Electromagnetic Theory I

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جامعة بيرزيت

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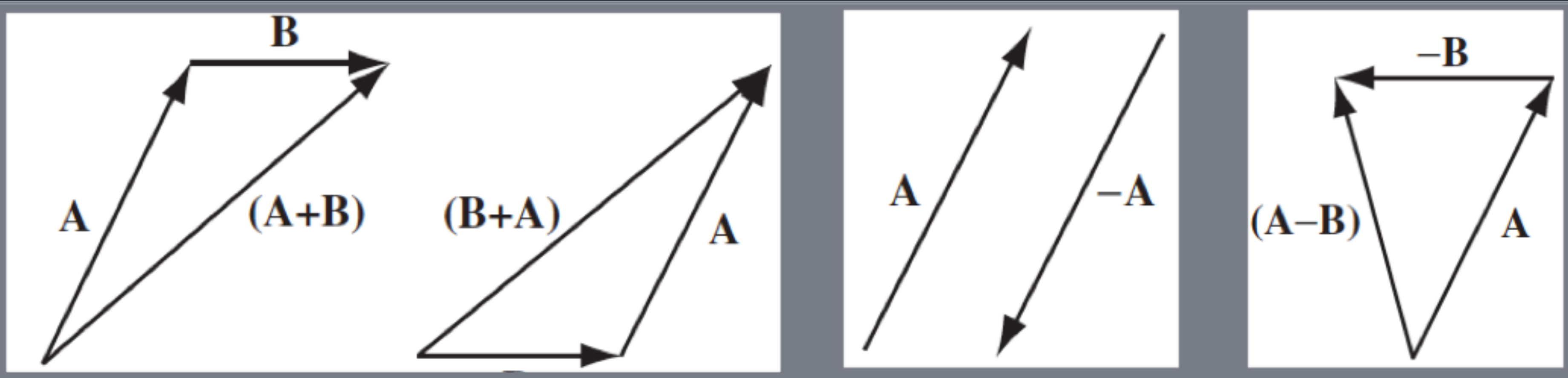
Chapter 1: Vector Analysis

- ✓ Vector Algebra
- ✓ Differential Calculus
- ✓ Integral Calculus
- ✓ Curvilinear Coordinates
- ✓ The Dirac Delta Function
- ✓ The Theory of Vector Fields

Vector Operations

(i) Addition of two vectors:

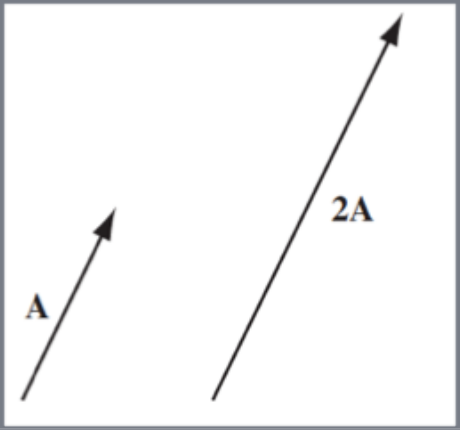
- Addition is **commutative**: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
- Addition is **associative**: $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$
- **Subtraction** is to add to its opposite: $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

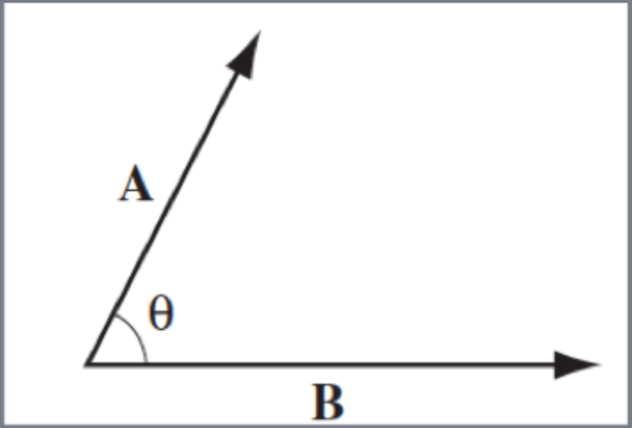


Vector Operations

(ii) Multiplication by a scalar

$$\alpha(\vec{A}+\vec{B}) = \alpha\vec{A}+\alpha\vec{B}$$





(iii) Dot product of two vectors (scalar product):

