$\boxed{8}$ $\sum_{n=0}^{\infty} D_n(x)\, t^n = \dfrac{1-tx}{1-2tx+t^2}$

(a) $D_0(x) + t\,D_1(x) + \sum_{n=2}^{\infty} D_n(x)\, t^n = (1-tx)\cdot \dfrac{1}{1+(t^2-2tx)}$

as power series: $= (1-tx)\left(1-(t^2-2tx)+(t^2-2tx)^2+\cdots\right)$

$= (1-tx)\left(1 - t^2 + 2tx + t^4 - 4t^3x + 4t^2x^2 + \cdots\right)$

$= 1 - tx + 2tx + \cdots$ → we need the terms 1, t

$= 1 + t(x) + \cdots$ corresponding to $D_0(x) + t\,D_1(x)$

Hence,

$\boxed{D_0(x) = 1}$, $\boxed{D_1(x) = x}$

(b) $\dfrac{\partial}{\partial t}\left\{\dfrac{1-tx}{1-2tx+t^2}\right\} = \dfrac{-2t+x+t^2x}{(1-2tx+t^2)^2} = \sum_{n=0}^{\infty} n\, D_n(x)\, t^{n-1}$

$\sum_{n=0}^{\infty} n\, D_n(x)\, t^{n-1}(1-2tx+t^2) = \dfrac{-2t+x+t^2x}{1-2tx+t^2}$

$= \dfrac{-x}{\cancel{1+t^2-2tx}} + (2x-2t)\sum_{n=0}^{\infty} D_n(x)\, t^n$

$n D_n t^{n-1} - 2x\, n D_n t^n + n D_n t^{n+1} = \cancel{(2x-2t)}$
$+ 2x\, D_n t^n - 2 D_n t^{n+1}$

$n D_n t^{n-1} + (2+n) D_n t^{n+1} = 2x\,(n+1) D_n t^n$

$(n+1) D_{n+1} t^n + (n+1) D_{n-1} t^n = 2x(n+1)\, D_n t^n$

Divide by $t^n(n+1)$

$$\boxed{D_{n+1} + D_{n-1} = 2x\, D_n}$$

(C) yes they are orthogonal

$\int \Phi_m(x)\, \Phi_n(x)\, dx = 0$

$\int_{-1}^{1}\left(\dfrac{1-tx}{1-2tx+t^2}\right)^2 dx \ldots$

$\boxed{1}$ $V_0 = k\cos^3\theta$

from Ex 1 and Ex 2 in the book

$$V(r,\theta) = \sum_{n=0}^{\infty}\left[A_n r^n P_n(\cos\theta) + \frac{B_n}{r^{n+1}} P_n(\cos\theta)\right]$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x) \Rightarrow P_3(\cos\theta) = \frac{1}{2}(5\cos^3\theta - 3\cos\theta)$$

$$P_1(x) = x \Rightarrow P_1(\cos\theta) = \cos\theta$$

$$V_0 = \frac{2k}{5}\left[\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta + \frac{3}{2}\cos\theta\right]$$

$$= \frac{2k}{5}\left[P_3(\cos\theta) + \frac{3}{2}P_1(\cos\theta)\right]$$

at the surface $(r = R)$ $(V_{in} = V_0)$

$$V_{in} = \sum_{n=0}^{\infty} A_n r^n P_n(\cos\theta) = \frac{2k}{5}P_3(\cos\theta) + \frac{3k}{5}P_1(\cos\theta)$$

at $r = R \Rightarrow V_{in} = V_0$

$$RA_1 = \frac{3k}{5} \Rightarrow \boxed{A_1 = \frac{3k}{5R}}$$

$$R^3 A_3 = \frac{2k}{5} \Rightarrow \boxed{A_3 = \frac{2k}{5R^3}}$$

$$\boxed{A_n = 0, \quad n \neq 1, n \neq 3}$$

$$\Rightarrow \boxed{V_{in} = \frac{3k}{5R} r P_1(\cos\theta) + \frac{2k}{5R^3} r^3 P_3(\cos\theta)}$$

