# Electromagnetic Theory I

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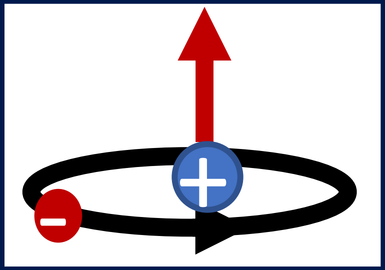
# Chapter 6: Magnetic Fields in Matter

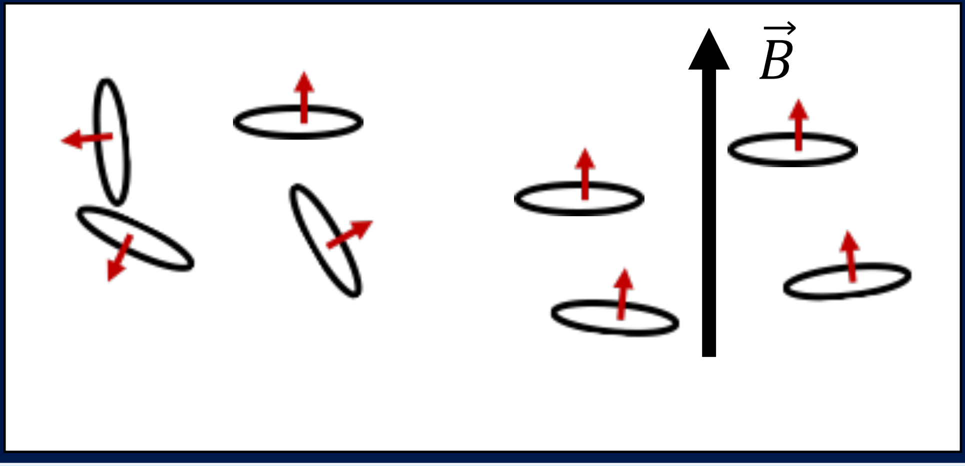
- Magnetization
- The Field of a Magnetized Object
- The Auxiliary Field $\vec{H}$
- Linear and Nonlinear Media

# 6.1 Magnetization

- All matters are composed of atoms, each with a positive charged nucleus and a number of orbiting electrons. In addition, both electrons and the nucleus of an atom have spin with corresponding magnetic dipole moments.



- In the absence of an external magnetic field, the magnetic dipoles of the atoms of most materials (excepts permanent magnets) have random orientations, resulting in no net magnetic moment. The application of an external magnetic fields causes alignment

