

11.8 $y = A(x^{-1} - 1) + Bx^2(1 - x + 3x^2/5 - 4x^3/15 + 2x^4/21 + \cdots)$

11.9 $y = A(1 - 3x^6/8 + 9x^{12}/320 - \cdots) + Bx^2(1 - 3x^6/16 + 9x^{12}/896 - \cdots)$

11.10 $y = A[1 + 2x - (2x)^2/2! + (2x)^3/(3\cdot 3!) - (2x)^4/(3\cdot 5\cdot 4!) + \cdots]$
$+ Bx^{3/2}[1 - 2x/5 + (2x)^2/(5\cdot 7\cdot 2!) - (2x)^3/(5\cdot 7\cdot 9\cdot 3!) + \cdots]$

11.11 $y = Ax^{1/6}[1 + 3x^2/2^5 + 3^2x^4/(5\cdot 2^{10}) + \cdots]$
$+ Bx^{-1/6}[x + 3x^3/2^6 + 3^2x^5/(7\cdot 2^{11}) + \cdots]$

11.12 $y = e^x(A + Bx^{1/3})$

15.9 $5^{-3/2}$

16.1 $y = x^{-3/2}Z_{1/2}(x)$

16.2 $y = x^{1/2}Z_{1/4}(x^2)$

16.3 $y = x^{-1/2}Z_1(4x^{1/2})$

16.4 $y = x^{1/6}Z_{1/3}(4x^{1/2})$

16.5 $y = xZ_0(2x)$

16.6 $y = x^{1/2}Z_1(x^{1/2})$

16.7 $y = x^{-1}Z_{1/2}(x^2/2)$

16.8 $y = x^{1/2}Z_{1/3}(\frac{2}{3}x^{3/2})$

16.9 $y = x^{1/3}Z_{2/3}(4\sqrt{x})$

16.10 $y = xZ_{2/3}(2x^{3/2})$

16.11 $y = x^{-2}Z_2(x)$

16.12 $y = x^{1/4}Z_{1/2}(\sqrt{x})$

16.14 $y = Z_3(2x)$

16.15 $y = Z_2(5x)$

16.16 $y = Z_1(4x)$

16.17 $y = Z_0(3x)$

17.7 (a) $y = x^{1/2}I_1(2x^{1/2})$ (b)$y = x^{1/2}I_{1/6}(x^3/3)$

Note that the factor i is not needed since any multiple of y is a solution.

17.9 $\frac{d}{dx}[x^pI_p(x)] = x^pI_{p-1}(x)$

$\frac{d}{dx}[x^{-p}I_p(x)] = x^{-p}I_{p+1}(x)$

$I_{p-1}(x) - I_{p+1}(x) = \frac{2p}{x}I_p(x)$

$I_{p-1}(x) + I_{p+1}(x) = 2I_p'(x)$

$I_p'(x) = -\frac{p}{x}I_p(x) + I_{p-1}(x) = \frac{p}{x}I_p(x) + I_{p+1}(x)$

18.9 Amplitude increases; outward swing takes longer.

18.10 $y = (Ax+B)^{1/2}J_{1/3}[\frac{2}{3A}(Ax+B)^{3/2}]$

18.11 1.7 m for steel, 0.67 m for lead

19.2 $\frac{1}{2}\sin^2\alpha$

19.3 $\int_0^1 x^2 j_n(\alpha x)j_n(\beta x)\,dx = \begin{cases} 0, & \alpha\neq\beta \\ \frac{1}{2}j_{n-1}^2(\alpha), & \alpha = \beta \end{cases}$ where $j_n(\alpha) = j_n(\beta) = 0$.

19.6 $\int_0^1 \cos^2(\alpha_n x)dx = \int_0^1 \frac{1}{2}\pi\alpha_n x N_{1/2}^2(\alpha_n x)\,dx = \frac{1}{2}$, where $\alpha_n = (n+\frac{1}{2})\pi$

20.1 $1/6$

20.2 1

20.3 $4/\pi$

20.4 $-1/(\pi p)$

20.5 $1/2$

20.6 $-1/(2n+1)$

20.7 $h_n^{(1)}(x) \sim \dfrac{1}{x}e^{i[x-(n+1)\pi/2]}$

20.8 $h_n^{(2)}(x) \sim \dfrac{1}{x}e^{-i[x-(n+1)\pi/2]}$

20.9 $h_n^{(1)}(ix) \sim -\dfrac{1}{i^n}\dfrac{1}{x}\,e^{-x}$

20.10 $h_n^{(2)}(ix) \sim i^n\dfrac{1}{x}e^x$

21.1 $y = Ax + B\left(x\sinh^{-1}x - \sqrt{x^2+1}\right)$
21.2 $y = A(1+x) + Bxe^{1/x}$
21.3 $y = A(1-\frac{2}{x}) + B(1+\frac{2}{x})e^{-x}$
21.4 $y = Ax - Be^x$
21.5 $y = A(x-1) + B[(x-1)\ln x - 4]$
21.6 $y = A\sqrt{x} + B[\sqrt{x}\ln x + x]$
21.7 $y = A\frac{x}{1-x} + B[\frac{x}{1-x}\ln x + \frac{1+x}{2}]$
21.8 $y = A(x^2+2x) + B[(x^2+2x)\ln x + 1 + 5x - x^3/6 + x^4/72 + \cdots]$
21.9 $y = Ax^2 + B[x^2\ln x - x^3 + x^4/(2\cdot 2!) - x^5/(3\cdot 3!) + x^6/(4\cdot 4!) + \cdots]$
21.10 $y = Ax^3 + B(x^3\ln x + x^2)$

22.4 $H_0(x) = 1$ $\quad H_3(x) = 8x^3 - 12x$
$H_1(x) = 2x$ $\quad H_4(x) = 16x^4 - 48x^2 + 12$
$H_2(x) = 4x^2 - 2$ $\quad H_5(x) = 32x^5 - 160x^3 + 120x$

22.13 $L_0(x) = 1$
$L_1(x) = 1 - x$
$L_2(x) = (2 - 4x + x^2)/2!$
$L_3(x) = (6 - 18x + 9x^2 - x^3)/3!$
$L_4(x) = (24 - 96x + 72x^2 - 16x^3 + x^4)/4!$
$L_5(x) = (120 - 600x + 600x^2 - 200x^3 + 25x^4 - x^5)/5!$
Note: The factor $1/n!$ is omitted in most quantum mechanics books but is included as here in most reference books.

22.20 $L_0^k(x) = 1$
$L_1^k(x) = 1 + k - x$
$L_2^k(x) = \frac{1}{2}(k+1)(k+2) - (k+2)x + \frac{1}{2}x^2$

22.28 $f_1 = xe^{-x/2}$
$f_2 = xe^{-x/4}(2 - \frac{x}{2})$
$f_3 = xe^{-x/6}(3 - x + \frac{x^2}{18})$

22.30 $R_n = -x^n D x^{-n}$, $L_n = x^{-n-1} D x^{n+1}$

23.6 $P_{2n+1}^1(0) = (2n+1)P_{2n}(0) = \dfrac{(-1)^n(2n+1)!!}{2^n n!}$

23.9 For $n \le l$, $\int_{-1}^{1} xP_l(x)P_n(x)dx = \begin{cases} \dfrac{2l}{(2l-1)(2l+1)}, & n = l-1 \\ 0, & \text{otherwise} \end{cases}$

23.18 (a) $y = Z_0(e^x)$ (b) $y = Z_p(e^{x^2}/2)$
23.23 $T_0 = 1$, $T_1 = x$, $T_2 = 2x^2 - 1$
23.30 $\pi/6$

Chapter 13

2.1 $T = \frac{20}{\pi}\sum_{1}^{\infty} \frac{(-1)^{n+1}}{n}\, e^{-n\pi y/10} \sin\frac{n\pi x}{10}$

2.2 $T = \frac{200}{\pi}\left(\sum_{\substack{1\\ \text{odd } n}}^{\infty} -2\sum_{\substack{2\\ n=2+4k}}^{\infty}\right)\frac{1}{n}\, e^{-n\pi y/20}\sin\frac{n\pi x}{20}$

2.3 $T = \frac{4}{\pi}\sum_{\substack{2\\ \text{even } n}}^{\infty} \frac{n}{n^2-1}\, e^{-ny}\sin nx$

2.4 $T = \frac{120}{\pi^2}\left(e^{-\pi y/30}\sin\frac{\pi x}{30} - \frac{1}{9}\, e^{-3\pi y/30}\sin\frac{3\pi x}{30} + \frac{1}{25}\, e^{-5\pi y/30}\sin\frac{5\pi x}{30}\cdots\right)$

