Homework: Dielectric sphere of radius R.

$$\vec{P} = \frac{-q}{R^3}(R-z)\,\hat{z}$$

$\boxed{1}$ $\sigma_b = \vec{P}\cdot\hat{n} = \frac{-q}{R^3}(R-z)\cos\theta$

$\boxed{2}$ $\rho_b = -\nabla\cdot\vec{P} = \frac{-q}{R^3}$

$\boxed{3}$ □ E_{in} : Using Gauss Law :

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\epsilon_0} \int_0^r \rho_b (4\pi r'^2)\, dr' = \left(\frac{4\pi}{\epsilon_0}\right)\left(\frac{-q}{R^3}\right) \frac{r'^3}{3} \Big|_0^r$$

$$\Rightarrow E = \frac{-q\, r}{3\epsilon_0 R^3} \Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{-q r}{3\epsilon_0 R^3} \hat{r} = \vec{E}_{in}(\vec{r})}$$

□ E_{out} :

$$Q_{encl} = \int_S \sigma_b\, da + \int_V \rho_b\, d\tau = \mathrm{I} + \mathrm{II}$$

$$\mathrm{I} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{-q}{R^3} (R - R\cos\theta) \cos\theta (R^2 \sin\theta)\, d\theta\, d\phi$$

$$= -2\pi q \int_{\theta=0}^{\pi} (1 - \cos\theta) \cos\theta \sin\theta\, d\theta = -2\pi q \left(\frac{-2}{3}\right) = \frac{4\pi q}{3}$$

$$\mathrm{II} = \frac{-q}{R^3} \int_V d\tau = \frac{-q}{R^3} \left(\frac{4}{3}\pi R^3\right) = \frac{-4\pi q}{3}$$

$$\Rightarrow Q_{encl} = \mathrm{I} + \mathrm{II} = \frac{4\pi q}{3} - \frac{4\pi q}{3} = \text{zero}$$

$$\Rightarrow \boxed{E_{out} = 0}$$

□ $\vec{P}_{in} = \frac{-q}{R^3} (R - z)\, \hat{z}$ — $r < R$

□ $\vec{P}_{out} =$ zero — $r > R$

□ $\vec{P}_{\text{on surface}} = \frac{-q}{R^2} (1 - \cos\theta)\, \hat{z}$ — $r = R$

□ $D = \epsilon_0 \vec{E} + \vec{P}$

$\Rightarrow$ To sum up:

	Inside the sphere	outside	on the Surface
$\vec{E}$	$\frac{-qr}{3\epsilon_0 R^3}\hat{r}$	zero	$\frac{-q}{3\epsilon_0 R^2}\hat{r}$
$\vec{P}$	$\frac{-q}{R^3}(R-z)\hat{z}$	zero	$\frac{-q}{R^2}(1-\cos\theta)\hat{z}$
$\vec{D}$	$\frac{-qr}{3R^3}\hat{r} - \frac{q}{R^3}(R-z)\hat{z}$	zero	$\frac{-q}{3R^2}\hat{r} - \frac{q}{R^2}(1-\cos\theta)\hat{z}$