Birzeit University
Department of Physics
Mathematical Physics, Phys330
Fall 2020
Midterm-Exam

1. Calculate the sum of the following series:
   (a) (5 points) $\sum_{n=0}^{n=N-1} cos(nx)$
   (b) (5 points) $\sum_{n=0}^{n=N-1} sin(nx)$
   (c) (5 points) $\sum_{n=0}^{n=\infty} p^n cos(nx)$, where $|p| < 1$
   (d) (5 points) $\sum_{n=0}^{n=\infty} p^n cos(nx)$, where $|p| < 1$
2. (10 points) Prove the following identity:
$$(\frac{ic-1}{ic+1})^{id} = e^{-2dcot^{-1}(c)}$$
   Both c and d are real
3. (10 points) Find the solution for the following equation:
$$z^3 + (3+i)z^2 + 2z + (5+i) = 0$$
4. (15 points) The electrostatic force is a conservative force, that is the work along a closed path is zero. Which of the following electric field can represent an electrostatic electric field:
$$\vec{E} = k[xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}]$$
$$\vec{E} = k[y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}]$$
5. (15 points) For the following Matrix
$$M = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
   Find cos(M),sin(M)
6. (10 points) Prove that the eigenvalues of Hermtian matrix are real
7. (10 points) Prove the following vector identity:
$$\nabla(\vec{U} \cdot \vec{V}) = \vec{U} \times (\nabla \times \vec{V}) + (\vec{U} \cdot \nabla)\vec{V} + \vec{V} \times (\nabla \times \vec{U}) + (\vec{V} \cdot \nabla)\vec{U}$$

8. (30 points) Calculate the following integral:
   (a) $\oint \vec{F} \cdot \vec{r}$ around the circle $x^2 + y^2 + 2x = 0$ for $\vec{F} = y\hat{\imath} - x\hat{\jmath}$
   (b) $\int \int \vec{V} \cdot \hat{n} d\sigma$ over the surface of the sphere $(x-3)^2+(y-2)^2+(z-1)^2 = 9$, $\vec{V} = (3x-yz)\hat{\imath}+(z^2-y^2)\hat{\jmath}+(2yz+x^2)\hat{k})$
   (c) Find the value of $\int \vec{F} \cdot d\vec{r}$ along the circle $x^2 + y^2 = 2)$ from (-1,1) to (1,1) for $\vec{F} = (2x - 3y)\hat{\imath} + (3x - 2y)\hat{\jmath}$

9. (15 points) define the following coordinates system:

$$x = acosh(u)cos(\nu)$$
$$y = asinh(u)sin(\nu)$$
$$z = z$$

Find the curl, divergence and lapalcian in this coordinate system