2.7 $T = \frac{4}{\pi}\sum_{\substack{2\\ \text{even } n}}^{\infty} \frac{n}{(n^2-1)\sinh n}\sinh n(1-y)\sin nx$

2.8 $T = \sum_{1}^{\infty}\frac{b_n}{\sinh\frac{4n\pi}{3}}\sinh\frac{n\pi}{30}(40-y)\sin\frac{n\pi x}{30}$

where $b_n = \frac{200}{n\pi}\left(1-\cos\frac{n\pi}{3}\right) = \frac{100}{n\pi}(1,\ 3,\ 4,\ 3,\ 1,\ 0,\ \text{and repeat})$

2.9 $T = \frac{200}{\pi}\left(\sum_{\substack{1\\ \text{odd } n}}^{\infty} -2\sum_{\substack{2\\ n=2+4k}}^{\infty}\right)\frac{1}{n\sinh\frac{n\pi}{2}}\sinh\frac{n\pi}{20}(10-y)\sin\frac{n\pi x}{20}$

2.10 $T = \sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{400}{n\pi\sinh n\pi}\sinh\frac{n\pi}{10}(10-y)\sin\frac{n\pi x}{10};\quad T(5,5)\cong 25^\circ$

2.11 $T = \sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{400}{n\pi\sinh n\pi}\left[\sinh\frac{n\pi}{10}(10-y)\sin\frac{n\pi x}{10} + \sinh\frac{n\pi}{10}(10-x)\sinh\frac{n\pi y}{10}\right]$

2.12 $$T = \sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{400}{n\pi\sinh 3n\pi}\sinh\frac{n\pi}{10}(30-y)\sin\frac{n\pi x}{10} + \sum_{\substack{1\\ \text{odd } n}}^{\infty}\frac{400}{n\pi\sinh(n\pi/3)}\sinh\frac{n\pi}{30}(10-x)\sin\frac{n\pi y}{30}$$

2.13 $$T(x,y) = \frac{20}{\pi}\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n\sinh 2n\pi}\sinh\frac{n\pi}{10}(20-y)\sin\frac{n\pi x}{10} + \frac{40}{\pi}\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n\sinh\frac{n\pi}{2}}\sinh\frac{n\pi}{20}(10-x)\sin\frac{n\pi y}{20}$$

2.14 For $f(x) = x - 5$: $T = -\frac{40}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos\frac{n\pi x}{10}\, e^{-n\pi y/10}$

For $f(x) = x$: Add 5 to the answer just given.

2.15 For $f(x) = 100$, $T = 100 - 10y/3$

For $f(x) = x$, $T = \frac{1}{6}(30 - y) - \frac{40}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2 \sinh 3n\pi} \sinh\frac{n\pi}{10}(30 - y) \cos\frac{n\pi x}{10}$

3.2 $u = \frac{400}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin\frac{n\pi x}{10}$

3.3 $u = 100 - \frac{100x}{l} - \frac{400}{\pi} \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{1}{n} e^{-(n\pi\alpha/l)^2 t} \sin\frac{n\pi x}{l}$

3.4 $u = \frac{40}{\pi} \left(\sum_{\substack{1 \\ \text{odd } n}}^{\infty} - 2 \sum_{\substack{2 \\ n=2+4k}}^{\infty} \right) \frac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin\frac{n\pi x}{10}$

3.5 $u = 100 + 400 \sum_{1}^{\infty} b_n e^{-(n\pi\alpha/2)^2 t} \sin\frac{n\pi x}{2}$ where $b_n = \begin{cases} 0, & \text{even } n \\ \frac{2}{n^2\pi^2} - \frac{1}{n\pi}, & n = 1 + 4k \\ \frac{-2}{n^2\pi^2} - \frac{1}{n\pi}, & n = 3 + 4k \end{cases}$

3.6 Add to (3.15) : $u_f = 20 + 30x/l$. Note: Any linear function added to both u_0 and u_f leaves the Fourier series unchanged.

3.7 $u = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos\frac{n\pi x}{l} e^{-(n\pi\alpha/l)^2 t}$

3.8 $u = 50x + \frac{200}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} e^{-(n\pi\alpha/2)^2 t} \sin\frac{n\pi x}{2}$

3.9 $u = 100 - \frac{400}{\pi} \sum_{0}^{\infty} \frac{(-1)^n}{2n+1} e^{-[(2n+1)\pi\alpha/4]^2 t} \cos\left(\frac{2n+1}{4}\pi x\right)$

3.11 $E_n = \frac{n^2\hbar^2}{2m}$, $\Psi(x,t) = \frac{4}{\pi} \sum_{\text{odd } n} \frac{\sin nx}{n} e^{-iE_n t/\hbar}$

3.12 $E_n = \frac{n^2\pi^2\hbar^2}{2m}$, $\Psi(x,t) = \frac{8}{\pi} \sum_{\text{odd } n} \frac{\sin n\pi x}{n(4 - n^2)} e^{-iE_n t/\hbar}$

4.2 $y = \frac{8h}{\pi^2} \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} \cos\frac{n\pi vt}{l}$, where $B_1 = \sqrt{2} - 1$, $B_2 = \frac{1}{2}$,

$B_3 = \frac{1}{9}(\sqrt{2} + 1)$, $B_4 = 0, \cdots$, $B_n = (2\sin n\pi/4 - \sin n\pi/2)/n^2$

4.3 $y = \frac{16h}{\pi^2} \sum_{1}^{\infty} B_n \sin\frac{n\pi x}{l} \cos\frac{n\pi vt}{l}$ where $B_n = \left(2\sin\frac{n\pi}{8} - \sin\frac{n\pi}{4}\right)/n^2$

4.4 $y = \frac{8h}{\pi^2} \sum_{1}^{\infty} \frac{1}{n^2} \sin\frac{n\pi}{2} \sin\frac{2n\pi x}{l} \cos\frac{2n\pi vt}{l}$

4.5 $y = \frac{8hl}{\pi^3 v} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^3} \sin\frac{n\pi}{2} \sin\frac{n\pi x}{l} \sin\frac{n\pi vt}{l}$

4.6 $y = \dfrac{4hl}{\pi^2 v} \displaystyle\sum_{\substack{1\\ \text{odd } n}}^{\infty} \frac{1}{n^2} \sin\frac{n\pi}{2} \sin\frac{n\pi w}{l} \sin\frac{n\pi x}{l} \sin\frac{n\pi vt}{l}$

4.7 $y = \dfrac{9hl}{\pi^3 v} \displaystyle\sum_{1}^{\infty} \frac{1}{n^3} \sin\frac{n\pi}{3} \sin\frac{n\pi x}{l} \sin\frac{n\pi vt}{l}$

4.8 $y = \dfrac{4l}{\pi^2 v}\left[\dfrac{1}{3}\sin\dfrac{\pi x}{l}\sin\dfrac{\pi vt}{l} + \dfrac{\pi}{16}\sin\dfrac{2\pi x}{l}\sin\dfrac{2\pi vt}{l} - \displaystyle\sum_{n=3}^{\infty}\frac{\sin(n\pi/2)}{n(n^2-4)}\sin\frac{n\pi x}{l}\sin\frac{n\pi vt}{l}\right]$

4.9 1. $n = 1$, $\nu = v/(2l)$
2. $n = 2$, $\nu = v/l$
3. $n = 3$, $\nu = 3v/(2l)$, and $n = 4$, $\nu = 2v/l$, have nearly equal intensity.
4. $n = 2$, $\nu = v/l$
5, 6, 7, 8. $n = 1$, $\nu = v/(2l)$

4.11 The basis functions for a string pinned at $x = 0$, free at $x = l$, and with zero initial string velocity, are $y = \sin\frac{(n+\frac{1}{2})\pi x}{l}\cos\frac{(n+\frac{1}{2})\pi vt}{l}$.
The solutions for Problems 2, 3, 4, parts (a) and (b) are:

(a) $y = \displaystyle\sum_{0}^{\infty} a_n \cos\frac{(n+\frac{1}{2})\pi x}{l}\cos\frac{(n+\frac{1}{2})\pi vt}{l}$

(b) $y = \displaystyle\sum_{0}^{\infty} b_n \sin\frac{(n+\frac{1}{2})\pi x}{l}\cos\frac{(n+\frac{1}{2})\pi vt}{l}$

where the coefficients are:

2(a) $a_n = \dfrac{128h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\cos\dfrac{(2n+1)\pi}{8}$

