

Phys331/Final Exam
Spring 2020/2021

- 1) Consider the electric field $\vec{E} = x\hat{x} + z\hat{y} + (f(x, y) + z^2)\hat{z}$.
- (5%) Determine the function $f(x, y)$.
 - (5%) Compute the total charge contained in a cube specified by $0 \leq x, y, z \leq 1$.
- 2) Answer the following two questions with complete reasoning and using basic equations/principles.

- (5%) If inside some region of space there was no currents, is it possible to have a magnetic field in that region of the form

$$\vec{B} = y\hat{x}$$

- (5%) If \vec{J}_0 is a constant vector, is it possible to realize a steady current of the form

$$\vec{J} = \vec{J}_0 e^{-r^2}$$

- 3) A thick spherical shell of inner radius a and outer radius b is made of dielectric material with uniform frozen-in polarization given by

$$\vec{P}(\vec{r}) = \begin{cases} P_0 \hat{z}, & a \leq r \leq b \\ 0 \text{ ..,} & \text{otherwise} \end{cases}$$

- (5%) Calculate all bound charges σ_b and ρ_b .
 - (10%) Find the electrostatic potential inside the spherical shell ($a \leq r \leq b$).
 - (5%) Find the electric field inside the spherical shell ($a \leq r \leq b$).
 - (5%) Determine the displacement vector inside the spherical shell ($a \leq r \leq b$).
- 4) An infinite conducting cylinder along the z-axis of radius R is placed in a uniform electric field $\vec{E} = E_0\hat{x}$.
- (15%) Find the electrostatic potential everywhere.
 - (10%) Find the induced surface charge density on the conducting cylinder.

- 5) If the vector potential due to a rotating uniformly charged sphere of radius R and volume charge density ρ_0 is given by

$$\vec{A}(\vec{r}) = \begin{cases} \frac{1}{2} \mu_0 \rho_0 \omega \left(\frac{rR^2}{3} - \frac{r^3}{5} \right) \sin \theta \hat{\phi}, & r \leq R \\ \frac{1}{15} \mu_0 \rho_0 \omega \frac{R^4}{r^2} \sin \theta \hat{\phi} & , \quad r > R \end{cases}$$

- (10%) Find the magnetic field inside and outside the sphere.
- (10%) Find the magnetic dipole moment.