$$\frac{\partial}{\partial t}=\frac{\partial}{\partial t'}+v\frac{\partial}{\partial x'}. \qquad \text{(A. 3b)}$$

The Schrödinger equation (for a free particle) obeyed by Φ is

$$i\hbar\frac{\partial\Phi}{\partial t}+\frac{\hbar^2}{2\mu}\frac{\partial^2\Phi}{\partial x^2}-U\Phi=0, \qquad \text{(A. 4)}$$

where μ is the mass of the particle and U its (constant) internal energy. The derivatives of the transformed wave function Φ', such as it is expressed in terms of the initial wave function Φ through (A. 1), are with respect to the transformed coordinates (A. 2)

$$\frac{\partial\Phi'}{\partial t'}=e^{if}\left[\frac{\partial\Phi}{\partial t}-v\frac{\partial\Phi}{\partial x}+i\left(\frac{\partial f}{\partial t}-v\frac{\partial f}{\partial x}\right)\Phi\right], \qquad \text{(A. 5a)}$$

$$\frac{\partial\Phi'}{\partial x'}=e^{if}\left(\frac{\partial\Phi}{\partial x}+i\frac{\partial f}{\partial x}\Phi\right), \qquad \text{(A. 5b)}$$

and

$$\frac{\partial^2\Phi'}{\partial x'^2}=e^{if}\left\{\frac{\partial^2\Phi}{\partial x^2}+2i\frac{\partial f}{\partial x}\frac{\partial\Phi}{\partial x}+\left[i\frac{\partial^2 f}{\partial x^2}-\left(\frac{\partial f}{\partial x}\right)^2\right]\right\}. \qquad \text{(A. 6)}$$

If we want the transformed wave function Φ' to obey the Schrödinger equation in the transformed coordinates, that is,

$$i\hbar\frac{\partial\Phi'}{\partial t'}+\frac{\hbar^2}{2\mu}\frac{\partial^2\Phi'}{\partial x'^2}-U\Phi=0 \qquad \text{(A. 7)}$$

whenever Φ obeys (A. 4), the coefficients of the independent terms in Φ and $\partial\Phi/\partial x$ coming from the substitution of (A. 1), (A. 5a), and (A. 6) into (A. 7), must vanish:

$$-i\hbar v+i\frac{\hbar^2}{\mu}\frac{\partial f}{\partial x}=0, \qquad \text{(A. 8)}$$

$$-\hbar\frac{\partial f}{\partial t}+\hbar v\frac{\partial f}{\partial x}+\frac{\hbar^2}{2\mu}\left[i\frac{\partial^2 f}{\partial x^2}-\left(\frac{\partial f}{\partial x}\right)^2\right]=0. \qquad \text{(A. 9)}$$

These equations imply the following simpler ones:

$$\frac{\partial f}{\partial x}=\frac{\mu v}{\hbar}, \qquad \text{(A. 10a)}$$

$$\frac{\partial f}{\partial t},=\frac{1}{2}\frac{\mu v_?^2}{\hbar}, \qquad \text{(A. 10b)}$$

so that the phase function is given by

$$f(x,t)=(\mu/\hbar)(vx+v^2t/2) \qquad \text{(A. 11)}$$

up to an irrelevant constant.

It is worth noting the similarity between the change in the wave function of a free quantum particle, as given by (A. 11), and the change in the action for a classical particle, which, as the Lagrangian itself, is not invariant under a Galilean transformation.[5]

*Postal address: Laboratoire de Physique Théorique et Hautes Energies, Université Paris VII, 2, place Jussieu, 75221 Paris Cedex 05, France.

[1] A. Landé, Am. J. Phys. **43**, 701 (1975). The present paper reached Professor Landé only a few days before his death in October 1975, so that he did not have time to respond. I wish to emphasize here the curious dialectical relationship of this article with Landé's: it intends to show that one of Landé's conclusions, namely his criticism of the wave–particle duality, is even more correct and deeper than he himself showed. By stressing a flaw in one of his arguments, precisely due to an uncritical use of a classical concept in a quantum context, I wish to turn a wrong technical argument into a correct epistemological one leading to the same conclusion. I feel it quite appropriate then to dedicate the present paper to the memory of Alfred Landé.