$$V_{out} = \sum_{n=0}^{\infty} \frac{B_n}{r^{n+1}} P_n(\cos\theta)$$

at $r = R$ (at the surface $\Rightarrow V_{out} = V_0$)

$$\sum_{n=0}^{\infty} \frac{B_n}{r^{n+1}} P_n(\cos\theta) = \frac{2k}{5}P_3(\cos\theta) + \frac{3k}{5}P_1(\cos\theta)$$

$$\frac{B_3}{R^4} = \frac{2k}{5} \Rightarrow \boxed{B_3 = \frac{2R^4 k}{5}}$$

$$\frac{B_1}{R^2} = \frac{3k}{5} \Rightarrow \boxed{B_1 = \frac{3R^2 k}{5}}$$

$$\boxed{B_n = 0, \quad n \neq 1, n \neq 3}$$

$$\Rightarrow \boxed{V_{out} = \frac{2R^4 k}{5r^4} P_3(\cos\theta) + \frac{3R^2 k}{5r^2} P_1(\cos\theta)}$$

$\boxed{4}$ $f(x) = \begin{cases} \delta(x - \frac{a}{2}), & 0 \le x \le a \\ 0, & \text{otherwise} \end{cases}$

$$f(x) = C_0 + \sum_{n=1}^{\infty} \sqrt{\frac{2}{a}}\, C_n \sin\left(\frac{n\pi x}{a}\right)$$

Let a Fourier Series of $f(x)$ defined on $[-a, a]$

$$\int_{-a}^{a} f(x)\,dx = C_0 \int_{-a}^{a} dx + \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{a}} \int_{-a}^{a} \sin\left(\frac{n\pi x}{a}\right) dx$$

$$= 2C_0 a + \sum C_n \sqrt{\frac{2}{a}} \Bigg(\underbrace{-\cos\left(\frac{n\pi x}{a}\right) \frac{a}{n\pi} \Bigg|_{-a}^{a}}_{(-1)^n - (-1)^n = 0} \Bigg)$$

$$\Rightarrow C_0 = \frac{1}{2a} \int_{-a}^{a} f(x)\,dx$$

$$= \frac{1}{2a} \int_{-a}^{a} \delta(x - \tfrac{a}{2})\,dx = \frac{1}{2a} \int_{-\infty}^{\infty} \delta(x - \tfrac{a}{2})\,dx = \frac{1}{2a}$$

(I used: $\int_{x_1}^{x_2} F(x)\,\delta(x - \frac{a}{2})\,dx = F(\frac{a}{2})$ if $\frac{a}{2} \in (x_1, x_2)$)

from HomeWork 5

$$\Rightarrow C_0 = \frac{1}{2a}$$

$$\int_{-a}^{a} f(x) \sin\left(\frac{m\pi x}{a}\right) dx = C_0 \underbrace{\int_{-a}^{a} \cancel{\sin\left(\frac{m\pi x}{a}\right) dx}}_{(-1)^n - (-1)^n = 0} + \sum_{n=1}^{\infty} \sqrt{\frac{2}{a}}\, C_n \int_{-a}^{a} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi x}{a}\right) dx$$

Now, we know that: (from Partial Course)

$$\int_{-a}^{a} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi x}{a}\right) dx = \begin{cases} 0, & m \ne n \text{ or } m = n = 0 \\ a, & m = n \ne 0 \end{cases}$$

$$\Rightarrow \int_{-a}^{a} f(x) \sin\left(\frac{n\pi x}{a}\right) dx = \sqrt{\frac{2}{a}}\, a\, C_n \qquad (n = m \ne 0)$$

$$C_n = \frac{1}{\sqrt{2a}} \int_{-a}^{a} f(x) \sin\left(\frac{n\pi x}{a}\right) dx = \frac{1}{\sqrt{2a}} \int_{-a}^{a} \delta(x - \tfrac{a}{2}) \sin\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{1}{\sqrt{2a}} \sin\left(\frac{n\pi}{2}\right)$$

$$C_n = \frac{1}{\sqrt{2a}} \begin{cases} 0, & n \text{ is even} \\ 1, & n = 1, 5, 9, \dots \\ -1, & n = 3, 7, 11, \dots \end{cases}$$

$$\Rightarrow f(x) = \frac{1}{2a} + \sum_{n=1}^{\infty} C_n \left(\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \right) = \frac{1}{2a} + \sum_{n=1}^{\infty} C_n \phi_n$$

where C_n is above.

