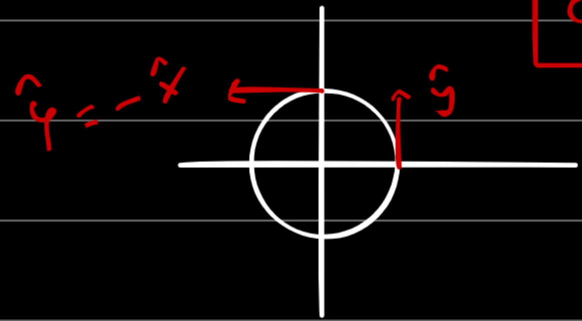
$$\boxed{\hat{\varphi} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}}$$



Example: Using spherical coordinate system, evaluate the volume and surface area of a sphere of radius R.

$$\boxed{\begin{array}{c} 0 \le r \le R \\ 0 \le \theta \le \pi \\ 0 \le \varphi \le 2\pi \end{array}}$$

$$V = \int_V d\tau = \int_0^R \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$= 2\pi \int_0^{\pi} \sin\theta \, d\theta \int_0^R r^2 dr$$

$$= 2\pi \left(-\cos\theta \Big|_0^{\pi}\right)\left(\frac{r^3}{3}\Big|_0^R\right)$$

$$= 2\pi\,(2)\left(\frac{R^3}{3}\right) = \frac{4}{3}\pi R^3$$

* Evaluate $\displaystyle\int_{-\infty}^{+\infty} \delta(x^2+1)\,dx = 0$

no roots

$$\int_{-\infty}^{+\infty} \delta(x^2-1)\,dx = \int_{-\infty} \frac{\delta(x-1)}{|f'(1)|}\,dx + \int_{-\infty} \frac{\delta(x+1)}{|f'(-1)|}\,dx$$

$f'(x) = 2x$

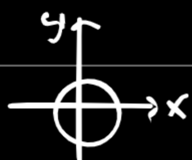
$x_1 = 1,\ x_2 = -1$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$\int_0^{\infty} \delta(x^2-1)\,dx = \frac{1}{2}$$

Example: write the charge distribution of a uniformly charged ring of radius and total charge Q in terms of δ-function. (ring is centered about the origin and lie in the xy-plane)

Cartesian,



$$\rho(x,y,z) = k_1\, \delta(z)\, \delta\left(\sqrt{x^2+y^2} - R\right)$$

$$\int \rho(x,y,z)\,dx\,dy\,dz = Q$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} k_1\, \delta(z)\, \delta\left(\sqrt{x^2+y^2} - R\right) dx\,dy\,dz$$

$$k_1 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta\left(\sqrt{x^2+y^2}-R\right) dx\, dy = Q$$

$$k_1 \int_{0}^{\infty}\int_{0}^{2\pi} \delta(\rho-R)\, \underline{\rho}\, d\rho\, d\varphi = Q$$

$$2\pi k_1 R = Q \Rightarrow k_1 = \frac{Q}{2\pi R}$$

$$\rho(x,y,z) = \frac{Q}{2\pi R}\, \delta\left(\sqrt{x^2+y^2}-R\right) \delta(z)$$