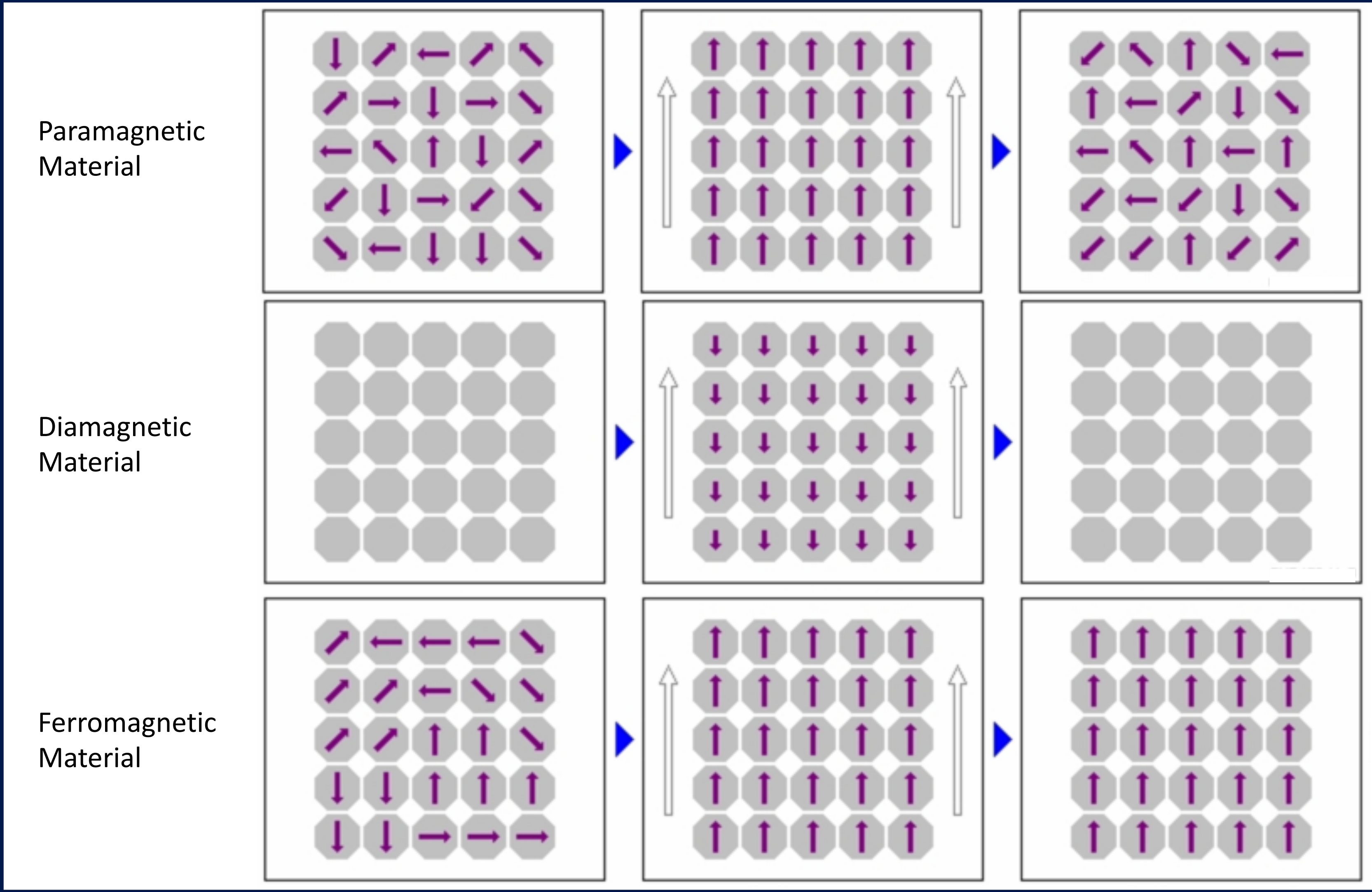


# 6.1 Magnetization

## Magnetic materials

| | |
|---|---|
| **Diamagnetic material** | • Magnetization opposite to external magnetic field<br>• Due to orbital motion of electrons<br>• e.g. Copper, Silver, Gold |
| **Paramagnetic material** | • Magnetization parallel to external magnetic field<br>• Magnetic dipole moments of the spinning electrons<br>• e.g. Aluminum, Magnesium, Titanium |
| **Ferromagnetic material** | • Retain magnetization even after external field turned off<br>• Magnetized domains (strong coupling forces between the magnetic dipole moments of the atoms)<br>• e.g. Nickel, Cobalt, Pure iron |