2(b) $b_n = \dfrac{128h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\sin\dfrac{(2n+1)\pi}{8}$

3(a) $a_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{32}\cos\dfrac{(2n+1)\pi}{16}$

3(b) $b_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{32}\sin\dfrac{(2n+1)\pi}{16}$

4(a) $a_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\sin\dfrac{(2n+1)\pi}{8}\sin\dfrac{(2n+1)\pi}{4}$

4(b) $b_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\sin\dfrac{(2n+1)\pi}{8}\cos\dfrac{(2n+1)\pi}{4}$

4.12 With $b_n = \dfrac{8}{n^3\pi^3}$, odd n, the six solutions on $(0, 1)$ are:
1. Temperature in semi-infinite plate: $T = \sum b_n e^{-n\pi y}\sin n\pi x$
2. Temperature in finite plate of height H:
$$T = \sum \frac{b_n}{\sinh(n\pi H)}\sinh n\pi(H-y)\sin n\pi x$$
3. 1-dimensional heat flow: $u = \sum b_n e^{-(n\pi\alpha)^2 t}\sin n\pi x$
4. Particle in a box: $\Psi = \sum b_n \sin n\pi x\, e^{-iE_n t/\hbar}$, $E_n = \dfrac{\hbar^2 n^2\pi^2}{2m}$
5. Plucked string: $y = \sum b_n \sin n\pi x \cos n\pi vt$
6. String with initial velocity: $y = \sum \dfrac{b_n}{n\pi v}\sin n\pi x \sin n\pi vt$

4.13 With $b_n = \dfrac{16}{n\pi(4-n^2)}$, n odd, the six solutions on $(0,\pi)$ are

1. $T = \sum b_n e^{-ny}\sin nx$
2. $T = \sum \dfrac{b_n}{\sinh nH}\sinh n(H-y)\sin nx$
3. $u = \sum b_n e^{(-n\alpha)^2 t}\sin nx$
4. $\Psi = \sum b_n \sin nx\, e^{-iE_n t/\hbar}, \quad E_n = \dfrac{\hbar^2 n^2}{2m}$
5. $y = \sum b_n \sin nx \cos nvt$
6. $y = \sum \dfrac{b_n}{nv}\sin nx \sin nvt$

4.14 Same as 4.12 with $b_n = \dfrac{12(-1)^{n+1}}{n^3\pi^3}$, all n, on $(0,1)$

5.1 (a) $u \cong 9.76°$ (b) $u \cong 9.76°$

5.2 (a) $\displaystyle\sum_{m=1}^{\infty} \frac{2}{k_m J_2(k_m)} J_1(k_m r)\; e^{-k_m z}\sin\theta,\; k_m = \text{zeros of } J_1$

(b) $\displaystyle\sum_{m=1}^{\infty} \frac{2a}{k_m J_2(k_m)} J_1(k_m r/a)\; e^{-k_m z/a}\sin\theta,\; k_m = \text{zeros of } J_1$

$u(r=1, z=1, \theta=\pi/2) \cong 0.211$

5.3 (a) $\displaystyle u = \sum_{m=1}^{\infty} \frac{200}{k_m J_1(k_m)\sinh(10k_m)} J_0(k_m r)\sinh k_m(10-z),\; k_m = \text{zeros of } J_0$

(b) $\displaystyle u = \sum_{m=1}^{\infty} \frac{200}{k_m J_1(k_m)\sinh(k_m H/a)} J_0(k_m r/a)\sinh\frac{k_m(H-z)}{a},$

$k_m = \text{zeros of } J_0$

5.4 $\displaystyle u = \sum_{m=1}^{\infty} \frac{200}{k_m J_1(k_m)} J_0(k_m r/a) e^{-(k_m\alpha/a)^2 t},\; k_m = \text{ zeros of } J_0$

5.5 $\displaystyle \sum_{m=1}^{\infty} \frac{200a}{k_m J_2(k_m)} J_1(k_m r/a) e^{-(k_m\alpha/a)^2 t}\sin\theta,\; k_m = \text{ zeros of } J_1$

5.6 $\displaystyle a_{mn} = \frac{2}{\pi a^2 J_{n+1}^2(k_{mn})}\int_0^a\int_0^{2\pi} f(r,\theta)J_n(k_{mn}r/a)\cos n\theta\, r\,dr\,d\theta$

$\displaystyle b_{mn} = \frac{2}{\pi a^2 J_{n+1}^2(k_{mn})}\int_0^a\int_0^{2\pi} f(r,\theta)J_n(k_{mn}r/a)\sin n\theta\, r\,dr\,d\theta$

5.7 $\displaystyle u = \frac{400}{\pi} + \sum_{\text{odd } n} \frac{1}{nI_0(3n\pi/20)} I_0\left(\frac{n\pi r}{20}\right)\sin\frac{n\pi z}{20}$

5.8 $\displaystyle u = 40 + \sum_{m=1}^{\infty}\frac{120}{k_m J_1(k_m)} J_0(k_m r)e^{-k_m^2\alpha^2 t},\; \text{ where } J_0(k_m) = 0$

5.9 $\displaystyle u = \frac{1600}{\pi^2}\sum_{\text{odd } m}\sum_{\text{odd } n}\frac{\sin(n\pi x/10)\sin(m\pi y/10)\sinh[\pi(n^2+m^2)^{1/2}(10-z)/10]}{mn\sinh[\pi(n^2+m^2)^{1/2}]}$

5.10 $\displaystyle u = \frac{6400}{\pi^3}\sum_{\text{odd } n}\sum_{\text{odd } m}\sum_{\text{odd } p}\frac{1}{nmp}\sin\frac{n\pi x}{l}\sin\frac{m\pi y}{l}\sin\frac{p\pi z}{l}e^{-(\alpha\pi/l)^2(n^2+m^2+p^2)t}$

5.11 $R = r^n,\ r^{-n},\ n \neq 0;\ R = \ln r,\ \text{const.},\ n = 0$

$R = r^l,\ r^{-l-1}$

5.12 $\displaystyle u = 50 + \frac{200}{\pi}\sum_{\text{odd } n}\left(\frac{r}{a}\right)^n\frac{\sin n\theta}{n}$

5.13 $\displaystyle u = \frac{400}{\pi}\sum_{\text{odd} n}\frac{1}{n}\left(\frac{r}{10}\right)^{4n}\sin 4n\theta$

5.14 $u = \dfrac{50\ln r}{\ln 2} + \dfrac{200}{\pi}\sum_{\text{odd } n}\dfrac{r^n - r^{-n}}{n(2^n - 2^{-n})}\sin n\theta$

5.15 $u = 50\left(1 - \dfrac{\ln r}{\ln 2}\right) - \dfrac{200}{\pi}\sum_{\text{odd } n}\dfrac{1}{n\,(2^n - 2^{-n})}\left[\left(\dfrac{r}{2}\right)^n - \left(\dfrac{r}{2}\right)^{-n}\right]\sin n\theta$

6.2 The first six frequencies are $\nu_{10},\ \nu_{11} = 1.593\nu_{10},\ \nu_{12} = 2.135\nu_{10}$
$\nu_{20} = 2.295\nu_{10},\ \nu_{13} = 2.652\nu_{10},\ \nu_{21} = 2.917\nu_{10}$.

6.4 $\nu_{nm} = \dfrac{v}{2}\sqrt{\left(\dfrac{l}{a}\right)^2 + \left(\dfrac{m}{b}\right)^2 + \left(\dfrac{n}{c}\right)^2}$

6.5 $z = \dfrac{64l^4}{\pi^6}\sum_{\text{odd } m}\sum_{\text{odd } n}\dfrac{1}{n^3m^3}\sin\dfrac{n\pi x}{l}\sin\dfrac{m\pi y}{l}\cos\dfrac{\pi v(m^2+n^2)^{1/2}t}{l}$

6.6 $\Psi_n = \sin\dfrac{n_x\pi x}{l}\sin\dfrac{n_y\pi y}{l}e^{-iE_nt/\hbar}, \quad E_n = \dfrac{\pi^2\hbar^2({n_x}^2 + {n_y}^2)}{2ml^2}$

6.7 See Problem 6.3. Some other examples of degeneracy:
$(n_x, n_y) = (1,8),\ (8,1),\ (4,7),\ (7,4)$, giving $E_n = 65\dfrac{\pi^2\hbar^2}{2ml^2}$;
similarly $2^2 + 9^2 = 6^2 + 7^2 = 85$; $2^2 + 11^2 = 5^2 + 10^2 = 125$, etc.

