Electromagnetic Theory II

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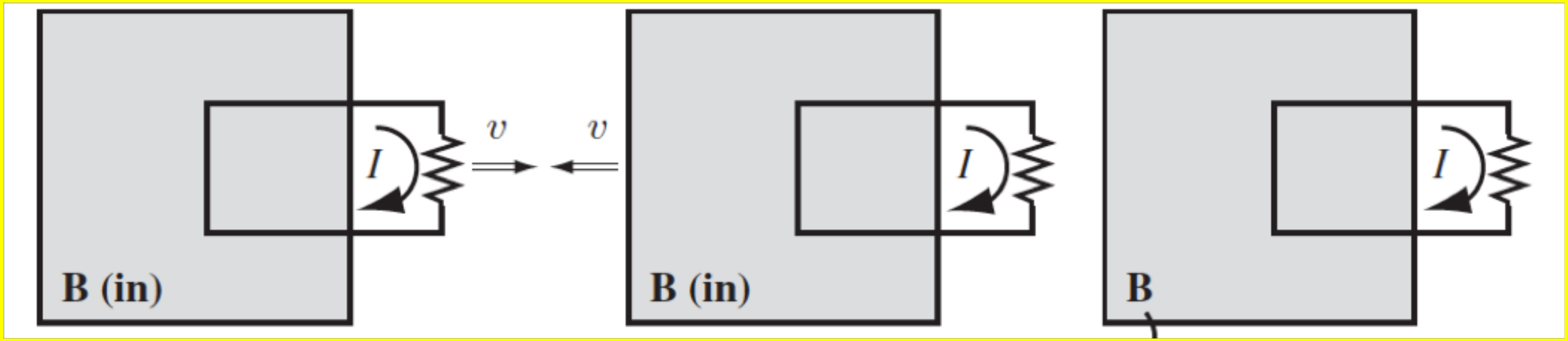
Chapter 7: Electrodynamics

- Electromagnetic Induction
  - Faraday's Law
  - The induced electric field
  - Energy in Magnetic fields

7.2.1 Faraday's Law

In 1831 Michael Faraday reported on a series of experiments, including three:



Experiment 1: He pulled a loop of wire to the right through a magnetic field. A current flowed in the loop.	**Experiment 2:** He moved the magnet to the left, holding the loop still.	**Experiment 3:** With both the loop and the magnet at rest, he changed the strength of the field.
It is just a motional emf	But if the loop is stationary, the force cannot be magnetic as stationary charges experience no magnetic forces. The force must be electric!!	

7.2.1 Faraday's Law

Michael Faraday concluded, A changing magnetic field induces an electric field

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a} \qquad \oint \vec{E} \cdot d\vec{l} = \int (\vec{\nabla}\times\vec{E}) \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

Quasistatic Approximation

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$(\vec{\nabla}\times\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$$

Characterizing electric fields induced by changing magnetic fields

The loop does not have to be a wire as induced electric field will exist even in vacuum. However, when a wire is there, the electrons respond to the emf making circular current.

The work done by the induced electric field for a charge q after one circulation:

$$W = \oint \vec{F} \cdot d\vec{l} = -q\oint \vec{E} \cdot d\vec{l} = -q\frac{d\Phi}{dt} \neq 0$$

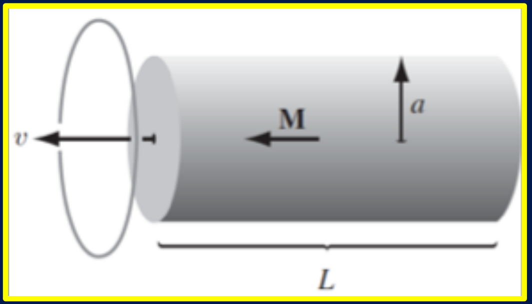
The induced electric field are not conservative and hence no potential can be defined.