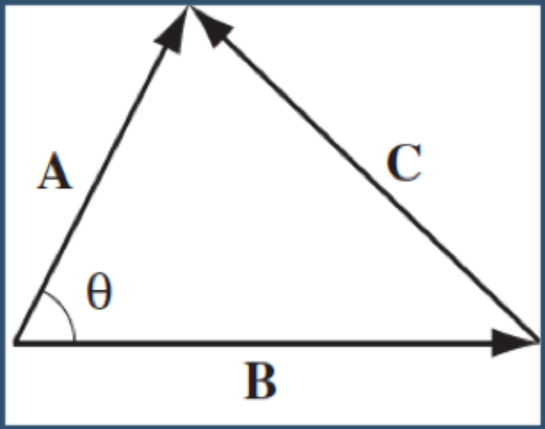
- $\vec{A}\cdot\vec{B} = AB\cos\theta$
- Dot product is commutative: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- Dot product is distributive: $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

Vector Operations

Example: Let $\vec{C} = \vec{A} - \vec{B}$, and calculate its dot product with itself

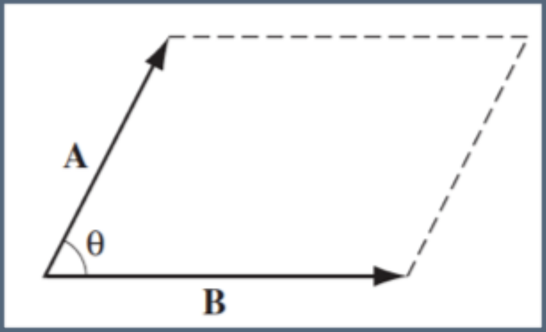
$$C^2 = |\vec{C}|^2 = \vec{C} \cdot \vec{C} = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$$
$$= \vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} - \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}$$
$$= A^2 + B^2 - 2AB \cos\theta$$

This the law of cosines



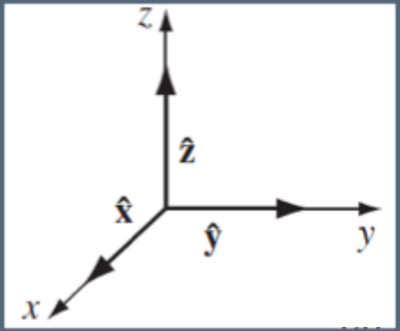
(iv) Cross product of two vectors (vector product):

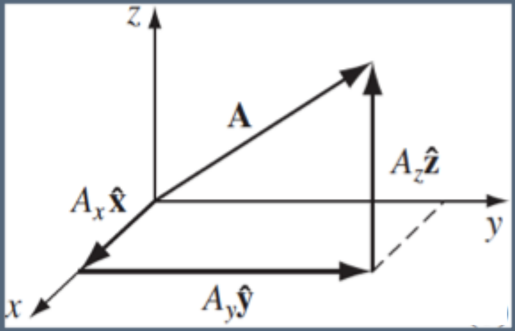
- $\vec{A} \times \vec{B} = AB \sin\theta \;\hat{n}$
- Cross product is not commutative: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- Cross product is distributive: $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$



Vector Algebra: component form

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$





Some rules

- $\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + \left(A_y + B_y\right)\hat{y} + (A_z + B_z)\hat{z}$
- $\alpha\vec{A} = \alpha A_x\hat{x} + \alpha A_y\hat{y} + \alpha A_z\hat{z}$
- $\vec{A}\cdot\vec{B} = (A_xB_x) + \left(A_yB_y\right) + (A_zB_z)$ $\quad \hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0\ ;\ \hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$
- $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left(A_yB_z - A_zB_y\right)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + \left(A_xB_y - A_yB_x\right)\hat{z}$

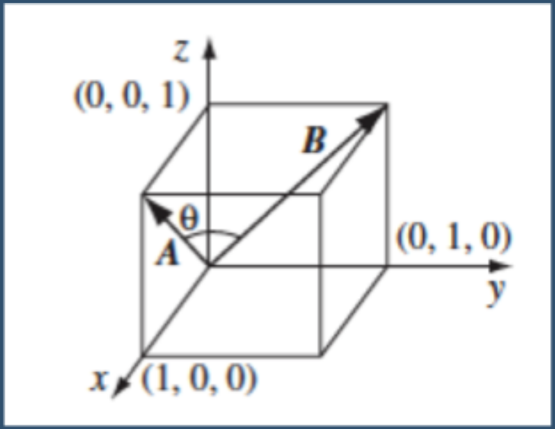
$$\hat{x}\times\hat{x} = \hat{y}\times\hat{y} = \hat{z}\times\hat{z} = 0$$

$$\hat{x}\times\hat{y} = \hat{z};\ \hat{y}\times\hat{z} = \hat{x};\ \hat{z}\times\hat{x} = \hat{y}$$