6.8 $\Psi_{mn} = J_n(k_{mn}r)\begin{Bmatrix}\sin n\theta\\ \cos n\theta\end{Bmatrix}e^{-iE_{mn}t/\hbar},\ E_{mn} = \dfrac{\hbar^2k_{mn}^2}{2ma^2}$

7.1 $u = 7P_0(\cos\theta) + 20r^2P_2(\cos\theta) + 8r^4P_4(\cos\theta)$
7.2 $u = \frac{2}{5}rP_1(\cos\theta) - \frac{2}{5}r^3P_3(\cos\theta)$
7.3 $u = -2P_0(\cos\theta) + rP_1(\cos\theta) + 2r^2P_2(\cos\theta)$
7.4 $u = -2P_0(\cos\theta) + 3rP_1(\cos\theta) + 2r^2P_2(\cos\theta) + 2r^3P_3(\cos\theta)$
7.5 $u = \frac{1}{2}P_0(\cos\theta) + \frac{5}{8}r^2P_2(\cos\theta) - \frac{3}{16}r^4P_4(\cos\theta)\cdots$
7.6 $u = \frac{\pi}{8}[3rP_1(\cos\theta) + \frac{7}{16}r^3P_3(\cos\theta) + \frac{11}{64}r^5P_5(\cos\theta)\cdots]$
7.7 $u = \frac{1}{4}P_0(\cos\theta) + \frac{1}{2}rP_1(\cos\theta) + \frac{5}{16}r^2P_2(\cos\theta) - \frac{3}{32}r^4P_4(\cos\theta)\cdots$
7.8 $u = 25[P_0(\cos\theta) + \frac{9}{4}rP_1(\cos\theta) + \frac{15}{8}r^2P_2(\cos\theta) + \frac{21}{64}r^3P_3(\cos\theta)\cdots]$
7.9 $u = r^2P_2^1(\cos\theta)\sin\phi$
7.10 $u = \frac{1}{15}r^3P_3^2(\cos\theta)\cos 2\phi - rP_1(\cos\theta)$
7.11 $u = 200[(3/4)]rP_1(\cos\theta) - (7/16)r^3P_3(\cos\theta) + (11/32)r^5P_5(\cos\theta) + \cdots]$
7.12 $u = \frac{3}{4}rP_1(\cos\theta) + \frac{7}{24}r^3P_3(\cos\theta) - \frac{11}{192}r^5P_5(\cos\theta)\cdots$
7.13 $u = E_0(r - a^3/r^2)P_1(\cos\theta)$
7.14 $u = 100[(1 - r^{-1})P_0(\cos\theta)$
$+ \frac{3}{7}(r - r^{-2})P_1(\cos\theta) - \frac{7}{127}(r^3 - r^{-4})P_3(\cos\theta)\cdots]$

7.15 $u = 100 + \dfrac{200a}{\pi r}\sum_1^\infty\dfrac{(-1)^n}{n}\sin\dfrac{n\pi r}{a}\,e^{-(\alpha n\pi/a)^2t}$

$= 100 + 200\sum_{n=1}^\infty(-1)^nj_0(n\pi r/a)e^{-(\alpha n\pi/a)^2t}$

7.17 $\Psi_n = \sin\dfrac{n_x\pi x}{l}\sin\dfrac{n_y\pi y}{l}\sin\dfrac{n_z\pi z}{l}e^{-iE_nt/\hbar},\ E_n = \dfrac{\pi^2\hbar^2({n_x}^2 + {n_y}^2 + {n_z}^2)}{2ml^2}$

7.19 $\Psi(r,\theta,\phi) = j_l(\beta r)P_l^m(\cos\theta)e^{\pm im\phi}e^{-iEt/\hbar}$,
where $\beta = \sqrt{2ME/\hbar^2}$, $\beta a =$ zeros of j_l, $E = \dfrac{\hbar^2}{2Ma^2}$(zeros of $j_l)^2$.

7.20 $\psi_n(x) = e^{-\alpha^2x^2/2}H_n(\alpha x),\ \alpha = \sqrt{m\omega/\hbar}$

7.21 $\psi_n(x) = e^{-\alpha^2(x^2+y^2+z^2)/2} H_{n_x}(\alpha x) H_{n_y}(\alpha y) H_{n_z}(\alpha z)$, $\alpha = \sqrt{m\omega/\hbar}$,
$E_n = (n_x + \frac{1}{2} + n_y + \frac{1}{2} + n_z + \frac{1}{2})\hbar\omega = (n + \frac{3}{2})\hbar\omega$.
Degree of degeneracy of E_n is $C(n+2, n) = \dfrac{(n+2)(n+1)}{2}$, $n = 0$ to ∞.

7.22 $\Psi(r,\theta,\phi) = R(r) Y_l^m(\theta,\phi)$, $R(r) = r^l e^{-r/(na)} L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)$, $E_n = -\dfrac{Me^4}{2\hbar^2 n^2}$

8.3 The second terms in (8.20) and (8.21) are replaced by

$$-q \sum_l \frac{a^l r^l}{R^{2l+1}} P_l(\cos\theta) = \frac{-qR/a}{\sqrt{r^2 - 2(rR^2/a)\cos\theta + (R^2/a)^2}}$$

Image charge -qR/a at$(0,0,R^2/a)$

8.4 Let$K$ = line charge per unit length. Then

$$V = -K\ln(r^2 + a^2 - 2ra\cos\theta) + K\ln a^2 - K\ln R^2 + K\ln\left[r^2 + \left(\frac{R^2}{a}\right)^2 - 2\frac{R^2}{a} r\cos\theta\right]$$

8.5 K at $(a,0)$, $-K$ at $(R^2/a, 0)$

9.2 $u = \dfrac{200}{\pi}\displaystyle\int_0^\infty k^{-2}(1-\cos 2k)e^{-ky}\cos kx\; dk$

9.4 $u(x,t) = \dfrac{200}{\pi}\displaystyle\int_0^\infty \frac{1-\cos k}{k} e^{-k^2\alpha^2 t}\sin kx\; dk$

9.7 $u(x,t) = 100\,\mathrm{erf}\left(\dfrac{x}{2\alpha\sqrt{t}}\right) - 50\,\mathrm{erf}\left(\dfrac{x-1}{2\alpha\sqrt{t}}\right) - 50\,\mathrm{erf}\left(\dfrac{x+1}{2\alpha\sqrt{t}}\right)$

10.1 $T = \dfrac{2}{\pi}\displaystyle\sum_1^\infty \frac{1}{n\sinh 2n\pi}\sinh n\pi(2-y)\sin n\pi x$

10.2 $T = \dfrac{2}{\pi}\displaystyle\sum_1^\infty \frac{1}{n\sinh 2n\pi}\sinh n\pi y\sin n\pi x$

10.3 $T = \dfrac{1}{4}(2-y) + \dfrac{4}{\pi^2}\displaystyle\sum_{\text{odd } n} \frac{1}{n^2\sinh 2n\pi}\sinh n\pi(2-y)\cos n\pi x$

10.4
$$T = 20 + \frac{40}{\pi}\sum_{\text{odd } n}\frac{1}{n\sinh\frac{3n\pi}{5}}\sinh\frac{n\pi y}{5}\sin\frac{n\pi x}{5} + \frac{40}{\pi}\sum_{\text{odd } n}\frac{1}{n\sinh\frac{5n\pi}{3}}\sinh\frac{n\pi(5-x)}{3}\sin\frac{n\pi y}{3}$$

10.5 $u = 20 - \dfrac{80}{\pi}\displaystyle\sum_{\text{odd } n}\frac{1}{n}\, e^{-(n\pi\alpha/l)^2 t}\sin\frac{n\pi x}{l}$

10.6 $u = 20 - \dfrac{80}{\pi}\displaystyle\sum_{n=0}^\infty \frac{(-1)^n}{2n+1}\, e^{-[(2n+1)\pi\alpha/(2l)]^2 t}\cos\left(\frac{2n+1}{2l}\pi x\right)$

10.7 $u = \dfrac{1}{4}y + \dfrac{4}{\pi^2}\displaystyle\sum_{\text{odd } n}\frac{1}{n^2\sinh 2n\pi}\sinh n\pi y\cos n\pi x$

10.8 $u = 20 - x - \dfrac{40}{\pi}\displaystyle\sum_{\substack{2\\ \text{even } n}}^\infty \frac{1}{n} e^{-(n\pi\alpha/10)^2 t}\sin\frac{n\pi x}{10}$

