Electromagnetic Theory II

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Chapter 7: Electrodynamics

- Electromotive Force
  - Ohm's Law
  - Electromotive Force
  - Motional Emf

7.1.1 Ohm's Law

Ohm's law states that the current density ($\vec{J}$) should be proportional to the force per unite charge ($\vec{f}$)

$$\vec{J} = \sigma \vec{f}$$

Conductivity: $\sigma (= \infty \; for \; a \; perfect \; conductor)$

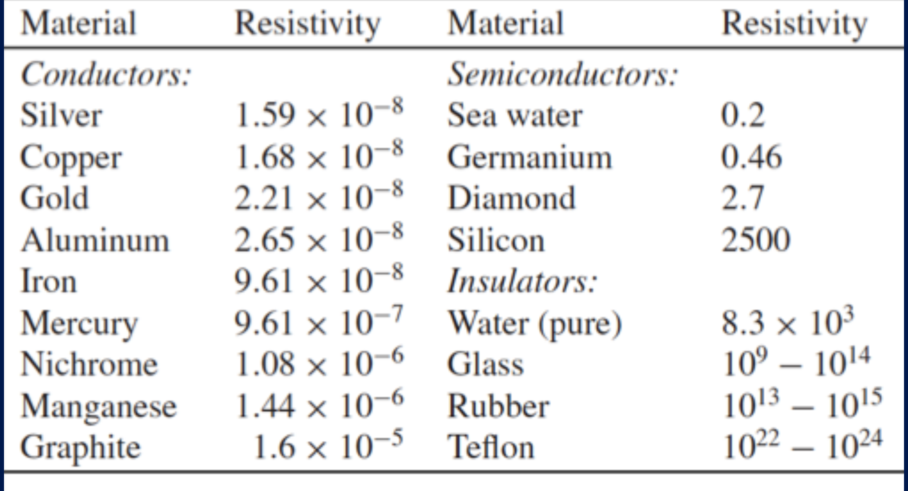
Resistivity: $\rho = 1/\sigma$

Electromotive force is then given by:

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

usually small

Ohm's Law $\vec{J} = \sigma \vec{E}$



Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Sea water	0.2
Copper	1.68×10^{-8}	Germanium	0.46
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2500
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.61×10^{-7}	Water (pure)	8.3×10^{3}
Nichrome	1.08×10^{-6}	Glass	$10^{9} - 10^{14}$
Manganese	1.44×10^{-6}	Rubber	$10^{13} - 10^{15}$
Graphite	1.6×10^{-5}	Teflon	$10^{22} - 10^{24}$

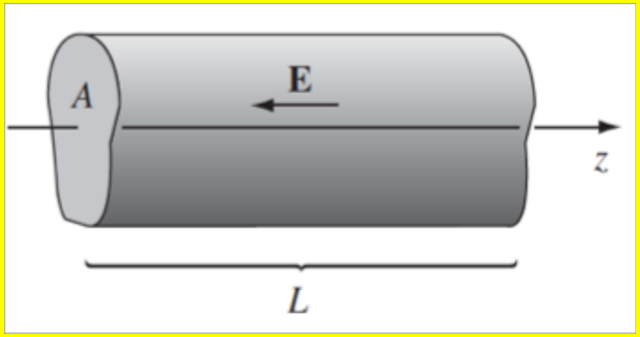
7.1.1 Ohm's Law

Example: A cylindrical resistor of cross-sectional area A and length L is made from material with conductivity σ. If the potential is constant over each end, and the potential difference between the ends is V, what current flows?

Electric Field is uniform through the wire (This can be shown by solving Laplace equation)

$$\frac{d^2V}{dz^2} = 0, V(0) = 0, V(L) = V_0 \rightarrow V(z) = \frac{V_0}{L} z$$

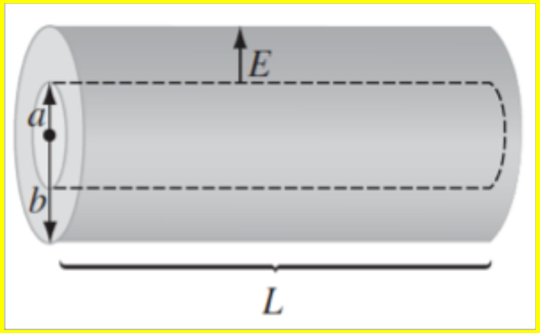
$$I = JA = \sigma EA = \frac{\sigma A}{L} V$$