Vector Operations

Example: Find the angle between the face diagonals of a cube

$$\vec{A} = \hat{x} + \hat{z}; \vec{B} = \hat{y} + \hat{z}$$
$$\vec{A} \cdot \vec{B} = 1; A = B = \sqrt{2}$$
$$\cos\theta = \frac{1}{2} \rightarrow \theta = 60°$$

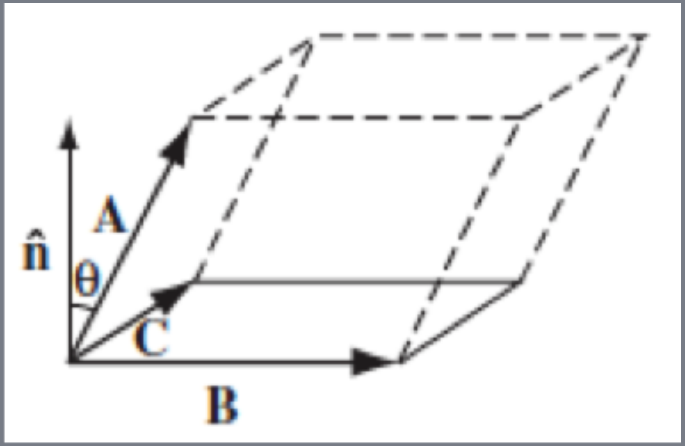


Triple products

(i) Scalar triple product:

- $\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

$$|\vec{A}\cdot(\vec{B}\times\vec{C})| = volume\ of\ the\ parallelpiped$$



Triple products

(ii) vector triple product:

- $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$ This the BAC-CAB rule

$$(\vec{A}\times\vec{B})\times\vec{C} = -\vec{C}\times(\vec{A}\times\vec{B}) = -\vec{A}(\vec{B}\cdot\vec{C}) + \vec{B}(\vec{A}\cdot\vec{C})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) + \vec{B}\times(\vec{C}\times\vec{A}) + \vec{C}\times(\vec{A}\times\vec{B}) = 0$$

Higher order products can be reduced the previous rules

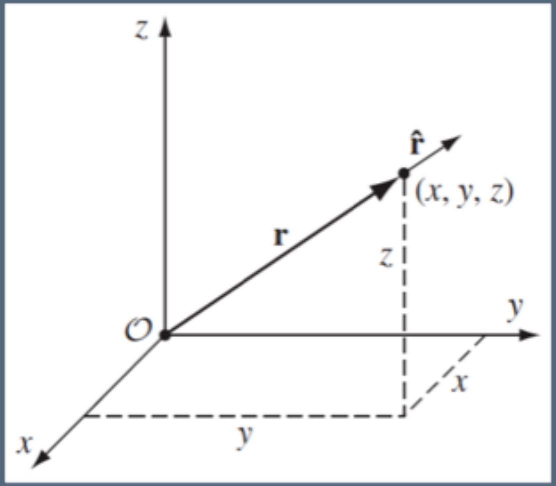
$$(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$$

$$\vec{A}\times\left(\vec{B}\times(\vec{C}\times\vec{D})\right) = \vec{B}\left(\vec{A}\cdot(\vec{C}\times\vec{D})\right) + (\vec{A}\cdot\vec{B})(\vec{C}\times\vec{D})$$

Position, Displacement and Separation vectors

(i) Position vector:

- $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z};\ \hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}$

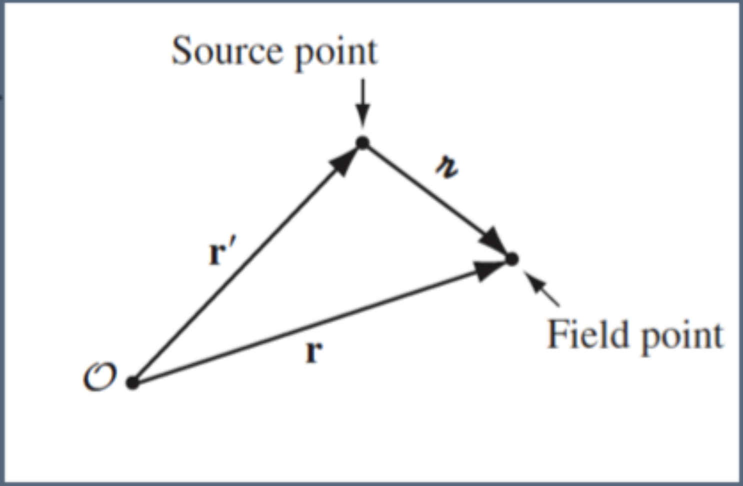


(ii) Infinitesimal Displacement vector:

