Birzeit University
Department of Physics
Mathematical Physics, Phys330
Fall 2020
Final-Exam

1. The potential at the surface of a sphere (radius R) is given by $V_0 = kcos^3(\theta)$, where k is a constant. Find the potential inside and outside the sphere.

2. Find the Laplacian for the following coordinate system, then do separation of variables.

$$\begin{aligned} x &= uvcos\phi \\ y &= uvsin\phi \\ z &= \frac{1}{2}(u^2 - v^2) \end{aligned}$$

3. Given

$$f(x) = \begin{cases} x & \text{if } 0 \leq \text{x} \leq 1 \\ 2 - x & \text{if } 1 \leq \text{x} \leq 2 \\ 0 & \text{if } 2 \leq \text{x} \end{cases}$$

Find the cosine transformation and evaluate the following integral

$$\int_0^\infty \frac{cos^2\alpha sin^2\alpha/2}{\alpha^2} d\alpha$$

4. Given

$$f(x) = \begin{cases} \delta(x - a/2) & \text{if } 0 \leq \text{x} \leq \text{a} \\ 0 & \text{otherwise} \end{cases}$$

Write the function as a linear combination of the complete set $\phi_n = \sqrt{\frac{2}{a}} sin(\frac{n\pi x}{a})$

5. Prove the following vector identity:

$$\nabla \cdot (\nabla\phi \times \nabla\psi) = 0$$

Both ϕ and ψ are scalar functions

6. Find out if the following functions are analytical or not, also find if they are harmonic or not.
   (a) $f(z) = \frac{iz}{|z|^2}$
   (b) $f(z) = ln(z)$

7. Use Cauchy's theorem to evaluate the following integral:

$$\oint_C \frac{e^{3z}}{(z - ln2)^4} dz$$

where C is a square with vertices $\pm 1, \pm i$

8. Given the following generating function of $D_n(x)$

$$\sum_{n=0}^{\infty} D_n(x) t^n = \frac{1 - tx}{1 - 2tx + t^2}$$

   (a) Find $D_n(x)$ for n =0 and 1
   (b) Prove the following recurrence relation

$$D_{n+1}(x) = 2xD_n(x) - D_{n-1}(x)$$

   (c) Are these polynomials orthogonal. Justify your answer