7.1.1 Ohm's Law

Example: Two long cylinders (radii a and b) are separated by material of conductivity σ. If they are maintained at a potential difference V, what current flows from one to the other, in a length L

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$



$$V = -\int_b^a \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}\cdot(ds\hat{s}+..) = -\int_b^a \frac{\lambda}{2\pi\epsilon_0 s}ds = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{a}$$

$$I = \int \vec{J}\cdot d\vec{a} = \int \sigma\vec{E}\cdot d\vec{a} = \sigma\int \vec{E}\cdot d\vec{a}$$

$$I = \sigma\int \vec{E}\cdot d\vec{a} = \sigma\int_0^L\int_0^{2\pi}\left(\frac{\lambda}{2\pi\epsilon_0 s}\hat{s}\right)\cdot(sd\phi dz\hat{s}) = \frac{\sigma}{\epsilon_0}\lambda L = \frac{2\pi\sigma L}{\ln\frac{b}{a}}V$$

Ohm's Law $V = IR$

7.1.1 Ohm's Law

Now if $\vec{E}$ produces a force $q\vec{E}$ (on a charge q), then Newton's second law implies that the charge will accelerate. If the charges are accelerating, the current will increase with time as the field exists. But Ohm's law implies, on the contrary, that a constant field produces a constant current???!!!

This can be resolved by noting that the charges are already moving fast in random directions due to their thermal energy.
In the presence of a field $\vec{E}$, the average drift velocity give rise to the current density.

$$\vec{J} = nfqv_{avg} = \frac{nfq\lambda}{2\mathrm{v}_{thermal}m}F = \frac{nfq^2\lambda}{2\mathrm{v}_{thermal}m}E = \sigma E$$

f	$Number\ of\ free\ electrons\ per\ molecule$
n	$Number\ of\ molecules\ per\ unit\ volume$

The work done by the electrical force is converted into heat in the resistor due to collisions.

Joule heating law $P = IV = I^2R$

7.1.2 Electromotive Force

For a current circuit, there are two forces driving current

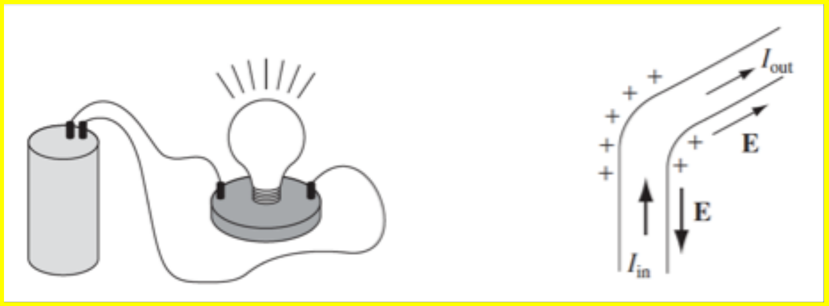
- the source, $\vec{f}_s$, which is act to one part of the circuit (a battery
- the electrostatic force, which smooth out the flow and propagate the influence of the source to other distant parts of the circuit.

$$\vec{f} = \overrightarrow{f_s} + \vec{E}$$

Now, the work done per unit charge (electromotive force emf) is given by

$$\mathcal{E} = \oint \vec{f} \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l}$$

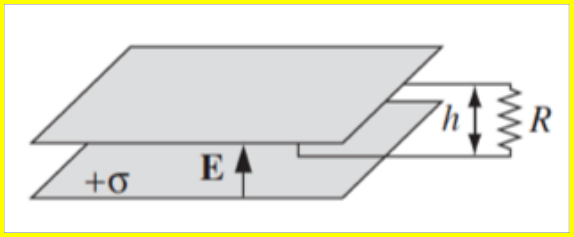
The static electric field keeps the current uniform in the circuit



7.1.2 Electromotive Force

Example: A rectangular loop of wire is situated so that one end (height h) is between the parallel-plate capacitor, oriented parallel to the field $\vec{E}$. The other end is way outside, where the field is essentially zero. What is the emf in this loop? If the total resistance is R what current flows?

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = \frac{\sigma}{\epsilon_0} h$$



if the field were $\frac{\sigma}{\epsilon_0}$ inside and zero outside.