- $d\vec{l} = d\vec{r} = dx\hat{x} + dy\hat{y} + dz\hat{z}$

(iii) Separation vector:

- $\vec{\mathscr{r}} = \vec{r} - \vec{r}' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$
- $\mathscr{r} = |\vec{r} - \vec{r}'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$
- $\hat{\mathscr{r}} = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} = \frac{(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$

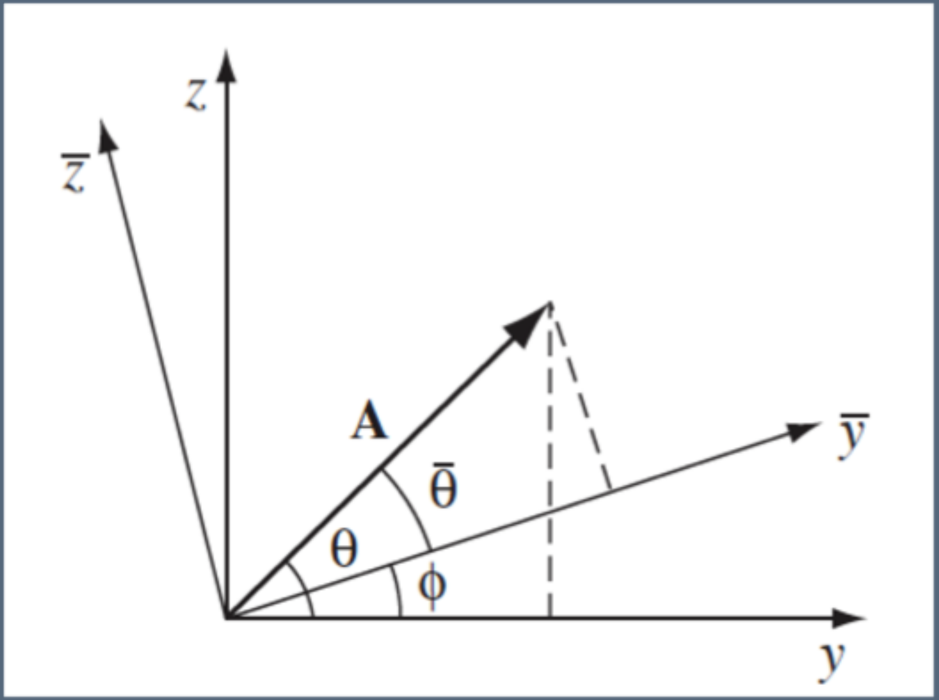


How do vectors transform?

$$A_y = A\cos\theta\,;\, A_z = A\sin\theta$$

$$\begin{aligned}\bar{A}_y &= A\cos\bar{\theta} = A\cos(\theta-\phi)\\ &= A(\cos\theta\cos\phi + \sin\theta\sin\phi)\\ &= A_y\cos\phi + A_z\sin\phi\end{aligned}$$

$$\begin{aligned}\bar{A}_z &= A\sin\bar{\theta} = A\sin(\theta-\phi)\\ &= A(\sin\theta\cos\phi - \cos\theta\sin\phi)\\ &= A_z\cos\phi - A_y\sin\phi\end{aligned}$$



$$\begin{pmatrix}\bar{A}_x\\ \bar{A}_y\\ \bar{A}_z\end{pmatrix} = \begin{pmatrix}0 & 0 & 1\\ \cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\end{pmatrix}\begin{pmatrix}A_x\\ A_y\\ A_z\end{pmatrix} \rightarrow R = \begin{pmatrix}R_{xx} & R_{xy} & R_{xz}\\ R_{yx} & R_{yy} & R_{yz}\\ R_{zx} & R_{zy} & R_{zz}\end{pmatrix} \rightarrow \bar{A}_i = \sum_{j=1}^{3} R_{ij}\,A_j$$