$\boxed{3}$ $f(x) = \begin{cases} x & , \quad 0 \le x \le 1 \\ 2-x & , \quad 1 \le x \le 2 \\ 0 & , \quad x \ge 2 \end{cases}$

- Cosine transformation:

$$g_c(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos \alpha x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 x \cos\alpha x \, dx + \int_1^2 (2-x) \cos\alpha x \, dx + \text{zero} \right]$$

Finding: $\int_0^1 x \cos \alpha x \, dx = I_1$

$u = x \qquad dv = \cos\alpha x \, dx$

$du = dx \qquad v = \frac{\sin\alpha x}{\alpha}$

$$I_1 = uv\Big|_0^1 - \int_0^1 v \, du = \frac{\sin\alpha}{\alpha} + \frac{\cos\alpha}{\alpha^2} \frac{-1}{\alpha^2}$$

$$= \frac{-1 + \cos\alpha + \alpha \sin\alpha}{\alpha^2}.$$

$$I_2 = \int_1^2 (2-x) \cos\alpha x \, dx$$

$u = 2-x \qquad dv = \cos\alpha x \, dx$

$du = -dx \qquad v = \frac{\sin\alpha x}{\alpha}$

$$I_2 = uv\Big|_1^2 - \int_1^2 v \, du = \frac{\cos\alpha}{\alpha^2} - \frac{\cos 2\alpha}{\alpha^2} - \frac{\sin\alpha}{\alpha}$$

$$= \frac{\cos\alpha - \cos 2\alpha - \alpha\sin\alpha}{\alpha^2}.$$

$$I_1 + I_2 = \frac{2\cos\alpha - \cos(2\alpha) - 1}{\alpha^2} = \frac{2\cos\alpha - \cos^2\alpha + \sin^2\alpha - \cos^2\alpha - \sin^2\alpha}{\alpha^2}$$

$$= \frac{2\cos\alpha - 2\cos^2\alpha}{\alpha^2} = \frac{2\cos\alpha(1 - \cos\alpha)}{\alpha^2}$$

$$= \frac{4\sin^2\left(\frac{\alpha}{2}\right)\cos\alpha}{\alpha^2}$$

$$\Rightarrow \boxed{g_c(\alpha) = \sqrt{\frac{2}{\pi}} * \frac{4\sin^2(\alpha/2)\cos\alpha}{\alpha^2}}$$

- Finding $\int_0^\infty \frac{\cos^2\alpha \sin^2(\alpha/2)}{\alpha^2} d\alpha$

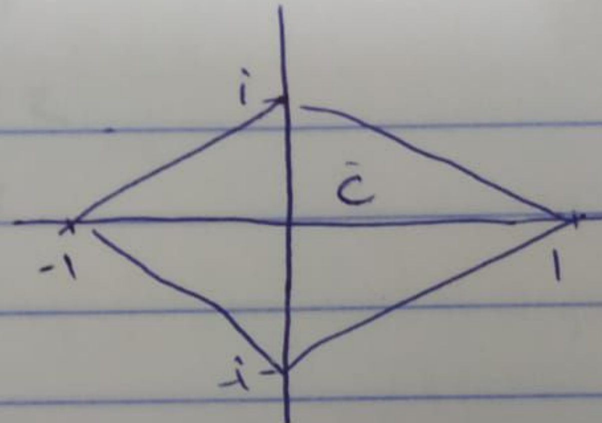
$$F(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty g_c(x) \cos\alpha x \, d\alpha$$

$$= \frac{8}{\pi} \int_0^\infty \frac{\cos\alpha \sin^2(\alpha/2)}{\alpha^2} \cos\alpha x \, d\alpha$$

put $\boxed{x = 1}$:

$$\frac{\pi}{8} f(1) = \boxed{\int_0^\infty \frac{\cos^2\alpha \sin^2(\alpha/2)}{\alpha^2} d\alpha = \frac{\pi}{8}}$$