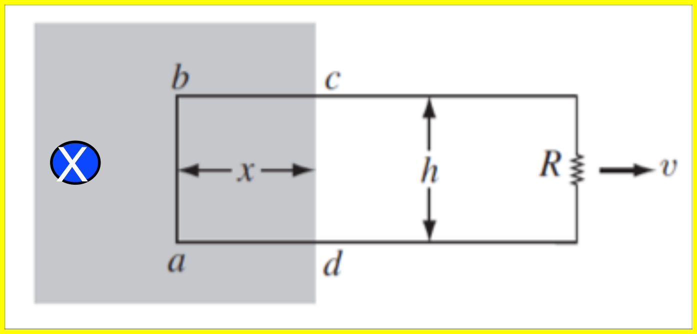
But we know that for $\vec{\nabla}\times\vec{E} = 0$ for electrostatics

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = 0$$

The fringing field is is just right to cancel off the contribution from the uniform field. We also conclude that the total resistance should be zero.

7.1.3 Motional emf

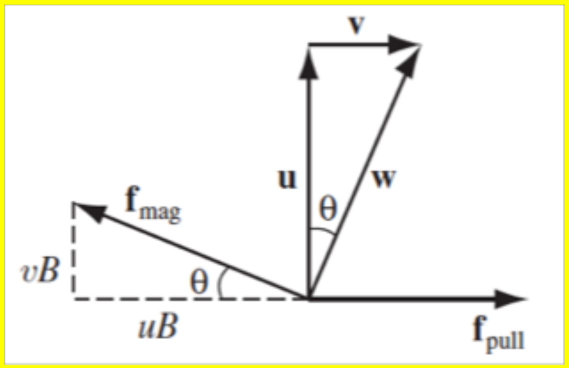
Motional emf arises when you move a wire in a magnetic field



$$\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = vBh$$

Magnetic forces never do work, But the person who is pulling on the loop is supplying the energy that heats the resistor

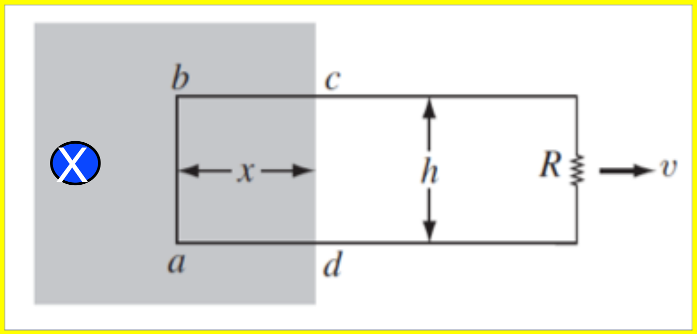
$$\mathcal{E} = \oint \vec{f}_{pull} \cdot d\vec{l} = uB \frac{h}{\cos\theta} \sin\theta = vBh$$



$$f_{bull} = uB$$

7.1.3 Motional emf

Flux rule for motional emf



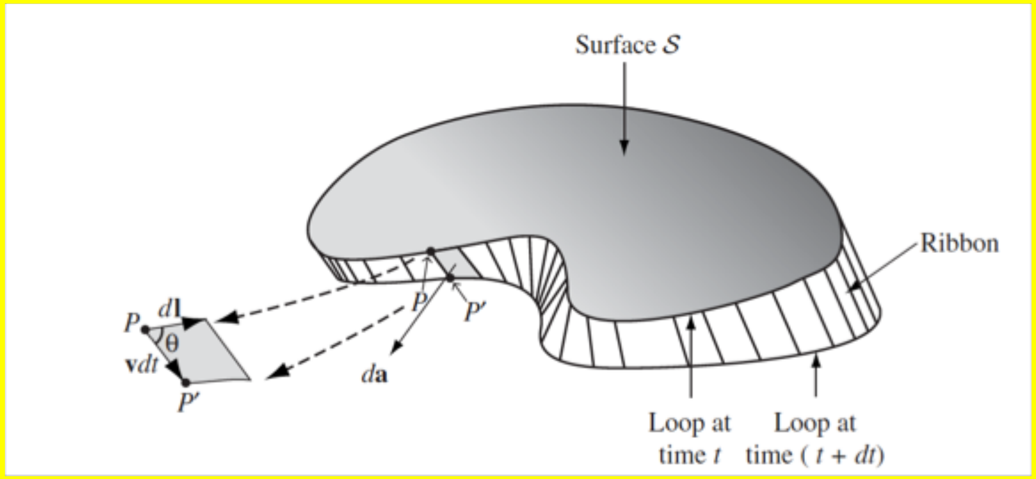
$$\Phi = \oint \vec{B} \cdot d\vec{a} = -Bxh$$

$$\frac{\mathrm{d}\Phi}{\mathrm{dt}} = -Bh\frac{dx}{dt} = -Bhv$$

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{dt}}$$

7.1.3 Motional emf

In general



$$d\Phi = \Phi(t+dt) - \Phi(t) = \Phi_{ribbon} = \oint \vec{B} \cdot d\vec{a} = \oint \vec{B} \cdot (\vec{v} \times d\vec{l} dt)$$

