$\rho_0 = \frac{3q}{4\pi(b^3-a^3)}$ (uniform)

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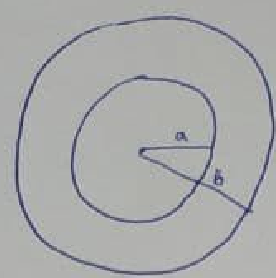
HW2
Electro



□ Let $r \leq a$:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0} q_{enc} = 0 \quad (q_{enc} = 0)$$

$\Rightarrow E = 0$

□ Let $a < r \leq b$:

we want to find q_{enc} in this region (Let volume of this region $= V_2$)

since ρ_0 is uniform:

$$q_{enl} = \rho_0 V_2 = \rho_0\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)$$

$$= \frac{4}{3}\pi(r^3-a^3)\rho_0$$

$$= \frac{\cancel{4}}{\cancel{3}}\cancel{\pi}(r^3-a^3)\frac{\cancel{3}q}{\cancel{4\pi}(b^3-a^3)}$$

$$= \frac{q(r^3-a^3)}{(b^3-a^3)}$$

$$\Rightarrow \oint \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0} q_{enc}$$

$$\int_0^{2\pi}\int_0^{\pi}(E(\vec{r})\hat{r}\cdot r^2\sin\theta\, d\theta\, d\phi\,\hat{r}) = \frac{1}{\epsilon_0} q_{enc}$$

$$\vec{E}(r)\int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\, d\theta\, d\phi = \frac{1}{\epsilon_0} q_{enc}$$

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\frac{(r^3-a^3)}{b^3-a^3}$$

□ Let $r > b$:

$q_{enc} = q$

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$$

$$E(r) = \frac{q}{4\pi\epsilon_0 r^3}$$

Results so far:

$$\vec{E}(r) = \begin{cases} \text{zero}\ \hat{r}, & r \leq a \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\frac{r^3-a^3}{b^3-a^3}\hat{r}, & a < r \leq b \\ \frac{q}{4\pi\epsilon_0 r^3}\hat{r}, & r > b \end{cases}$$

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{L} = -\int_{\infty} E(r)\hat{r} \cdot (dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi})$$

$$= \int_{-\infty}^{r} E(r)\, dr$$

□ Region 3 : $r \geq b$:

$$V_3 = -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 r'^2} dr' = \frac{q}{4\pi\epsilon_0 r}$$

□ Region 2 : $a < r \leq b$:

$$V_2 = -\int_{\infty}^{b} \frac{q}{4\pi\epsilon_0 r^2} dr - \int_{b}^{r} \frac{q}{4\pi\epsilon_0} \frac{r^3 - a^3}{r^2(b^3 - a^3)} dr$$

$$= \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 (b^3 - a^3)} \int_{b}^{r} r - \frac{a^3}{r^2}\, dr \qquad \frac{r^2}{2} + \frac{a^3}{r}$$

$$= \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 (b^3 - a^3)} \left[\frac{r^2}{2} + \frac{a^3}{r} - \frac{b^2}{2} - \frac{a^3}{b} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{b^3 - a^3} \left(\frac{r^2}{2} + \frac{a^3}{r} - \frac{b^2}{2} - \frac{a^3}{b} \right) \right]$$

□ Region 1 :

$$V_1 = -\int_{\infty}^{b} \frac{q}{4\pi\epsilon_0 r^2} dr - \int_{b}^{a} \frac{q}{4\pi\epsilon_0} \frac{r^3 - a^3}{r^2(b^3 - a^3)} dr - \int_{a}^{r} \text{zero} \cdot dr$$

$$= \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 (b^3 - a^3)} \left[\frac{a^2}{2} + \frac{a^{\cancel{3}\,2}}{\cancel{a}} - \frac{b^2}{2} - \frac{a^3}{b} \right]$$