7.2.1 Faraday's Law

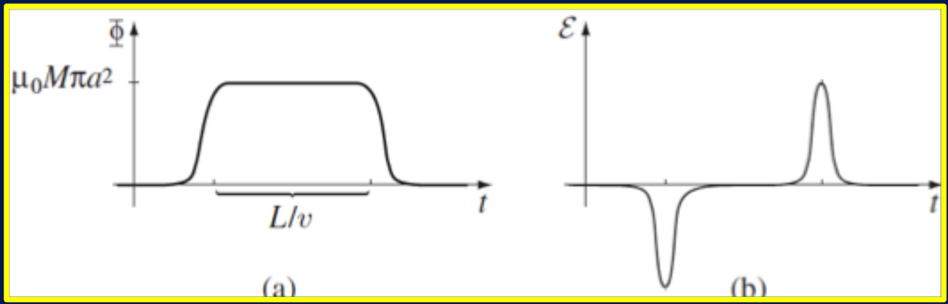
Example: A long cylindrical magnet of length L and radius a carries a uniform magnetization $\vec{M}$ parallel to its axis. It passes at constant velocity $\vec{v}$ through a circular wire ring of slightly larger diameter. Graph the emf induced in the ring, as a function of time.

$$\vec{K}_b = M\hat{\phi}$$

$$\vec{B} = \mu_0 \vec{M}$$



The flux through the ring is zero when the magnet is far away; it builds up to a maximum as the leading end passes through. It drops again to zero as the other end leaves the ring.

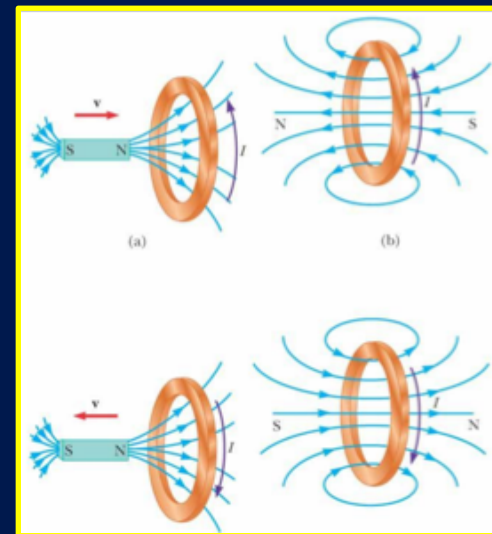


Which way around the ring does the induced current flow?

7.2.1 Lenz's Law

Getting the signs in Faraday's law can be difficult.

$$\mathcal{E} = -\frac{d\Phi}{dt}$$



Lenz's law can help in getting directions of the induced current right. It is summarized by Nature oppose a change in flux. OR the induced current will flow in such a direction that the flux it produces tends to cancel the change.

7.2.1 Faraday's Law

Example: A square loop of wire, with sides of length a, lies in the first quadrant of the xy plane, with one corner at the origin. In this region, there is a nonuniform time-dependent magnetic field $B(y,t) = ky^3t^2\,\hat{z}$ (where k is a constant). Find the emf induced in the loop.

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

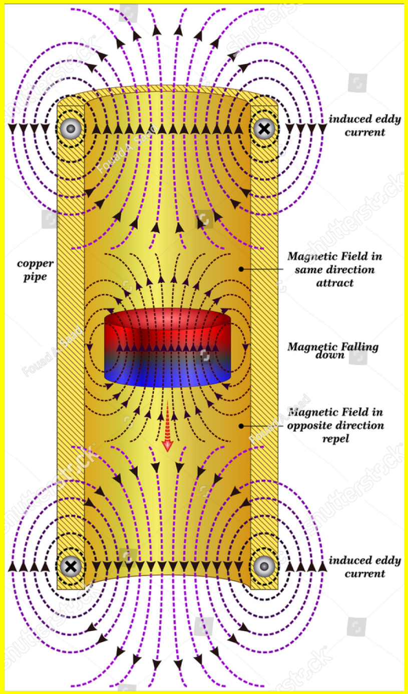
$$= -\int_0^a\int_0^a (2ky^3t\hat{z})\cdot dxdy\hat{z}$$