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Point charge between two parallel grounded planes

Markus Zahn

Department of Electrical Engineering, University of Florida, Gainesville, Florida 32611

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The problem of a point charge q between two parallel conducting planes has been of interest because a straightforward application of the method of images leads to an infinite set of images within each conductor of alternating signs and alternating spacings of $2a$ and $2b$ as illustrated in Fig. 1. The total charge q_c on each conductor is given by the nonconvergent series of the form

$$q_c=-q(1-1+1-1+1-1+1-1+\cdots) \qquad (1)$$

and cannot be directly evaluated.

The method of images can be used by considering the problem of a point charge between concentric spheres[1] of radius R and $R+d$ and letting R go to infinity, or the similar problem of two adjacent spheres of equal radii R with centers separated by the distance $D=2R+d$ with the point charge between and again taking the limit as the radii of the spheres become infinite but with the distance d remaining finite. In each case an infinite series of images result, but the series converges because each successive image decreases in magnitude.

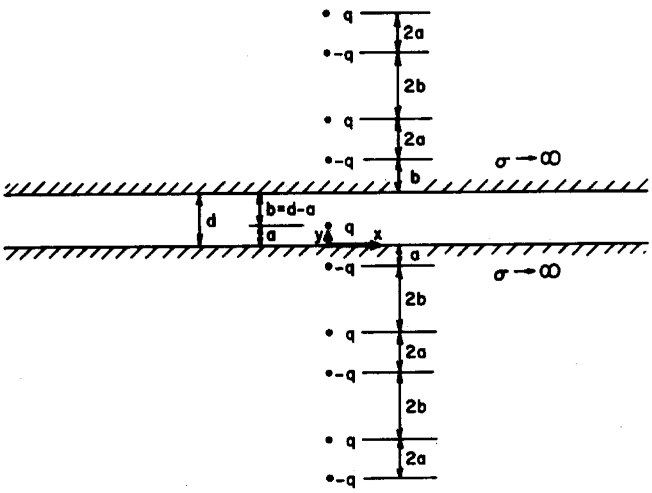

Fig. 1. A point charge q between two parallel conducting planes leads to an infinite set of image charges of alternating signs within each conductor.

A more difficult method is to solve the problem by Green's functions techniques.[2] The easiest method is to use Green's reciprocity theorem, which yields the total charge on each conductor but not the electric field or surface charge distribution.[3]

This note considers an alternative method using the method of images applied to the well-known problem of two conducting planes which intersect at an angle which is an integer submultiple of π.[4] If the angle of intersection is π/n, there will be $2n-1$ image charges of alternating signs and alternating angular spacings of 2α and 2β distributed about the circle of radius ρ_0 as shown in Fig. 2. for a point charge q a radial distance ρ_0 from the intersection of planes at angle α from the lower plane and angle $\beta = \pi/n - \alpha$ from the upper plane. We will solve this problem for any value of n and then take the limit as $n \to \infty$ to model the original parallel plane problem with the intersection of planes at $-\infty$.

The potential at any point P between the conducting

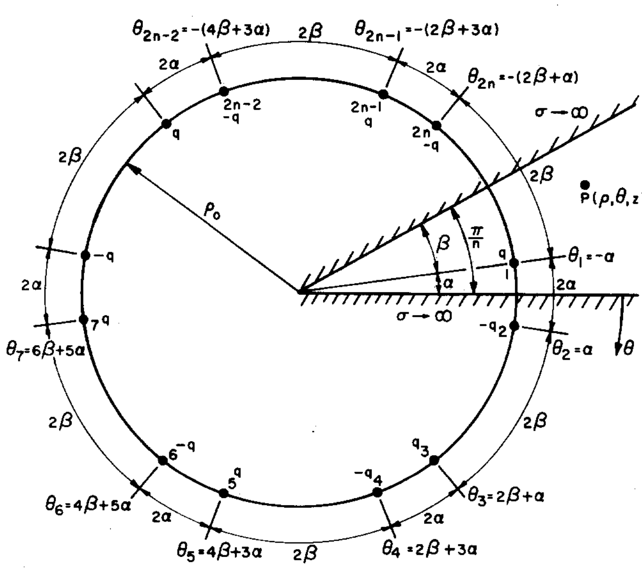

Fig. 2. A point charge q between two planes intersecting at an angle which is an integer submultiple of π gives rise to a finite number of image charges.