# 6.1 Magnetization

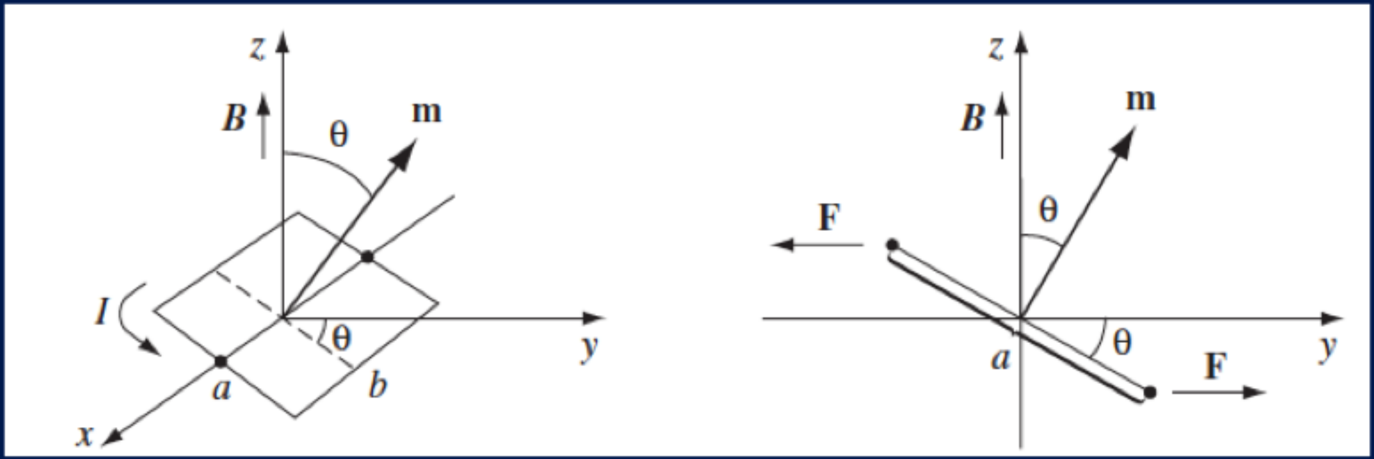


# 6.1 Magnetization

## Torques and forces on a magnetic dipole

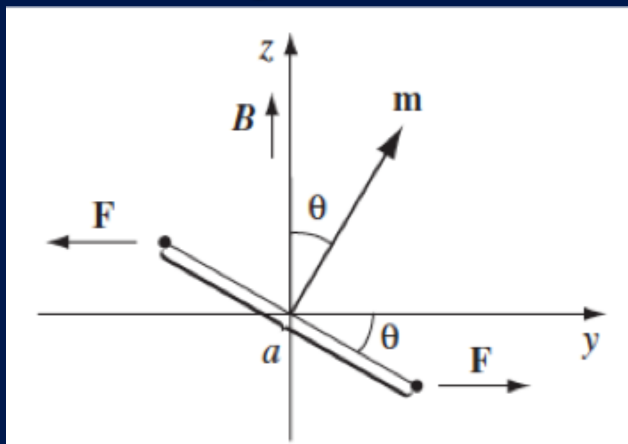
- In a uniform magnetic field the net force on a current loop is zero

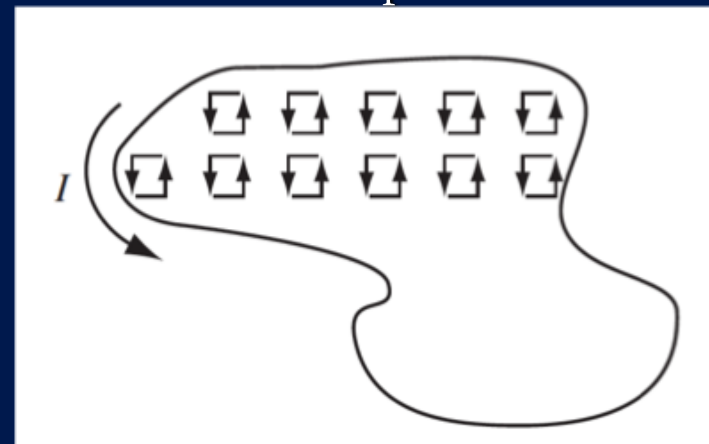
$$\vec{F} = \oint I d\vec{l} \times \vec{B} = I\left(\oint d\vec{l}\right) \times \vec{B} = 0$$