$$= -2kt\int_0^a\int_0^a y^3dxdy = -\frac{1}{2}kta^5$$

7.2.1 Faraday's Law

Example: A short cylindrical bar magnet is dropped down a vertical aluminum pipe of slightly larger diameter, about 2 meters long. It takes several seconds to emerge at the bottom, Whereas an otherwise identical piece of unmagnetized iron makes the trip in a fraction of a second. Explain why the magnet falls more slowly.

A ring of pipe below the magnet experiences an increasing upward flux as the magnet approaches, and hence (by Lenz's law) a current (Iind) will be induced in it such as to produce a downward flux. Thus Iind must flow clockwise, which is opposite to the current in the magnet. Since opposite currents repel, the force on the magnet is upward. Meanwhile, a ring above the magnet experiences a decreasing (upward) flux, so its induced current is parallel to I, and it attracts the magnet upward.



7.2.1 Lenz's Law

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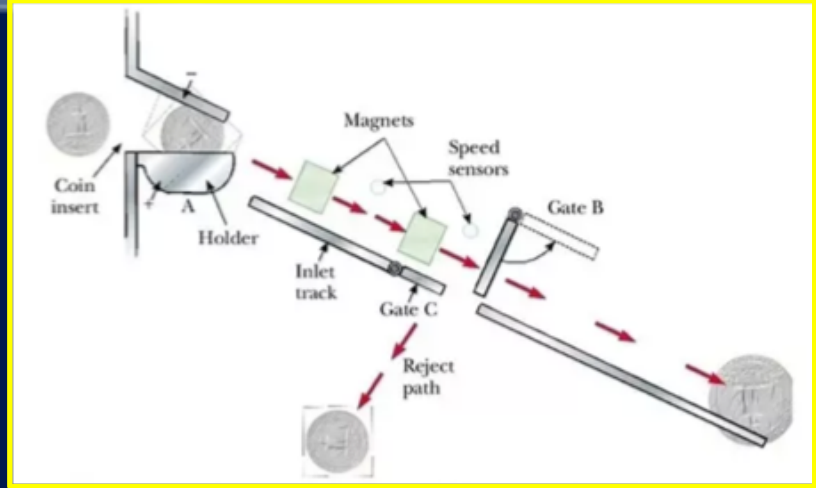
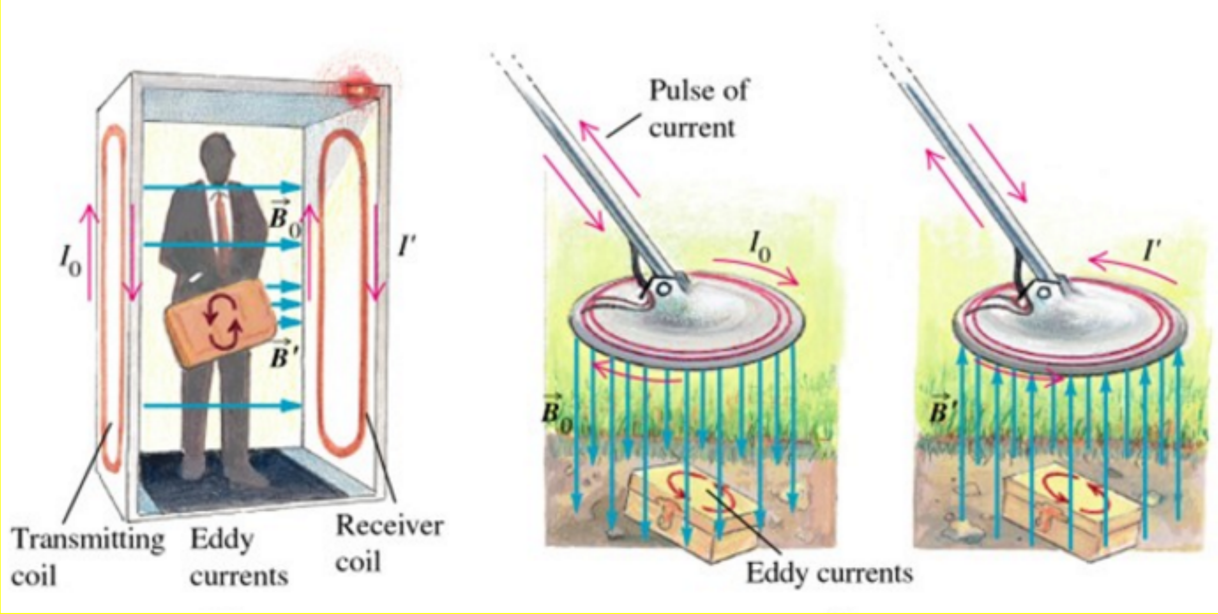
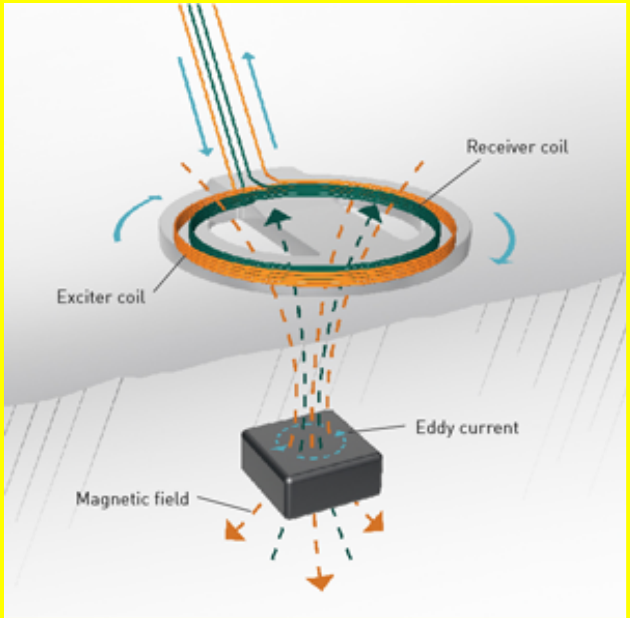
7.2.1 Eddy Currents