planes at radial distance ρ from the intersection of planes, angular distance θ from the lower plane, and an axial distance z from the point charge is given by

$$V = \frac{-q}{4\pi\epsilon_0} \sum_{m=1}^{2n} \frac{(-1)^m}{s_m}, \tag{2}$$

where s_m is the distance from each charge to the point P,

$$s_m = [\rho^2 + \rho_0^2 + z^2 - 2\rho\rho_0 \cos(\theta - \theta_m)]^{1/2}. \tag{3}$$

The angles θ_m are the angles as measured from the lower plane to each of the $2n$ charges:

$$\theta_m = \begin{cases} (m-2)\beta + (m-1)\alpha, & m \text{ even}, \\ (m-1)\beta + (m-2)\alpha, & m \text{ odd}. \end{cases} \tag{4}$$

The electric field is then

$$\bar{E} = -\nabla V = \frac{-q}{4\pi\epsilon_0} \sum_{m=1}^{2n} \frac{(-1)^m}{s_m{}^3} \{[\rho - \rho_0 \cos(\theta - \theta_m)]\bar{i}_\rho + \rho_0 \sin(\theta - \theta_m)\bar{i}_\theta + z\bar{i}_z\}. \tag{5}$$

The surface charge on each electrode is given by

$$\begin{aligned} \sigma_f(\theta = 0) &= -\epsilon_0 E_\theta(\theta = 0), \\ \sigma_f(\theta = -\pi/n) &= \epsilon_0 E_\theta(\theta = -\pi/n). \end{aligned} \tag{6}$$

The total charge on each conductor is then found by integrating (6) over the whole plane. For the lower electrode

$$\begin{aligned} q_c(\theta = 0) &= \int_{\rho=0}^{\infty} \int_{z=-\infty}^{+\infty} \sigma_f(\theta = 0)\, d\rho\, dz \\ &= \frac{-q\rho_0}{4\pi} \int_{\rho=0}^{\infty} d\rho \int_{z=-\infty}^{+\infty} \left(\sum_{m=1}^{2n} (-1)^m \sin\theta_m\, dz \right) (\rho^2 + \rho_0^2 + z^2 - 2\rho\rho_0 \cos\theta_m)^{-3/2} \\ &= \frac{-q}{2\pi} \sum_{m=1}^{2n} (-1)^m \left(\tan^{-1} \frac{\rho}{\rho_0 \sin\theta_m} \Big|_{\rho=\infty} + \tan^{-1} \cot\theta_m \right). \end{aligned} \tag{7}$$

Care must be taken in the valuation of the inverse tangent terms because the principal value of the angle must be used which only extends over the range $-\pi/2 < \theta < \pi/2$. The inverse tangent terms are equal to

$$\tan^{-1} \frac{\rho}{\rho_0 \sin\theta_m}\Big|_{\rho=\infty} = \begin{cases} \pi/2, & 0 < \theta_m < \pi \\ -\pi/2, & -\pi < \theta_m < 0 \end{cases} \tag{8}$$

and

$$\tan^{-1} \cot\theta_m = \begin{cases} \pi/2 - \theta_m, & 0 < \theta_m < \pi \\ -(\pi/2 + \theta_m), & -\pi < \theta_m < 0 \end{cases} \tag{9}$$

so that the solution of (7) can be reduced to

$$q_c(\theta = 0) = \begin{cases} \dfrac{q}{2\pi} \displaystyle\sum_{m=1}^{2n} (-1)^m \theta_m, & n \text{ even} \\ \dfrac{-q}{2\pi} \left(2\pi - \displaystyle\sum_{m=1}^{2n} (-1)^m \theta_m \right), & n \text{ odd} \end{cases} \tag{10}$$

which for both cases, n even or odd, adds up to

$$q_c(\theta = 0) = -q \left(1 - \frac{\alpha}{\pi/n} \right) = -\frac{q\beta n}{\pi}. \tag{11}$$

Since the total charge on the planes must equal $-q$,

$$\begin{aligned} q_c(\theta = -\pi/n) &= -q - q_c(\theta = 0) \\ &= -q\alpha n/\pi. \end{aligned} \tag{12}$$