# 6.1 Magnetization

- In a uniform magnetic field the net torque on a current loop is not zero



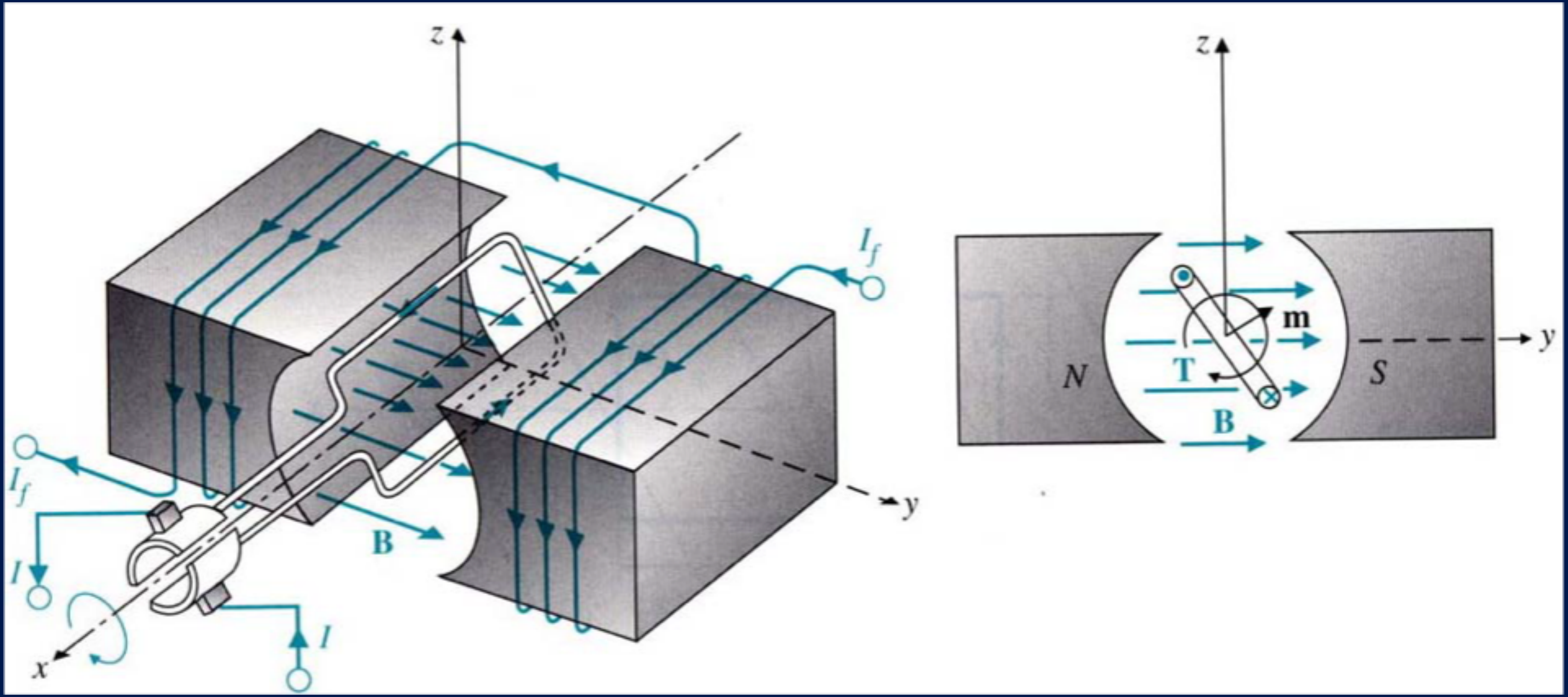

$$\vec{N}=\oint\vec{r}\times d\vec{F}=\oint\vec{r}\times(Id\vec{l}\times\vec{B})=I\left(\oint\vec{r}\times d\vec{l}\times\vec{B}\right)$$

$$=\left(I\oint(\vec{r}\cdot\vec{B})d\vec{l}-I\vec{B}\oint(\vec{r}\cdot d\vec{l})\right)=\left(-I\int\vec{\nabla}(\vec{r}\cdot\vec{B})\times d\vec{a}-I\vec{B}\oint\frac{1}{2}\vec{\nabla}(r^2)\cdot d\vec{l}\right)$$

$$=-I\int\vec{B}\times d\vec{a}=\left(I\int d\vec{a}\right)\times\vec{B}=\vec{m}\times\vec{B}$$

# 6.1 Magnetization

## Direct current motor



# 6.1 Magnetization

Rail Gun

$$B(s) = \frac{1}{2}\frac{\mu_0 I}{2\pi}\left[\frac{1}{s} + \frac{1}{d-s}\right]$$

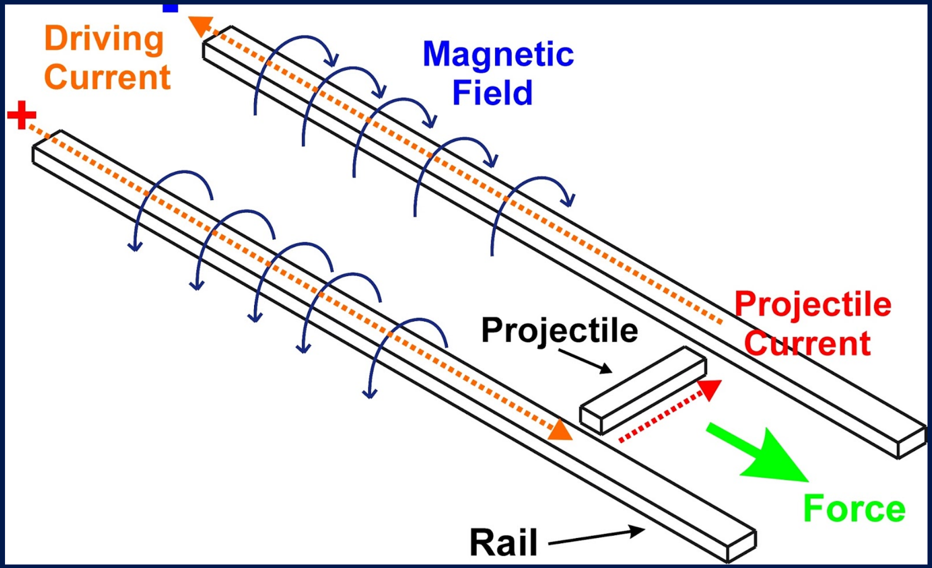
$$F = \frac{1}{2}\frac{\mu_0 I^2}{2\pi}\int_r^{d-r}\left[\frac{1}{s} + \frac{1}{d-s}\right] ds$$