10.9 $y = \dfrac{8l^2}{\pi^3}\displaystyle\sum_{\text{odd } n}\frac{1}{n^3}\cos\frac{n\pi vt}{l}\sin\frac{n\pi x}{l}$

10.10 $u = \dfrac{1600}{\pi^2}\displaystyle\sum_{\text{odd } n}\sum_{\text{odd } m}\frac{1}{nmI_n(3m\pi/20)} I_n\left(\frac{m\pi r}{20}\right)\sin n\theta\sin\frac{m\pi z}{20}$

10.12 $u = \dfrac{400}{\pi} \sum_{\text{odd } n} \dfrac{1}{n} \left(\dfrac{r}{a}\right)^{2n} \sin 2n\theta$

10.14 Same as 9.12

10.15 $u = \dfrac{400}{\pi} \sum_{\text{odd } n} \dfrac{1}{n} \left(\dfrac{r}{10}\right)^{6n} \sin 6n\theta = \dfrac{200}{\pi} \arctan \dfrac{2(10r)^6 \sin 6\theta}{10^{12} - r^{12}}$

10.16 $v\sqrt{5}/(2\pi)$

10.17 ν_{mn}, $n \neq 0$; the lowest frequencies are: .
$\nu_{11} = 1.59\nu_{10},\ \nu_{12} = 2.14\nu_{10},\ \nu_{13} = 2.65\nu_{10},\ \nu_{21} = 2.92\nu_{10},\ \nu_{14} = 3.16\nu_{10}$

10.18 $\nu_{mn},\ n = 3, 6, \cdots$; the lowest frequencies are:
$\nu_{13} = 2.65\ \nu_{10},\ \nu_{23} = 4.06\ \nu_{10},\ \nu_{16} = 4.13\ \nu_{10},\ \nu_{33} = 5.4\ \nu_{10}$

10.19 $u = E_0 \left(r - \dfrac{a^2}{r}\right) \cos\theta$

10.20 $\nu = \dfrac{v\lambda_l}{2\pi a}$ where λ_l = zeros of j_l, a = radius of sphere, v = speed of sound

10.21 $u = \frac{2}{3} P_0(\cos\theta) + \frac{3}{5} r P_1(\cos\theta) - \frac{2}{3} r^2 P_2(\cos\theta) + \frac{2}{5} r^3 P_3(\cos\theta)$

10.22 $u = 1 - \frac{1}{2} r P_1(\cos\theta) + \frac{7}{8} r^3 P_3(\cos\theta) - \frac{11}{16} r^5 P_5(\cos\theta) \cdots$

10.23 $u = 100 \sum_{\text{odd } l} (a_l r^l + b_l r^{-l-1}) P_l(\cos\theta)$ where

$a_l = \dfrac{2A+1}{2A^2-1} c_l, \quad b_l = -\dfrac{2A(A+1)}{2A^2-1} c_l, \quad A = 2^l,$

$c_l = (2l+1) \int_0^1 P_l(x)\, dx$ (Chapter 12, Problem 9.1).

The first few terms are

$u = (107.1r - 257.1r^{-2}) P_1(\cos\theta)$
$-(11.7r^3 - 99.2r^{-4}) P_3(\cos\theta) + (2.2r^5 - 70.9r^{-6}) P_5(\cos\theta) \cdots$

10.24 $T = A + \sum_{\text{odd } n} \dfrac{4(D-A)}{n\pi \sinh(n\pi b/a)} \sinh \dfrac{n\pi}{a}(b-y) \sin \dfrac{n\pi x}{a}$
$+ \sum_{\text{odd } n} \dfrac{4(C-A)}{n\pi \sinh(n\pi a/b)} \sinh \dfrac{n\pi}{b}(a-x) \sin \dfrac{n\pi y}{b}$
$+ \sum_{\text{odd } n} \dfrac{4(B-A)}{n\pi \sinh(n\pi b/a)} \sinh \dfrac{n\pi y}{a} \sin \dfrac{n\pi x}{a}$

10.26 $\nu = \dfrac{v}{2\pi} \sqrt{(k_{mn}/a)^2 + \lambda^2}$ where k_{mn} is a zero of J_n

10.27 $\nu = \dfrac{v}{2} \sqrt{\left(\dfrac{n}{a}\right)^2 + \left(\dfrac{m}{b}\right)^2 + \left(\dfrac{\lambda}{\pi}\right)^2}$

10.28 $u(x, y) = \dfrac{200}{\pi} \int_0^\infty \dfrac{\sin k}{k \cosh k} \cos kx \cosh ky\, dk$

Chapter 14

1.1	$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$	1.2	$u = x,\ v = y$
1.3	$u = x,\ v = -y$	1.4	$u = (x^2+y^2)^{1/2},\ v = 0$
1.5	$u = x,\ v = 0$	1.6	$u = e^x \cos y,\ v = e^x \sin y$

1.7 $u = \cos y \cosh x,\ v = \sin y \sinh x$
1.8 $u = \sin x \cosh y,\ v = \cos x \sinh y$
1.9 $u = x/(x^2+y^2),\ v = -y/(x^2+y^2)$
1.10 $u = (2x^2+2y^2+7x+6)/[(x+2)^2+y^2],\ v = y/[(x+2)^2+y^2]$
1.11 $u = 3x/[x^2+(y-2)^2],\ v = (-2x^2-2y^2+5y-2)/[x^2+(y-2)^2]$
1.12 $u = x(x^2+y^2+1)/[(x^2-y^2+1)^2+4x^2y^2]$,
$v = y(1-x^2-y^2)/[(x^2-y^2+1)^2+4x^2y^2]$

1.13	$u = \ln(x^2+y^2)^{1/2},\ v = 0$	1.14	$u = x(x^2+y^2),\ v = y(x^2+y^2)$
1.15	$u = e^x \cos y,\ v = -e^x \sin y$	1.16	$u = 0,\ v = 4xy$

1.17 $u = \cos x \cosh y,\ v = \sin x \sinh y$
1.18 $u = \pm 2^{-1/2}[(x^2+y^2)^{1/2}+x]^{1/2},\ v = \pm 2^{-1/2}[(x^2+y^2)^{1/2}-x]^{1/2}$,
where the $\pm$ signs are chosen so that uv has the sign of y.
1.19 $u = \ln(x^2+y^2)^{1/2},\ v = \arctan(y/x)$ [angle is in the quadrant of the point (x, y)].
1.20 $u = x^2 - y^2 - 4xy - x - y + 3,\ v = 2x^2 - 2y^2 + 2xy + x - y$
1.21 $u = e^{-y}\cos x,\ v = e^{-y}\sin x$

In 2.1 to 2.24, A = analytic, N = not analytic

2.1	A	2.2	A	2.3	N	2.4	N
2.5	N	2.6	A	2.7	A	2.8	A
2.9	A, $z \neq 0$	2.10	A, $z \neq -2$	2.11	A, $z \neq 2i$	2.12	A, $z \neq \pm i$
2.13	N	2.14	N	2.15	N	2.16	N
2.17	N	2.18	A, $z \neq 0$	2.19	A, $z \neq 0$	2.20	A
2.21	A	2.22	N	2.23	A, $z \neq 0$	2.24	N

2.34 $-z - \frac{1}{2}z^2 - \frac{1}{3}z^3 \cdots,\ |z| < 1$
2.35 $1 - (z^2/2!) + (z^4/4!) \cdots$, all z
2.36 $1 + \frac{1}{2}z^2 - \frac{1}{8}z^4 \cdots,\ |z| < 1$
2.37 $z - \frac{1}{3}z^3 + \frac{2}{15}z^5 \cdots,\ |z| < \pi/2$
2.38 $-\frac{1}{2}i + \frac{1}{4}z + \frac{1}{8}iz^2 - \frac{1}{16}z^3 \cdots,\ |z| < 2$
2.39 $(z/9) - (z^3/9^2) + (z^5/9^3) \cdots,\ |z| < 3$
2.40 $1 + z + z^2 + z^3 \cdots,\ |z| < 1$
2.41 $1 + iz - z^2/2 - iz^3/3! + z^4/4! \cdots$, all z
2.42 $z + z^3/3! + z^5/5! \cdots$, all z

2.48	Yes, $z \neq 0$	2.49	No	2.50	Yes, $z \neq 0$	2.51	Yes
2.52	No	2.53	Yes, $z \neq 0$	2.54	$-iz$	2.55	$-iz^3$
2.56	$-iz^2/2$	2.57	$(1-i)z$	2.58	$\cos z$	2.59	e^z
2.60	$2\ln z$	2.61	$1/z$	2.62	$-ie^{iz}$	2.63	$-i/(1-z)$