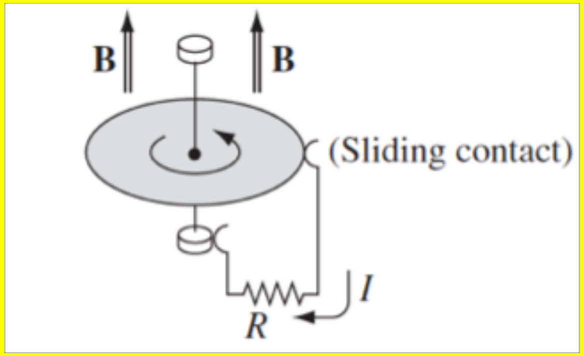
$$\frac{d\Phi}{dt} = \oint \vec{B} \cdot (\vec{v} \times d\vec{l}) = -\oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = -\oint \vec{f}_{\text{mag}} \cdot d\vec{l} = -\mathcal{E}$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

7.1.3 Motional emf

Example: A metal disk of radius a rotates with angular velocity ω about a vertical axis, through a uniform field B, pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact, which touches the outer edge of the disk. Find the current in the resistor.

$$v = s\omega \qquad \vec{f}_{mag} = \vec{v}\times\vec{B} = s\omega B\hat{s}$$



$$\mathcal{E} = \oint \vec{f}_{mag}\cdot d\vec{l} = \frac{\omega B a^2}{2}$$

$$I = \mathcal{E}/R = \frac{\omega B a^2}{2R}$$

Note: (the Faraday disk, or Faraday dynamo) involves a motional emf that you can't calculate (at least, not directly) from the flux rule. The flux rule assumes the current flows along a well-defined path, whereas in this example the current spreads out over the whole disk.

7.1.3 Motional emf

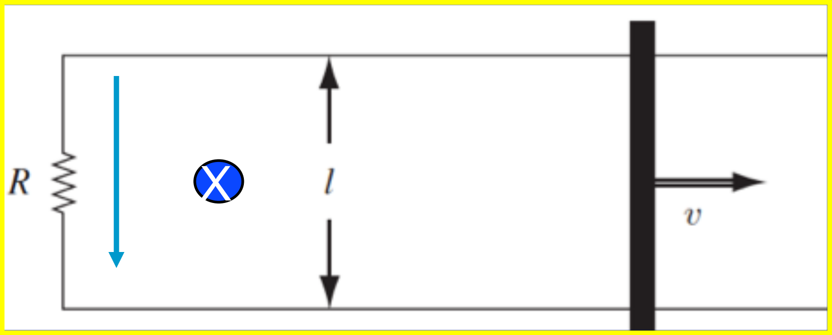
Example: A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance *l* apart. A resistor *R* is connected across the rails and a uniform magnetic field *B*, pointing into the page, fills the entire region.

$$\Phi = Blx$$

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{dt}} = Bl\frac{dx}{dt} = Blv$$

$$I = \mathcal{E}/R = \frac{Blv}{R}$$

$$F = IlB = \frac{B^2l^2Iv}{R}$$



If the bar starts out with speed v_0 at time t = 0, and is left to slide, what is its speed at a later time t?

$$F = ma = m\frac{dv}{dt} = -\frac{B^2l^2Iv}{R} \rightarrow v(t) = v_0 \exp - \frac{B^2l^2}{mR}t$$

7.1.3 Motional emf

Example: A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $B(t) = B_0 \cos \omega t$. A circular loop of wire, of radius $\frac{a}{2}$ and resistance R is paced inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

$$\Phi = \frac{\pi B a^2}{4}$$

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{dt}} = -\frac{\pi a^2}{4}\frac{dB}{dt} = \frac{\pi a^2 B_0 \omega}{4} \sin \omega t$$

$$I = \frac{\mathcal{E}}{\mathrm{R}} = \frac{\pi a^2 B_0 \omega}{4R} \sin \omega t$$