Eddy currents: are loops of electrical current induced within conductors induced by a changing magnetic field in the conductor . They flow in closed loops within conductors, in planes perpendicular to the magnetic field.







7.2.2 The Induced Electric Field

There are two distinct kinds of electric fields:

Static Electric Fields	**produced by static electric charges**	$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0; \vec{\nabla}\times\vec{E} = 0$
Induced Electric Fields	produced by changing magnetic fields	$\vec{\nabla}\cdot\vec{E} = 0; \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

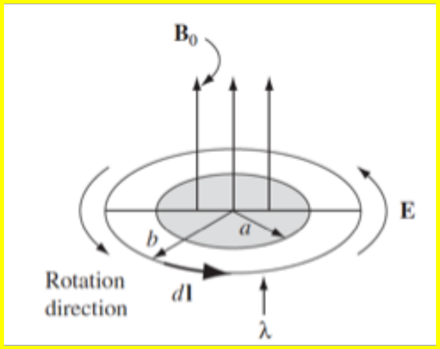
Recall, $\vec{\nabla}\cdot\vec{B} = 0; \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Faraday-induced electric fields are determined by $-\frac{\partial\vec{B}}{\partial t}$ in similar way as magnetostatics are determined by $\mu_0\vec{J}$. We can use all what we learned about Ampere's law in integral form to find the induced field by Faraday's law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \qquad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$

Example: A line charge λ is glued onto the rim of a wheel of radius b, which is then suspended horizontally so that it is free to rotate (the spokes are made of some nonconducting material—wood, maybe). In the central region, out to radius a, there is a uniform magnetic field B_0, pointing up. Now someone turns the field off. What happens?