3.1	$\frac{1}{2}+i$	3.2	$-(2+i)/3$	3.3	0	3.4	$i\pi/2$
3.5	-1	3.6	$-1,\ -1$	3.7	$\pi(1-i)/8$	3.8	$i/2$
3.9	1	3.10	$(2i-1)e^{2i}$	3.11	$2\pi i$		
3.12	(a) $\frac{5}{3}(1+2i)$	(b) $\frac{1}{3}(8i+13)$				3.16	0
3.17	(a) 0	(b) $i\pi$		3.18	$i\pi\sqrt{3}/6$	3.19	$16i\pi$
3.20	(a) 0	(b) $-17\pi i/4$		3.22	$-i\pi\sqrt{3}/108$		
3.23	$72i\pi$	3.24	$-17i\pi/96$				

4.3 For $0<|z|<1$: $\frac{1}{2}z^{-1}+\frac{3}{4}+\frac{7}{8}z+\frac{15}{16}z^2\cdots$; $R(0)=\frac{1}{2}$
For $1<|z|<2$: $-(\cdots z^{-4}+z^{-3}+z^{-2}+\frac{1}{2}z^{-1}+\frac{1}{4}+\frac{1}{8}z+\frac{1}{16}z^2+\frac{1}{32}z^3\cdots)$
For $|z|>2$: $z^{-3}+3z^{-4}+7z^{-5}+15z^{-6}\cdots$

4.4 For $0<|z|<1$: $-\frac{1}{4}z^{-1}-\frac{1}{2}-\frac{11}{16}z-\frac{13}{16}z^2\cdots$; $R(0)=-\frac{1}{4}$
For $1<|z|<2$: $\cdots+z^{-3}+z^{-2}+\frac{3}{4}z^{-1}+\frac{1}{2}+\frac{5}{16}z+\frac{3}{16}z^2\cdots$
For $|z|>2$: $z^{-4}+5z^{-5}+17z^{-6}+49z^{-7}\cdots$

4.5 For $0<|z|<2$: $\frac{1}{2}z^{-3}-\frac{1}{4}z^{-2}-\frac{1}{8}z^{-1}-\frac{1}{16}-\frac{1}{32}z-\frac{1}{64}z^2$; $R(0)=-\frac{1}{8}$
For $|z|>2$: $z^{-3}+z^{-4}+2z^{-5}+4z^{-6}+8z^{-7}\cdots$

4.6 For $0<|z|<1$: $z^{-2}-2z^{-1}+3-4z+5z^2\cdots$; $R(0)=-2$
For $|z|>1$: $z^{-4}-2z^{-5}+3z^{-6}\cdots$

4.7 For $|z|<1$: $2-z+2z^2-z^3+2z^4-z^5\cdots$; $R(0)=0$
For $|z|>1$: $z^{-1}-2z^{-2}+z^{-3}-2z^{-4}\cdots$

4.8 For $|z|<1$: $-5+\frac{25}{6}z-\frac{175}{36}z^2\cdots$; $R(0)=0$
For $1<|z|<2$: $-5(\cdots+z^{-3}-z^{-2}+z^{-1}+\frac{1}{6}z+\frac{1}{36}z^2+\frac{7}{216}z^3\cdots)$
For $2<|z|<3$: $\cdots+3z^{-3}+9z^{-2}-3z^{-1}+1-\frac{1}{3}z+\frac{1}{9}z^2-\frac{1}{27}z^3\cdots$
For $|z|>3$: $30(z^{-3}-2z^{-4}+9z^{-5}\cdots)$

4.9	(a) regular	(b) pole of order 3
	(c) pole of order 2	(d) pole of order 1
4.10	(a) simple pole	(b) pole of order 2
	(c) pole of order 2	(d) essential singularity
4.11	(a) regular	(b) pole of order 2
	(c) simple pole	(d) pole of order 3
4.12	(a) pole of order 3	(b) pole of order 2
	(c) essential singularity	(d) pole of order 1

6.1 $z^{-1}-1+z-z^2\cdots$; $R=1$
6.2 $(z-1)^{-1}-1+(z-1)-(z-1)^2\cdots$; $R=1$
6.3 $z^{-3}-\frac{1}{6}z^{-1}+\frac{1}{120}z\cdots$; $R=-\frac{1}{6}$
6.4 $z^{-2}+(1/2!)+(z^2/4!)\cdots$; $R=0$
6.5 $\frac{1}{2}e[(z-1)^{-1}+\frac{1}{2}+\frac{1}{4}(z-1)\cdots]$; $R=\frac{1}{2}e$
6.6 $z^{-1}-(1/3!)z^{-3}+(1/5!)z^{-5}\cdots$; $R=1$
6.7 $\frac{1}{4}\left[\left(z-\frac{1}{2}\right)^{-1}-1+(1-\pi^2/2)\left(z-\frac{1}{2}\right)+\cdots\right]$, $R=\frac{1}{4}$
6.8 $1/2-(z-\pi)^2/4!+(z-\pi)^4/6!-\cdots$, $R=0$
6.9 $-\left[(z-2)^{-1}+1+(z-2)+(z-2)^2+\cdots\right]$; $R=-1$

6.14	$R(-2/3)=1/8$, $R(2)=-1/8$	6.15	$R(1/2)=1/3$, $R(4/5)=-1/3$
6.16	$R(0)=-2$, $R(1)=1$	6.17	$R(1/2)=5/8$, $R(-1/2)=-3/8$
6.18	$R(3i)=\frac{1}{2}-\frac{1}{3}i$	6.19	$R(\pi/2)=1/2$
6.20	$R(i)=1/4$	6.21	$R[\sqrt{2}\,(1+i)]=\sqrt{2}\,(1-i)/16$
6.22	$R(i\pi)=-1$	6.23	$R(2i/3)=-ie^{-2/3}/12$
6.24	$R(0)=2$	6.25	$R(0)=2$
6.26	$R(e^{2\pi i/3})=\frac{1}{6}(i\sqrt{3}-1)e^{-\pi\sqrt{3}}$	6.27	$R(\pi/6)=-1/2$

6.28	$R(3i) = -\frac{1}{16} + \frac{1}{24}i$	6.29	$R(\ln 2) = 4/3$
6.30	$R(0) = 1/6!$	6.31	$R(0) = 9/2$
6.32	$R(2i) = -3ie^{-2}/32$	6.33	$R(\pi) = -1/2$
6.34	$R(0) = -7$, $R(1/2) = 7$	6.35	$R(i) = 0$

6.14′	$\pi i/4$	6.15′	0	6.16′	$-2\pi i$	6.17′	$\pi i/2$
6.18′	0	6.19′	0	6.20′	0	6.21′	0
6.22′	0	6.23′	$-\frac{\pi}{3}\sinh\frac{2}{3}$	6.24′	$4\pi i$	6.25′	$4\pi i$
6.26′	$-\frac{2}{3}\pi i(1 + \cosh\pi\sqrt{3} + i\sqrt{3}\sinh\pi\sqrt{3})$						
6.27′	$-\pi i$	6.28′	$\frac{1}{4}\pi i$	6.29′	$5\pi i/2$	6.30′	$\pi i/360$
6.31′	$9\pi i$	6.32′	0	6.33′	0	6.34′	0
6.35′	0	6.36	$-\frac{1}{4}$	6.37	$R(-n) = (-1)^n/n!$		

7.1	$\pi/6$	7.2	$\pi/2$	7.3	$2\pi/3$	7.4	$2\pi/9$
7.5	$\pi/(1-r^2)$	7.6	$2\pi/3^{3/2}$	7.7	$\pi/6$	7.8	$\pi/18$
7.9	$2\pi/\lvert\sin\alpha\rvert$	7.10	π	7.11	$3\pi/32$	7.12	$\pi\sqrt{2}/8$
7.13	$\pi/10$	7.14	$-(\pi/e)\sin 2$	7.15	$\pi e^{-4/3}/12$	7.16	$\pi e^{-2/3}/18$
7.17	$(\pi/e)(\cos 2 + 2\sin 2)$			7.18	$\frac{1}{2}\pi e^{-\pi\sqrt{3}/2}$	7.19	$\pi e^{-3}/54$
7.20	$\pi e^{-1/3}/9$	7.22	$-\pi/2$	7.23	$\pi/8$	7.24	π
7.25	$\pi/36$	7.26	$-\pi/2$	7.27	$\pi/4$	7.28	$\pi/4$
7.29	$\pi/2$ for $a > 0$, 0 for $a = 0$, $-\pi/2$ for $a < 0$						
7.30	$\pi/(2\sqrt{2})$	7.31	$\pi/3$	7.32	$\frac{3}{16}\pi\sqrt{2}$	7.33	$\pi\sqrt{2}/2$
7.34	$\pi/2$	7.35	$2\pi(2^{1/3} - 1)/\sqrt{3}$			7.36	$-\pi^2\sqrt{2}$
7.38	$\pi\cot p\pi$	7.39	2	7.40	$\pi^2/4$	7.41	$(2\pi)^{1/2}/4$