$$= \frac{1}{2}\frac{\mu_0 I^2}{2\pi}\ln\left[\frac{s}{d-s}\right]_r^{d-r}$$

$$= \frac{1}{2}\frac{\mu_0 I^2}{2\pi}\ln\left[\frac{(d-r)^2}{r^2}\right]$$

$$= \frac{\mu_0 I^2}{2\pi}\ln\left[\frac{(d-r)}{r}\right] = 9.2\frac{\mu_0 I^2}{2\pi}$$

$for\ r = 0.0001d$

# 6.1 Magnetization

- In a non-uniform magnetic field the net force on a current loop is not zero

$$\vec{F} = \oint I d\vec{l} \times \vec{B} \neq 0$$

$$\vec{F} = \oint I d\vec{l} \times \vec{B} = I \oint d\vec{l}' \times [\vec{B}(0) + (\vec{r}' \cdot \vec{\nabla})\vec{B}(0) + HOT]$$

$$= I \oint d\vec{l}' \times \left[\vec{\nabla}\left(\vec{r}' \cdot \vec{B}(0)\right) - \left(\vec{r}' \times (\vec{\nabla} \times \vec{B})_{\vec{r}=0}\right) - \vec{B}(0) \times (\vec{\nabla} \times \vec{r}') - (\vec{B}(0) \cdot \vec{\nabla})\vec{r}'\right]$$

$$= I \oint d\vec{l}' \times \vec{\nabla}\left(\vec{r}' \cdot \vec{B}(0)\right) = -I\vec{\nabla} \times \oint \left(\vec{r}' \cdot \vec{B}(0)\right) d\vec{l}'$$

$$= I\vec{\nabla} \times \oint \vec{B}(0) \times (\vec{r}' \times d\vec{l}') + I\vec{\nabla} \times \oint \left(\vec{B}(0) \cdot d\vec{l}'\right)\vec{r}' = I/2 \ \vec{\nabla} \times \oint \vec{B}(0) \times (\vec{r}' \times d\vec{l}')$$

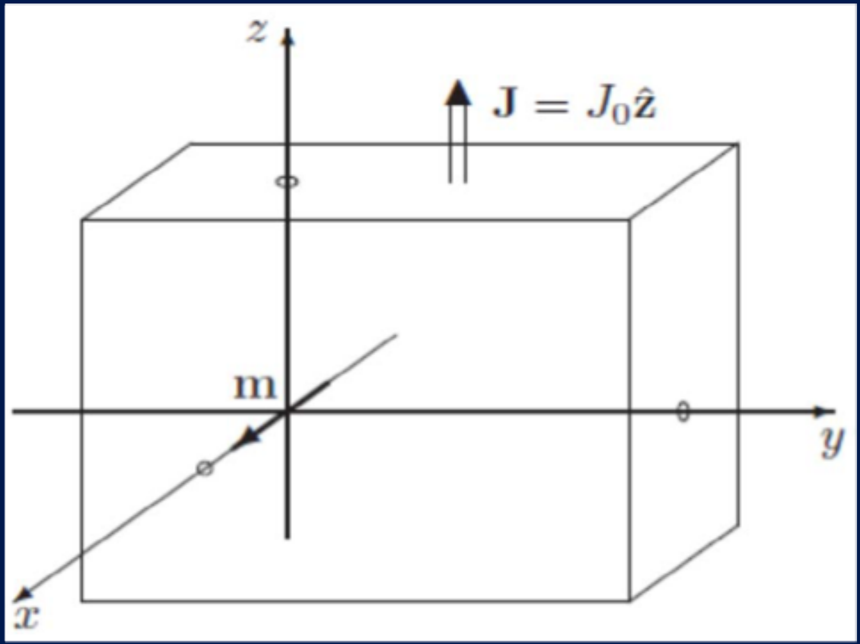
$$= \ \vec{\nabla} \times (\vec{B}(0) \times I/2 \oint (\vec{r}' \times d\vec{l}') \ = \vec{\nabla} \times (\vec{B}(0) \times \vec{m}) = \vec{\nabla}(\vec{m} \cdot \vec{B})$$