$\boxed{7}$ $\displaystyle\oint_C \frac{e^{3z}}{(z-\ln 2)^4}\,dz$,



$$f^{(n)}(a) = \frac{n!}{2\pi i}\oint_C \frac{f(z)\,dz}{(z-a)^{n+1}}$$

$$\oint_C \frac{e^{3z}\,dz}{(z-\ln 2)^4} = \frac{2\pi i}{3!}\left[e^{3z}\right]^{(3)}\Big|_{\text{at } z = \ln 2}$$

here $n = 3$, $f(z) = e^{3z}$, $a = \ln 2$

$(e^{3z})^{(1)} = 3e^{3z}$

$\Rightarrow (e^{3z})^{(3)} = 27\, e^{3z}$

$$\Rightarrow \oint_C \frac{e^{3z}\,dz}{(z-\ln 2)^4} = \frac{2\pi i}{3!}\left(27\, e^{\overset{8}{3\ln 2}}\right) = \frac{2\times 27\times 8}{6}\pi i = 72\pi i$$

$\boxed{8}$ $\displaystyle\sum_{n=0}^{\infty} P_n(x)\, t^n = \frac{1 - tx}{1-2tx+t^2}$

$\boxed{5}$ Prove : $\nabla \cdot (\vec{\nabla} \phi \times \vec{\nabla} \Psi) = 0$

let $\vec{\nabla} \phi = \phi_x \hat{i} + \phi_y \hat{j} + \phi_z \hat{k}$

$\vec{\nabla} \Psi = \Psi_x \hat{i} + \Psi_y \hat{j} + \Psi_z \hat{k}$

where $\phi_x = \frac{\partial \phi}{\partial x}$ and so on...

$$\vec{\nabla} \phi \times \vec{\nabla} \Psi = (\phi_y \Psi_z - \phi_z \Psi_y)\hat{i} + (\phi_z \Psi_x - \phi_x \Psi_z)\hat{j} + (\phi_x \Psi_y - \phi_y \Psi_x)\hat{k}$$

$$\vec{\nabla} \cdot (\vec{\nabla} \phi \times \vec{\nabla} \Psi) = \frac{\partial (\quad)}{\partial x} + \frac{\partial (\quad)}{\partial y} + \frac{\partial (\quad)}{\partial z}$$

$$\Rightarrow = \cancel{\phi_y \Psi_{zx}} + \cancel{\phi_{yx} \Psi_z} - \cancel{\phi_z \Psi_{yx}} - \cancel{\phi_{zx} \Psi_y} + \cancel{\phi_z \Psi_{xy}} + \cancel{\phi_{zy} \Psi_x}$$
$$- \cancel{\phi_x \Psi_{zy}} - \cancel{\phi_{xy} \Psi_z} + \cancel{\phi_x \Psi_{yz}} + \cancel{\phi_{xz} \Psi_y} - \cancel{\phi_y \Psi_{xz}} - \cancel{\phi_{yz} \Psi_x}$$
$$= \text{zero}$$

بشرط $\rightarrow$ ($\Psi_{zx} = \Psi_{xz}$, $\phi_{xy} = \phi_{yx}$, ... and so on)

continuoues functions.

All terms are cancelled.

$\Rightarrow \nabla \cdot (\vec{\nabla} \phi \times \vec{\nabla} \Psi) = \text{zero}$ ✓

2 continue:

... And since u and v are independent;

we get:

$$\boxed{\begin{array}{l} \dfrac{u^2 U''}{U} + \dfrac{u U'}{U} = \lambda \quad -(2) \\ \dfrac{v^2 V''}{V} + \dfrac{v V'}{V} = \lambda \quad -(3) \end{array}}$$

2) $x = uv\cos\phi$ $\qquad u = q_1$

$y = uv\sin\phi$ $\qquad v = q_2$

$z = \frac{1}{2}(u^2 - v^2)$ $\qquad \phi = q_3$

$$\nabla^2 = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2}\frac{\partial}{\partial v}\right) + \frac{\partial}{\partial \phi}\left(\frac{h_1 h_2}{h_3}\frac{\partial}{\partial \phi}\right)\right]$$