These results are true for any integer value of n. To apply them to the original parallel plane problem, we let ρ_0 and n go to infinity, so that

$$\lim_{\substack{n\to\infty \\ \rho_0\to\infty}} \begin{cases} a = \rho_0\alpha, \\ b = \rho_0\beta, \\ d = \rho_0\pi/n. \end{cases} \tag{13}$$

So that (11) and (12) become the familiar results

$$q_c(x=0) = -qb/d, \quad q_c(x=d) = -qa/d. \tag{14}$$

The results of (11) and (12) happen also to be correct for any opening angle, not just those which are integer submultiples of π. This can be easily shown using Green's reciprocity theorem.

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Treatment of nonspecular reflection in the single-particle model of an ideal gas

Arthur M. Lesk
Department of Chemistry, Fairleigh Dickinson University, Teaneck, New Jersey 07666

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It is a common practice in elementary courses to introduce the kinetic theory of gases by means of a simple model consisting of a single particle moving in an enclosed volume. In deriving the equation of state of an ideal gas from this model, it is convenient to take the enclosure to have spherical shape. Then, under the usual assumption of elastic and specular reflection upon collision with the wall, the time between collisions in a spherical enclosure is constant. It has been pointed out that this simplifies the computation of the force on the wall and the pressure of the "gas"[1,2]:

Force = rate of change of momentum
= momentum change per collision
× number of collisions per unit time.

Following Ref. 1, a particle of mass m moving at velocity v in a spherical enclosure of radius R, on a trajectory making an angle θ with the normal to the wall at the point of collision, will exert a force on the container of

$$F = (2mv\cos\theta)\, v/2R\cos\theta = mv^2/R.$$

The pressure will be

$$P = \frac{F}{A} = \left(\frac{mv^2}{R}\right)(4\pi R^2) = \frac{(2/3)\,(mv^2/2)}{4\pi R^3/3},$$

leading to the familiar result

$$PV = 2/3 \cdot \text{kinetic energy}.$$

However, a more realistic picture of the interaction between particle and wall involves the absorption of the particle followed by remission some time later at an angle uncorrelated with the angle of incidence.[3-6] But, if the angle between the trajectory of the particle and the normal to the wall is not constant, then the number of collisions per unit time and the momentum transfer per collision are also not constant. The derivation of the equation $PV = 2/3$ (kinetic energy) must therefore be reexamined.

Now it has been known for a long time that the second law of thermodynamics ensures that no pressure differential can arise from different modes of interaction between particles and surfaces.[7-9] For if surfaces with two different coatings, A and B, were to feel different pressures upon exposure to some gas G, it would be possible to construct a perpetual motion machine of the second kind: Coat the two surfaces of a plane rotor blade with the materials A and B, mount the blade on a fixed shaft parallel to the plane of the blade, and surround the assembly with an atmosphere of gas G. In fact, the actual distribution of particles emerging from a surface must follow a cosine law.[9]

However, it is a fundamental defect of the single-particle model of an ideal gas that it cannot give a proper account of any property of a gas that depends on the existence of molecular chaos or upon equilibrium properties of large numbers of interacting particles.[10] The purpose of this note is to point out that the choice of an enclosure of spherical shape permits a simple kinematic demonstration, in the context of the single-particle model, of the fact that the

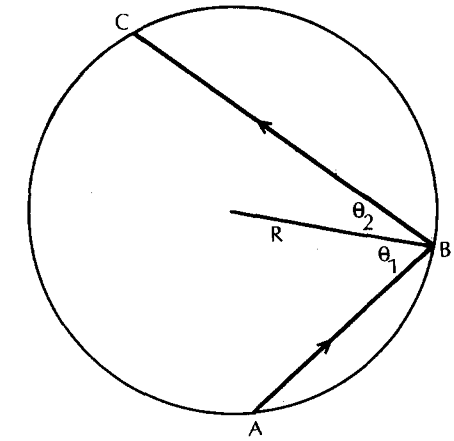

Fig. 1. Trajectory of a particle in a spherical enclosure of radius R, undergoing nonspecular reflection at point B. $|AB| = 2R\cos\theta_1$; $|BC| = 2r\cos\theta_2$.