$$f(x,y,z) = r = \sqrt{x^2+y^2+z^2}$$

$$\vec{\nabla} r = \frac{\partial r}{\partial x}\hat{x} + \frac{\partial r}{\partial y}\hat{y} + \frac{\partial r}{\partial z}\hat{z}$$

$$\frac{\partial r}{\partial x} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}, \quad \frac{\partial r}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\vec{\nabla} r = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{x} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{y} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{z}$$

$$= \frac{1}{\sqrt{x^2+y^2+z^2}}\left[x\hat{x} + y\hat{y} + z\hat{z}\right]$$

$$= \frac{1}{r}\vec{r} = \hat{r}$$

* Evaluate the divergence of the following vector fields:

① $\vec{v} = \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\vec{\nabla}.\vec{v} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1$$

$$= 3$$

② $\vec{v} = \hat{z}$

$$\vec{\nabla}.\vec{v} = \frac{\partial(0)}{\partial x} + \frac{\partial(0)}{\partial y} + \frac{\partial(1)}{\partial z} = 0 + 0 + 0$$

$$= 0$$

③ $\vec{v} = y\hat{x} - x\hat{y}$

$$\vec{\nabla}.\vec{v} = \frac{\partial(y)}{\partial x} + \frac{\partial(-x)}{\partial y} + \frac{\partial(0)}{\partial z}$$

$$= 0 + 0 + 0 = 0$$

* Evaluate the curl of the following vector fields:

① $\vec{V} = \vec{r}$

$$\vec{\nabla} \times \vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right)\hat{x} + \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}\right)\hat{y}$$

$$+ \left(\frac{\partial x}{\partial y} - \frac{\partial y}{\partial x}\right)\hat{z} = 0$$

② $\vec{V} = x\,\hat{y}$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x & 0 \end{vmatrix}$$

$$= (\ 0\ )\hat{x} + (\ 0\ )\hat{y} + (1 - 0)\hat{z}$$

$\hat{z}$

③ $\vec{v} = -y\hat{x} + x\hat{y}$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$$= (\quad 0 \quad)\hat{x} + (\quad 0 \quad)\hat{y}$$

$$+ (1 - -1 \quad)\hat{z} = 2\hat{z}$$

Show that $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$

$$\vec{\nabla}(fg) = \frac{\partial(fg)}{\partial x}\hat{x} + \frac{\partial(fg)}{\partial y}\hat{y} + \frac{\partial(fg)}{\partial z}\hat{z}$$

$$= \left[g\frac{\partial f}{\partial x} + f\frac{\partial g}{\partial x}\right]\hat{x} + \text{---} + \text{--}$$

$$= g\left[\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}\right] + f\left[\frac{\partial g}{\partial x}\hat{x} + \dots\right]$$

$$= g\vec{\nabla}f + f\vec{\nabla}g$$

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{\nabla}(A_x B_x) + \vec{\nabla}(A_y B_y) + \vec{\nabla}(A_z B_z$$

$\vdots$

Note: $(\vec{A}\cdot\vec{\nabla})\vec{B} \neq \vec{A}\cdot \cancel{\vec{\nabla}\vec{B}}$

$$\underline{(\vec{A}\cdot\vec{\nabla})\vec{B}} = \left[A_x\frac{\partial B_x}{\partial x} + A_y\frac{\partial B_x}{\partial y} + A_z\frac{\partial B_x}{\partial z}\right]\hat{x}$$
$$+ \left[A_x\frac{\partial B_y}{\partial x} + A_y\frac{\partial B_y}{\partial y} + A_z\frac{\partial B_y}{\partial z}\right]\hat{y}$$

$$+ \left[A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z} \right] \hat{z}$$

$$\vec{\nabla}\left(\frac{f}{g}\right) = \frac{g\vec{\nabla}f - f\vec{\nabla}g}{g^2}$$

$$\vec{\nabla}\cdot\left(\frac{\vec{A}}{f}\right)$$

Show that $\vec{\nabla} \times \vec{\nabla} f = 0$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\vec{\nabla} \times \vec{\nabla} f = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$$

$$= \overbrace{\left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}\right)}^{0}\hat{x} + \overbrace{\left(\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z}\right)}^{0}\hat{y}$$

$$+ \underbrace{\left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}\right)}_{0}\hat{z} = 0$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{V}) = \vec{\nabla}\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)$$

$$= \left[\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_y}{\partial x \partial y} + \frac{\partial^2 V_z}{\partial x \partial z}\right]\hat{x}$$

$$+ \cdots \hat{y} + \cdots \hat{z}$$

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