- Finding the h_i's : $\quad h_i^2 = \frac{\partial \vec{r}}{\partial q_i}\cdot\frac{\partial \vec{r}}{\partial q_i}$ (where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$)

- $h_1^2 = h_u^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$

$= v^2\cos^2\phi + v^2\sin^2\phi + u^2 = v^2 + u^2$

$\Rightarrow h_1 = h_u = (v^2 + u^2)^{1/2}$

- $h_2^2 = h_v^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$

$= u^2\cos^2\phi + u^2\sin^2\phi + v^2 = v^2 + u^2$

$\Rightarrow h_2 = h_v = (v^2 + u^2)^{1/2}$

- $h_3^2 = h_\phi^2 = \left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \phi}\right)^2 + \left(\frac{\partial z}{\partial \phi}\right)^2$

$= (-uv\sin\phi)^2 + (uv\cos\phi)^2 + 0 = u^2v^2$

$\Rightarrow h_3 = h_\phi = uv$

$$\Rightarrow \boxed{\nabla^2} = \frac{1}{uv(v^2+u^2)}\left[\frac{\partial}{\partial u}\left(uv\frac{\partial}{\partial u}\right) + \frac{\partial}{\partial v}\left(uv\frac{\partial}{\partial v}\right) + \frac{\partial}{\partial \phi}\left(\frac{v^2+u^2}{uv}\frac{\partial}{\partial \phi}\right)\right]$$ The Laplacian

- Separation of variables:

Let $F(u, v, \phi) = U(u)V(v)\Phi(\phi)$

- $\frac{\partial}{\partial u}\left(uv\frac{\partial F}{\partial u}\right) = \frac{\partial}{\partial u}(uvU'V\Phi) = uvU''V\Phi + vU'V\Phi$

- $\frac{\partial}{\partial v}\left(uv\frac{\partial F}{\partial v}\right) = \frac{\partial}{\partial v}(uvUV'\Phi) = uvUV''\Phi + uUV'\Phi$

- $\frac{\partial}{\partial \phi}\left(\frac{v^2+u^2}{uv}\frac{\partial F}{\partial \phi}\right) = \frac{\partial}{\partial \phi}\left(\frac{v^2+u^2}{uv}UV\Phi'\right) = \frac{v^2+u^2}{uv}UV\Phi''$

$$\Rightarrow \nabla^2 F = \frac{1}{uv(v^2+u^2)}\left[uvU''V\Phi + vU'V\Phi + uvUV''\Phi + uUV'\Phi + \frac{v^2+u^2}{uv}UV\Phi''\right] = 0$$

Dividing by F : $\frac{1}{uv(v^2+u^2)}\left[\frac{uvU''}{U} + \frac{vU'}{U} + \frac{uvV''}{V} + \frac{uV'}{V} + \frac{v^2+u^2}{uv}\frac{\Phi''}{\Phi}\right] = 0$

$$\Rightarrow \frac{1}{u^2v^2}\left[\underbrace{\left(\frac{u^2v^2U''}{U} + \frac{uv^2U'}{U} + u^2v^2\frac{V''}{V} + \frac{vu^2V'}{V}\right) * \frac{1}{u^2+v^2}}_{\lambda} + \underbrace{\frac{\Phi''}{\Phi}}_{-\lambda}\right] = 0$$

$\Rightarrow \boxed{\frac{\Phi''}{\Phi} + \lambda = 0 \quad (1)}$

$$\frac{1}{u^2+v^2}\left[v^2\left(\frac{u^2U''}{U} + u\frac{U'}{U}\right) + u^2\left(v^2\frac{V''}{V} + v\frac{V'}{V}\right)\right] = \lambda$$

$$\Rightarrow v^2\left(u^2\frac{U''}{U} + u\frac{U'}{U}\right) + u^2\left(v^2\frac{V''}{V} + v\frac{V'}{V}\right) = \lambda v^2 + \lambda u^2$$