7.45 One negative real, one each in quadrants I and IV
7.46 One negative real, one each in quadrants II and III
7.47 One negative real, one each in quadrants I and IV
7.48 Two each in quadrants I and IV
7.49 Two each in quadrants I and IV
7.50 Two each in quadrants II and III

7.51	$4\pi i$, $8\pi i$	7.52	πi
7.53	πi	7.54	$8\pi i$
7.55	$\cosh t\cos t$	7.56	$(\sinh t - \sin t)/2$
7.57	$1 + \sin t - \cos t$	7.58	$(\cos 2t + \cosh 2t)/2$
7.59	$2e^t\cos t\sqrt{3} + e^{-2t}$	7.60	$t + e^{-t} - 1$
7.61	$\frac{1}{3}(\cosh 2t + 2\cosh t\cos t\sqrt{3})$	7.62	$1 - 4te^{-t}$
7.63	$(\cosh t - \cos t)/2$	7.64	$\frac{2}{3}\sinh 2t - \frac{1}{3}\sinh t$
7.65	$(\cos 2t + 2\sin 2t - e^{-t})/5$		

8.3	Regular, $R = -1$	8.4	Regular, $R = -2$
8.5	Regular, $R = -1$	8.6	Simple pole, $R = -5$
8.7	Simple pole, $R = -2$	8.8	Regular, $R = 0$
8.9	Regular, $R = 0$	8.10	Regular, $R = 2$
8.11	Regular, $R = -1$	8.12	Regular, $R = -2$
8.14	$-2\pi i$	8.15	πi

9.1 $x^2 = \frac{1}{2}[u + (u^2 + v^2)^{1/2}]$, $y^2 = \frac{1}{2}[-u + (u^2 + v^2)^{1/2}]$
9.2 $u = y/2$, $v = -(x+1)/2$
9.3 $u = x/(x^2 + y^2)$, $v = -y/(x^2 + y^2)$
9.4 $u = e^x\cos y$, $v = e^x\sin y$
9.5 $u = (x^2 + y^2 - 1)/[x^2 + (y+1)^2]$, $v = -2x/[x^2 + (y+1)^2]$
9.7 $u = \sin x\cosh y$, $v = \cos x\sinh y$
9.8 $u = \cosh x\cos y$, $v = \sinh x\sin y$

10.4 $T = 200\pi^{-1} \arctan(y/x)$

10.5 $V = 200\pi^{-1} \arctan(y/x)$

10.6 $T = 100y/(x^2 + y^2)$; isothermals $y/(x^2 + y^2) = \text{const.}$;
flow lines $x/(x^2 + y^2) = \text{const.}$

10.7 Streamlines $xy = \text{const.}$; $\Phi = (x^2 - y^2)V_0$, $\Psi = 2xy\mathbf{V}_0$, $\mathbf{V} = (2\mathbf{i}x - 2\mathbf{j}y)V_0$

10.9 Streamlines $y - y/(x^2 + y^2) = \text{const.}$

10.10 $\cos x \sinh y = \text{const.}$

10.11 $(x - \coth u)^2 + y^2 = \operatorname{csch}^2 u$
$x^2 + (y + \cot v)^2 = \csc^2 v$

10.12 $T = (20/\pi) \arctan[2y/(1 - x^2 - y^2)]$, arc tan between $\pi/2$ and $3\pi/2$

10.13 $V = \dfrac{V_2 - V_1}{\pi} \arctan \dfrac{2y}{1 - x^2 - y^2} + \dfrac{3V_1 - V_2}{2}$, arc tan between $\pi/2$ and $3\pi/2$

10.14 $\phi = \dfrac{1}{2} V_0 \ln \dfrac{(x+1)^2 + y^2}{(x-1)^2 + y^2}$

$$\psi = V_0 \arctan \frac{2y}{1 - x^2 - y^2}, \quad \text{arc tan between } \pi/2 \text{ and } 3\pi/2.$$

$$V_x = \frac{2V_0(1 - x^2 + y^2)}{(1 - x^2 + y^2)^2 + 4x^2y^2}, \qquad V_y = \frac{-4V_0 xy}{(1 - x^2 + y^2)^2 + 4x^2y^2}$$

11.1 $\ln(1 + z)$

11.2 $-i \ln(1 + z)$

11.5 $R(i) = (1 - i\sqrt{3})/4$
$R(-i) = -1/2$

11.6 $R(-1/2) = i/(6\sqrt{2})$
$R(e^{i\pi/3}/2) = R(e^{5\pi i/3}/2) = -i/(6\sqrt{2})$

11.7 $R(i) = \pi/4$, $R(-i) = R(e^{3\pi i/2}) = -3\pi/4$

11.8 $R(1/2) = 1/2$

11.9 $-1/6$

11.10 -1

11.12 $1/2$

11.13 (a) $1/96$ (b) -5 (c) $-1/80$ (d) $1/2$

11.14 (a) 2 (b) $-\sin 5$ (c) $1/16$ (d) -2π

11.15 $\pi/6$

11.16 $-\pi/6$

11.17 $\pi(e^{-1/2} - \frac{1}{6}e^{-3})/35$

11.18 $\pi e^{-\pi/2}/4$

11.19 $3(2^{-1} - e^{-\pi})/(10\pi)$

11.20 $\pi(e^{-1} + \sin 1)/2$

11.28 π

11.29 $\pi^3/8$ (Caution: $-\pi^3/8$ is wrong.)

11.31 One in each quadrant

11.32 One negative real, one each in quadrants II and III

11.33 One each in quadrants I and IV, two each in II and III

11.34 Two each in quadrants I and IV, one each in II and III

11.40 $\dfrac{2a^2p}{p^4 + 4a^4}$

11.41 $\pi^2/8$

Chapter 15

1.1	1/10, 1/9	1.2	3/8, 1/8, 1/4
1.3	1/3, 5/9	1.4	1/2, 1/52, 2/13, 7/13
1.5	1/4, 3/4, 1/3, 1/2	1.6	27/52, 16/52, 15/52
1.7	9/26, 1/2, 1/13	1.8	9/100, 1/10, 3/100, 1/10
1.9	3/10, 1/3	1.10	3/8

2.12 (a) 3/4 (b) 1/5 (c) 2/3 (d) 3/4 (e) 3/7

2.14 (a) 3/4 (b) 25/36 (c) 37, 38, 39, 40

2.15 (a) 1/6 (b) 1/2 (c) 1/3 (d) 1/3 (e) 1/9

2.17 (a) 3 to 9 with $p(5) = p(7) = 2/9$; others, $p = 1/9$
(b) 5 and 7 (c) 1/3

2.18 (a) 1/2, 1/2 (b) 1/2, 1/4, 1/4 (c) Not a sample space

2.19 1/3, 1/3; 1/7, 1/7

3.3 2^{-6}, 2^{-3}, 2^{-3}

3.4 (a) 8/9, 1/2 (b) 3/5, 1/11, 2/3, 2/3, 6/13

3.5 1/33, 2/9

3.6 4/13, 1/52

3.10 (a) 1/6 (b) 2/3 (c) $P(A) = P(B) = 1/3$, $P(A + B) = 1/2$, $P(AB) = 1/6$

3.11 1/8

3.12 (a) 1/49 (b) 68/441 (c) 25/169 (d) 15 times (e) 44/147

3.13 (a) 1/4 (b) 25/144, 1/16, 1/16

3.14 $n > 3.3$, so 4 tries are needed.