The changing magnetic field will induce an electric field, This electric field exerts a force on the charges at the rim, and the wheel starts to tum. According to Lenz's law, it will rotate in counterclockwise direction.



$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = -\frac{dB}{dt}\pi a^2$$

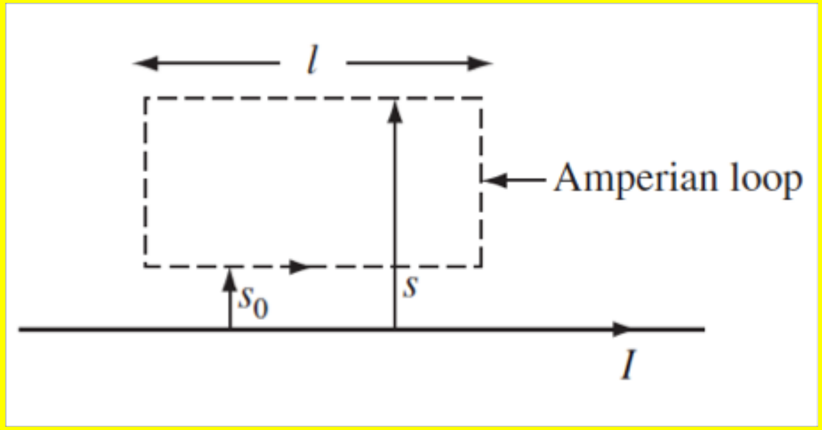
$$2\pi E = -\frac{dB}{dt}\pi a^2 \rightarrow \vec{E} = \frac{1}{2}\frac{dB}{dt}a^2\hat{\phi}$$

$$\vec{N} = \oint \vec{r}\times d\vec{F} = \oint \vec{r}\times\lambda dl\vec{E} = \oint \lambda bEdl\hat{z} = \lambda bE\left(\oint dl\right)\hat{z} = -\frac{dB}{dt}\pi a^2 b\hat{z}$$

$$\vec{L} = \int \vec{N}dt = \pi a^2 b\hat{z}\int_0^{t_0} -\frac{dB}{dt}dt = \pi a^2 b\hat{z}\int_{t_0}^{0} dB = \pi a^2 b\vec{B}_0$$

Example: An infinitely long straight wire carries a slowly varying current $I\ (t)$. Determine the induced electric field, as a function of the distance s from the wire.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a} =$$



$$E(s_0)l - E(s)l = -\frac{d}{dt} \int_0^l \int_{s_0}^{s} \frac{\mu_0 I}{2\pi s} ds dz = \frac{\mu_0 l}{2\pi} \frac{dI}{dt} (\ln s_0 - \ln s)$$

Electric field gets very large as s goes to infinity. That can't be true . What is going on?

This example is artificial, and not just in the obvious sense of involving infinite wires, but in a more subtle respect. It assumes that the current is the same (at any given instant) all the way down the line. This is a safe assumption for the short wires in typical electric circuits, but not for long wires (transmission lines), unless you supply a distributed and synchronized driving mechanism. But never mind—the problem doesn't inquire how you would produce such a current; it only asks what fields would result if you did. Variations on this problem are discussed by M. A. Heald, Am. J. Phys. 54, 1142 (1986).

Homework: An long solenoid with radius a and n turns per unit length carries a time-dependent current $I\,(t) = I_0 te^{-\alpha t}$ in the $\hat{\phi}$ direction. 1) Find the electric field (magnitude and direction) at time $t > 0$ and a distance s from the axis (both inside and outside the solenoid), in the quasistatic approximation. 2) At distance $d = 2a$, plot the magnitude of the electric field as a function of time. 3) Discuss the behavior of the field over time, noting extreme values and their occurrence times.