# 6.1 Magnetization

**Example:** A uniform current density $\vec{J} = J_0\hat{z}$ fills a slab straddling the yz plane, from $x = -a$ to $x = +a$. Find the force on (a) a magnetic dipole $\vec{m} = m_0\hat{x}$ is situated at the origin. (a) a magnetic dipole $\vec{m} = m_0\hat{y}$ is situated at the origin.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\vec{B} = \begin{cases} x\mu_0 J_0 \hat{y} & -a < x < a \\ a\mu_0 J_0 \hat{y} & x > a \\ -a\mu_0 J_0 \hat{y} & x < -a \end{cases}$$



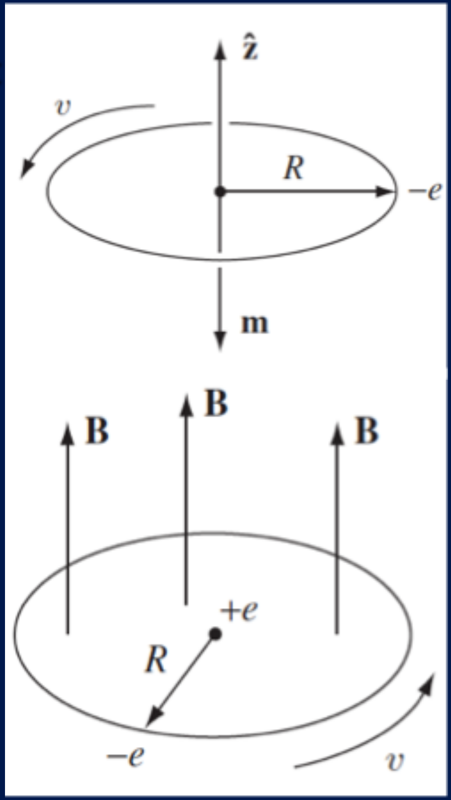

$$(a)\ \vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}) = 0$$

$$(b)\ \vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}) = \vec{\nabla}(x\mu_0 J_0 m_0) = \mu_0 J_0 m_0 \hat{x}$$

# 6.1 Magnetization

## Effects of magnetic fields on atomic orbits

- Electrons not only spin; they also revolve around the nucleus
- The orbital motion with the period $T = \frac{2\pi R}{v}$ → a steady current




$$I = \frac{e}{T} = \frac{ev}{2\pi R} \rightarrow \quad \vec{m} = -\frac{ev}{2\pi R}\pi R^2 \hat{z} = -\frac{evR}{2}\hat{z}$$

When the atom is placed in a magnetic field,

*The magnetic dipole is subject to a torque. But it is hard to tilt the entire orbit than the spin. This means the orbital contribution to paramagnetism is negligible

*A more significant effect on the orbital motion is that the electron speeds up or slows down, depending on the orientation of the external field

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = \frac{m_e v^2}{R} \rightarrow \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B = \frac{m_e \bar{v}^2}{R}$$

# 6.1 Magnetization

$$e\bar{v}B = \frac{m_e\bar{v}^2}{R} - \frac{m_e v^2}{R} \rightarrow \frac{eBR}{m_e}\bar{v} = \bar{v}^2 - v^2 = (\bar{v}+v)(\bar{v}-v) \sim 2\bar{v}\Delta v$$

$$\Delta v = \frac{eBR}{2m_e} \rightarrow electrom\ speeds\ up$$

$$\Delta\vec{m} = -\frac{1}{2}\Delta veR\hat{z} = -\frac{e^2BR^2}{4m_e}\hat{z}$$

- The change is antiparallel to the field.
- This is the mechanism for diamagnetism.
- This effect present in all atoms.

# 6.1 Magnetization

## Magnetization

Summary: In the presence of an external magnetic field, matter becomes magnetized:

- **Paramagnetism:** the dipoles associated with the spins of unpaired electrons experience a torque tending to line them up parallel to the field.
- **Diamagnetism:** the orbital speed of the electrons is altered in such a way as to change the orbital dipole moment in a direction opposite to the field.
- Atoms with odd number of atoms, paramagnetism is dominant.

*Magnetization, $\vec{M}$ is the total magnetic dipole moment per unit volume*

$$\vec{M} = \lim_{v \to 0} \frac{1}{v} \sum_{\vec{m}_i \in v} \vec{m}_i$$