3.15 (a) 1/3 (b) 1/7

3.16 9/23

3.17 (a) 39/80, 5/16, 1/5, 11/16 (b) 374/819 (c) 185/374

3.18 (a) 15/34 (b) 2/15

3.19 1/3

3.20 5/7, 2/7, 11/14

3.21 2/3, 1/3

3.22 6/11, 5/11

4.1 (a) $P(10, 8)$ (b) $C(10, 8)$ (c) 1/45

4.3 3, 7, 31, $2^n - 1$

4.4 1.98×10^{-3}, 4.95×10^{-4}, 3.05×10^{-4}, 1.39×10^{-5}

4.5	2^8, 2^{-8}, 7/32	4.6	15
4.7	1/26	4.8	1/221, 1/33, 1/17

4.9 25/102, 25/77, 49/101, 12/25
4.11 0.097, 0.37, 0.67; 13
4.12 5
4.14 $n!/n^n$
4.17 MB: 16, FD: 6, BE: 10
4.18 MB: 125, FD: 10, BE: 35
4.21 $C(n+2,n)$
4.22 0.135
4.23 0.30

5.1 $\mu = 0$, $\sigma = \sqrt{3}$
5.2 $\mu = 7$, $\sigma = \sqrt{35/6}$
5.3 $\mu = 2$, $\sigma = \sqrt{2}$
5.4 $\mu = 1$, $\sigma = \sqrt{21}/2$
5.5 $\mu = 1$, $\sigma = \sqrt{7/6}$
5.6 $\mu = 3$, $\sigma = \sqrt{284/13} = 4.67$
5.7 $\mu = 3(2p-1)$, $\sigma = 2\sqrt{3p(1-p)}$
5.8 $E(x) = \$12.25$
5.12 $E(x) = 7$
5.15 $\bar{x} = 3(2p-1)$
5.17 Problem 5.2: $E(x^2) = 329/6$, $\sigma^2 = 35/6$
Problem 5.6: $E(x^2) = 401/13$, $\sigma^2 = 284/13$
Problem 5.7: $E(x^2) = 24p^2 - 24p + 9$, $\sigma^2 = 12p(1-p)$

6.1 (a) $f(x) = \pi^{-1}(a^2 - x^2)^{-1/2}$ (c) $\bar{x} = 0$, $\sigma = a/\sqrt{2}$
6.2 $e^{-2} = 0.135$
6.3 $f(h) = 1/(2\sqrt{l}\sqrt{l-h}\,)$
6.4 $f(x) = \alpha e^{-\alpha^2 x^2}/\sqrt{\pi}$, $\bar{x} = 0$, $\sigma = 1/(\alpha\sqrt{2}\,)$
6.5 $f(t) = \lambda e^{-\lambda t}$, $F(t) = 1 - e^{-\lambda t}$, $\bar{t} = 1/\lambda$, half life $= \bar{t}\ln 2$
6.6 $F(r) = r^2$, $f(r) = 2r$, $\bar{r} = 2/3$, $\sigma = \sqrt{2}/6$
6.7 (a) $F(s) = 2[1 - \cos(s/R)]$, $f(s) = (2/R)\sin(s/R)$
(b) $F(s) = [1 - \cos(s/R)]/[1 - \cos(1/R)] \cong s^2$,
$f(s) = R^{-1}[1 - \cos(1/R)]^{-1}\sin(s/R) \cong 2s$
6.8 $f(r) = 3r^2$; $\bar{r} = 3/4$, $\sigma = \sqrt{3/80} = 0.19$
6.9 $f(r) = 4a^{-3}r^2 e^{-2r/a}$

	n	Exactly 7 h	At most 7 h	At least 7 h	Most probable number of h	Expected number of h
7.1	7	0.0078	1	0.0078	3 or 4	7/2
7.2	12	0.193	0.806	0.387	6	6
7.3	15	0.196	0.500	0.696	7 or 8	15/2
7.4	18	0.121	0.240	0.881	9	9

7.5 0.263

8.3 $\mu = 0$, $\sigma^2 = kT/m$, $f(v) = \dfrac{1}{\sqrt{2\pi kT/m}} e^{-mv^2/(2kT)}$

In (8.11) to (8.20), the first number is the binomial result and the second number is the normal approximation using whole steps at the ends as in Example 2.

8.11 0.0796, 0.0798
8.12 0.03987, 0.03989
8.13 0.9598, 0.9596
8.14 0.9546, 0.9546
8.15 0.03520, 0.03521
8.16 0.4176, 0.4177
8.17 0.0770, 0.0782
8.18 0.372, 0.376
8.19 0.0946, 0.0967
8.20 0.462, 0.455
8.25 C: 38.3%, B and D: 24.2%, A and F: 6.7%
In $\mu + \frac{1}{2}\sigma$ and $\mu + \frac{3}{2}\sigma$, change $\frac{1}{2}$ to 0.5244, and $\frac{3}{2}$ to 1.2816.

9.3 Number of particles: 0 1 2 3 4 5
Number of intervals: 406 812 812 541 271 108

9.4 $P_0 = 0.018$, $P_1 = 0.073$, $P_4 = 0.195$

9.5 $P_0 = 0.37$, $P_1 = 0.37$, $P_2 = 0.18$, $P_3 = 0.06$

9.6 Exactly 5: 64 days. Fewer than 5: 161 days. Exactly 10: 7 days. More than 10: 5 days. Just 1: 12 days. None at all: 2 or 3 days

9.7 0.238 9.8 3, 10, 3

9.9 $P_2 = 0.022$, $P_6 = P_7 = 0.149$, $P_{n>10} = 0.099$

9.11 Normal: 0.08, Poisson: 0.0729, (binomial: 0.0732)

10.8 $\bar{x} = 5$, $\bar{y} = 1$, $s_x = 0.122$, $s_y = 0.029$,
$\sigma_x = 0.131$, $\sigma_y = 0.030$, $\sigma_{mx} = 0.046$, $\sigma_{my} = 0.0095$,
$r_x = 0.031$, $r_y = 0.0064$,
$\overline{x+y} = 6$ with $r = 0.03$, $\overline{xy} = 5$ with $r = 0.04$,
$\overline{x^3 \sin y} = 105$ with $r = 2.00$, $\overline{\ln x} = 1.61$ with $r = 0.006$

10.9 $\bar{x} = 100$ with $r = 0.47$, $\bar{y} = 20$ with $r = 0.23$,
$\overline{x-y} = 80$ with $r = 0.5$, $\overline{x/y} = 5$ with $r = 0.06$,
$\overline{x^2y^3} = 8 \cdot 10^7$ with $r = 2.9 \cdot 10^6$, $\overline{y \ln x} = 92$ with $r = 1$

10.10 $\bar{x} = 6$ with $r = 0.062$, $\bar{y} = 3$ with $r = 0.067$,
$\overline{2x-y} = 9$ with $r = 0.14$, $\overline{y^2-x} = 3$ with $r = 0.4$,
$\overline{e^y} = 20$ with $r = 1.3$, $\overline{x/y^2} = 0.67$ with $r = 0.03$

11.1 (a) 11/30 (b) 19.5 cents (c) 6/11 (d)7/11

11.2 (b) $E(x) = 5$, $\sigma = \sqrt{3}$ (c) 0.0767 (d) 0.0807 (e) 0.0724

11.3 20/47

11.4 5/8

11.6 MB: 25 FD: 10 BE: 15

11.7 $\bar{x} = 1/4$, $\sigma = \sqrt{3}/4$

11.8 (b) $\bar{x} = 4/3$, $\sigma = 2/3$ (c) 1/5

11.9 (a)

x:	0	1	2
p:	$55/72 = 0.764$	$16/72 = 0.222$	$1/72 = 0.139$

(b) $17/72 = 0.236$
(c) $6/17 = 0.353$
(d) $\bar{x} = 1/4$, $\sigma = \sqrt{31}/12 = 0.463$

11.10 (a) 0.7979, 0.7979 (b) 0.9123, 0.9123

11.11 (a) 0.0347, 0.0352 (b) 0.559, 0.562

11.12 (a) 0.00534, 0.00540 (b) 0.503, 0.500

11.13 30, 60 11.14 1

11.15 binomial: 0.2241, normal: 0.195, Poisson: 0.2240

11.16 (a) binomial: 0.0439, normal: 0.0457, Poisson: 0.0446
(b) binomial: 0.0946, normal: 0.0967, Poisson: 0.0846

11.17 $\bar{x} = 2$ with $r = 0.073$, $\bar{y} = 1$ with $r = 0.039$, $\overline{x-y} = 1$ with $r = 0.08$,
$\overline{xy} = 2$ with $r = 0.11$, $\overline{x/y^3} = 2$ with $r = 0.25$

11.18 $\bar{x} = 5$ with $r = 0.134$, $\bar{y} = 60$ with $r = 0.335$, $\overline{x+y} = 65$ with $r = 0.36$,
$\overline{y/x} = 12$ with $r = 0.33$, $\overline{x^2} = 25$ with $r = 1.3$