$\longrightarrow$ Continue next page

46. Find the Cauchy-Riemann eqs in Polar coord.:

$z = re^{i\theta}$

$f(z) = u(r,\theta) + i\, v(r,\theta)$

$$\frac{\partial f}{\partial r} = \frac{df}{dz}\frac{\partial z}{\partial r} = \frac{df}{dz} e^{i\theta} \Rightarrow \frac{df}{dz} = e^{-i\theta}\frac{\partial f}{\partial r} \quad -(1)$$

$$\frac{\partial f}{\partial \theta} = \frac{df}{dz}\frac{\partial z}{\partial \theta} = \frac{df}{dz}(ire^{i\theta}) \Rightarrow \frac{df}{dz} = \frac{-ie^{-i\theta}}{r}\frac{\partial f}{\partial \theta} \quad -(2)$$

$$\frac{\partial f}{\partial r} = \frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta}$$

Using (1) and (2), substitute $\partial f/\partial r$ and $\partial f/\partial \theta$:

$$\frac{df}{dz} = e^{-i\theta}\frac{\partial u}{\partial r} + ie^{-i\theta}\frac{\partial v}{\partial r} \quad -(1)'$$

$$\frac{df}{dz} = \frac{-ie^{-i\theta}}{r}\frac{\partial u}{\partial \theta} + \frac{\cancel{i}e^{-i\theta}}{r}\frac{\partial v}{\partial \theta} \quad -(2)'$$

$(1)' = (2)' \Rightarrow$ divide both by $e^{-i\theta} \Rightarrow$

$$\boxed{\begin{aligned} \frac{\partial u}{\partial r} &= \frac{1}{r}\frac{\partial v}{\partial \theta} \\ \frac{\partial v}{\partial r} &= \frac{-1}{r}\frac{\partial u}{\partial \theta} \end{aligned}}$$

— Cauchy-Riemann in Polar coord.

6 (a) $f(z) = \frac{iz}{|z|^2}$ ~~$= \left(\frac{1}{x^2+y^2} + x\right) + iy$~~

$$f(z) = \frac{i(x+iy)}{x^2+y^2} = \frac{-y}{x^2+y^2} + \frac{x}{x^2+y^2} i$$

$$u(x,y) = \frac{-y}{x^2+y^2}, \quad v(x,y) = \frac{x}{x^2+y^2}$$

$$\frac{\partial u}{\partial x} = \frac{2xy}{(x^2+y^2)^2}, \quad \frac{\partial v}{\partial y} = \frac{-2xy}{(x^2+y^2)^2}$$

$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y} \Rightarrow f(z)$ is not analytic

$\Rightarrow$ There are not harmonic since $f(z)$ is not analytic

(b) $f(z) = \ln(z) = \ln|z| + i(\theta + 2n\pi)$; $\tan\theta = \frac{y}{x}$

for each z, $\ln z$ has ~~an inf~~ an infinte set of values

But if $0 \leq \theta \leq 2\pi \Rightarrow \ln z$ has one value.

Suppose the interval is $0 \leq \theta \leq 2\pi$.

Method 1: Using Cauchy-Riemann in Polar coordinates:

$$\left.\begin{aligned} \frac{\partial u}{\partial r} &= \frac{1}{r}\frac{\partial v}{\partial \theta} \\ \frac{\partial v}{\partial r} &= \frac{-1}{r}\frac{\partial u}{\partial \theta} \end{aligned}\right\}$$

$f(z) = \ln|r| + i\theta$

$u(r,\theta) = \ln|r|$, $\frac{\partial u}{\partial r} = \frac{1}{r} \Rightarrow \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ ✓

$v(r,\theta) = \theta$, $\frac{\partial v}{\partial \theta} = 1$

$\frac{\partial v}{\partial r} = 0 = \frac{\partial u}{\partial \theta} \Rightarrow \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta} = 0$ ✓

$\Rightarrow f(z)$ is analytic

$\Rightarrow$ by defult: u and v are harmonic functions

$\Rightarrow \nabla^2 u = \nabla^2 v = 0$ (since $f(z)$ is analytic)